Some thoughts regarding Mitsui *et al* paper "100-kyr ice age cycles as a timescale matching problem"

by Mikhail Verbitsky

Introduction

The authors propose "the hypothesis that the ice-sheet climate system responds to astronomical forcing at the ∼100-kyr periodicity because the intrinsic timescale of the system is closer to 100 kyr than to other major astronomical periods". In terms of the similarity theory, this hypothesis suggests that the period of the system response P is a function of the intrinsic timescale τ_{int} and of the amplitude and period of the astronomical forcing, ε, T:

$$P = \varphi(\tau_{int}, \varepsilon, T) \qquad \text{(I)}$$

Since τ_{int} and ε are parameters with independent dimensions then according to π-theorem:

$$\frac{P}{\tau_{int}} = \Phi\left(\frac{T}{\tau_{int}}\right) \qquad \text{(II)}$$

Thus the presented paper advances a theory that the period of the system response to astronomical forcing is largely described by a similarity parameter formed by the ratio of the astronomical-forcing period to the intrinsic timescale $\frac{T}{\tau_{int}}$ ("timescale matching") and ***is independent of the amplitude of the astronomical forcing***. The numerical experiments with *phenomenological* SO and G24-3 models are mostly supportive of this theory. As it could be expected (phenomenological models are designed to produce 100-kyr periodicity), the vertically oriented strips in Fig. 4 (a, c) tell us that the period of the system response is mostly independent of forcing amplitude and is largely defined by the intrinsic timescale. At the same time - and this is what I would like to bring to the authors' attention - the results of their numerical experiments with *physical* VCV model are not consistent with the proposed theory. Even a superficial look at Fig. 4 (b) would tell us that in this case the period of the system response to astronomical forcing ***depends on the forcing amplitude***. In the following paragraphs I will try to explain why the proposed theory does not work well for ice physics.

1**. Preliminary physical considerations**

Ice-sheet physics is defined by vertical advection of ice and temperature. The timescale of this process is

$$\tau_{adv} = \frac{H}{a} \qquad (1)$$

where H is ice thickness and a is mass influx. From scaling consideration of ice motion equations

$$H = \zeta S_0^{1/4} \qquad (2)$$

where $\zeta = \left[\frac{\mu a}{(\rho g)^n}\right]^{1/(2n+2)}$, n = 3 is the power degree of Glen's rheological law, μ is ice viscosity, ρ is ice density, g acceleration of gravity, S_0 is ice area (Verbitsky, 1992, Bahr *et al*, 2015).

It can be seen that for all practical purposes ζ may be assumed to be a constant and since the area is involved in a power degree 1/4, the thickness of an ice sheet will not dramatically change with or without astronomical forcing. At the same time, the mass influx will change radically. Therefore the "hypothesis that the ice-sheet climate system responds to astronomical forcing at the ~100-kyr periodicity because the intrinsic timescale of the system is closer to 100 kyr than to other major astronomical periods" causes immediate concern because for the changed a the intrinsic timescale may be simply irrelevant.

Let us proceed with the more rigorous reasoning.

2. Intrinsic period of relaxation oscillations.

We suggest that the intrinsic period of relaxation oscillations depends on ice thickness, mass influx, and the balance between positive and negative feedbacks, V:

$$P_{int} = \varphi(a, \zeta S_0^{1/4}, V) \tag{3}$$

If we take a (m/s) and $\zeta S_0^{1/4}$ (m) as parameters with independent dimensions, then according to π-theorem:

$$\frac{P_{int}}{\tau_{int}} = \Phi(V) \tag{4}$$

$$\tau_{int} = \tau_{adv} = \frac{\zeta S_0^{1/4}}{a} \tag{5}$$

If we approximate $\Phi(V)$ as $1/(1-V)$ implying that weak negative feedbacks (stronger V) provide longer relaxation periods, then

$$P_{int} = \frac{\zeta S_0^{1/4}}{a(1-V)} \tag{6}$$

For reference values of VCV model parameters, $V = 0.74$ and $P_{int} = 110$ kyr. Therefore authors' observation that the intrinsic period of VCV relaxation oscillations is close to the eccentricity period is correct.

3. Period of the system response to orbital forcing.

We suggest that the period of the system response, in addition to ice thickness, mass influx, and the balance between positive and negative feedbacks, depends also on the amplitude and period of the astronomical forcing, ε, T:

$$P = \varphi(a, \zeta S_0^{1/4}, V, \varepsilon, T) \tag{7}$$

If we take again a, and $\zeta S_0^{1/4}$ as parameters with independent dimensions, then

$$\frac{P}{\tau_{int}} = \Phi(\frac{\varepsilon}{a}, \frac{T}{\tau_{int}}, V) \tag{8}$$

We know from experiments with VCV model (these experiments are not part of the proposed paper, but the authors can easily replicate them) that for T = 35 – 50 kyr (and reference values of other parameters) the system responds with the period-doubling. This means that $\frac{P}{\tau_{int}}$ depends linearly on T, i.e.

$$\frac{P}{\tau_{int}} = \frac{T}{\tau_{int}} \Phi(\frac{\varepsilon}{a}, V) \text{ or}$$

$$\frac{P}{T} = \Phi(\frac{\varepsilon}{a}, V) \tag{9}$$

We can see that ***astronomical forcing makes the intrinsic timescale irrelevant.*** This does not mean though that terrestrial properties of ice-climate system have no role in ice-age periodicity, but they manifest themselves in other then $\frac{T}{\tau_{int}}$ similarity parameters, i.e., the ratio of orbital and terrestrial mass influx amplitudes $\frac{\varepsilon}{a}$ and the ratio of amplitudes of positive and negative feedbacks, V.

The same conclusion becomes apparent from a closer look at Fig. 4b adopted below. The timescale control parameter r modifies ζ such that $\zeta' = r\zeta$ (see VCV equations), and the horizontal scale of fig. 4b has a physical meaning of relative changes of ζ, i.e. $r = \zeta'/\zeta$. It can be observed that the results are independent on ζ for $0.8 \leq r \leq 1.2$ (framed below in the green rectangle). It means that, around its reference value, parameter ζ is not part of the equation (7) and the scaling law takes form of the equation (9).

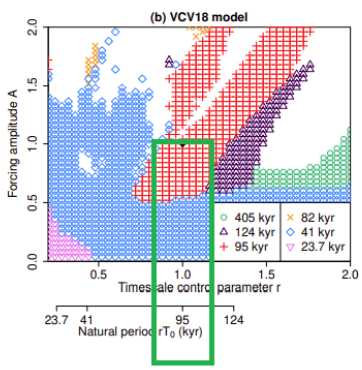


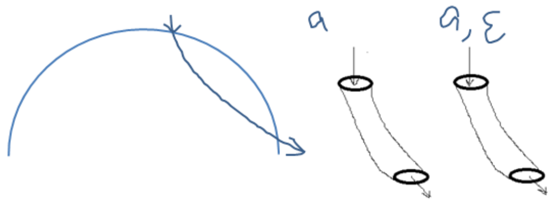
Fig. 4(b). The green frame was absent in the original figure.

In other words, the period of the system response is not defined by the similarity parameter $\frac{T}{\tau_{int}}$ and therefore the VCV model does not support the hypothesis advanced by the authors. Instead, this paper provides a comprehensive support to the scaling law (9) that has been first suggested by Verbitsky and Crucifix (2020).

4. Back to physics

There is no physical similarity between ice sheets with and without orbital forcing. Formally, it can be concluded by comparing the scaling laws (4) and (9). Physically, it may be illustrated by the following.

Sometimes parameter ζ is called "the shape factor" because it reflects ice rheology and defines the shape of an ice sheet. Since ζ ~ constant (I am not even sure that $r < 0.8$ or $r > 1.2$ are physically feasible for large ice sheets) the shape of an ice sheet remains the same with and without orbital forcing, and the mass influx is the only factor that matters for the advection timescale.



This may be envisioned as a fluid going through a pipe. Though the pipe is the same, there is no physical similarity between these two flows.

The intrinsic timescale has no role in the presented results (around reference values) simply because the advection flow of forced ice sheet is different from the advection flow of "intrinsic" ice sheet. When the forcing amplitude is small (A = 0.5, other parameters being at their reference values), the negative feedbacks dominate, and the effective mass influx is about two times smaller than the reference (intrinsic) a. When the forcing amplitude is larger (A = 1, other parameters being at their reference values), positive feedbacks are engaged, and the characteristic mass influx becomes about 2 times larger than a, and the advection becomes much faster. Here we have interplay between faster advection and the obliquity period (i.e., obliquity-period doubling) that may resemble a non-linear resonance. Demodulation of the eccentricity period from the precession forcing (mostly done by the heat-advection equation) shifts the spectrum pick toward 95 kyr. Its closeness to the intrinsic period is just a coincidence.

5. Conclusions

The authors applied phenomenological thinking (P_{int} has the same numerical value as P) to explain response of the VCV model to the orbital forcing, but phenomenology does not imply physical similarity and therefore may lead to questionable interpretations.

Nevertheless, I do believe that this paper should be published because (a) it has wealth of results that need to be studied (e.g., Fig. 4 is precious), and (b) it comprehensively exposes the difference between phenomenological (SO, G24-3) and physical (VCV) models. Having author's thoughts on what it means for ice-ages studies will be most valuable.

References

Bahr, D. B., Pfeffer, W. T., and Kaser, G.: A review of volume-area scaling of glaciers, Rev. Geophys., 53, 95–140, doi:10.1002/2014RG000470, 2015.

Verbitsky, M.Y.: Equilibrium ice sheet scaling in climate modeling, Climate Dynamics, 7, 105–110, https://doi.org/10.1007/BF00209611, 1992.

Verbitsky, M. Y. and Crucifix, M.: π-theorem generalization of the ice-age theory, Earth Syst. Dynam., 11, 281–289, https://doi.org/10.5194/esd-11-281-2020, 2020.