

Nicely done. The paper makes its key point clear and offers a nice addition to the literature on the carbon budget. I congratulate the author for their efforts on the paper and on this last set of revisions in particular.

I have included a number of technical corrections below. There are only a few that are critical (i.e., I couldn't understand the sentences as currently written). The rest are mostly tidying. These can be included or not. It is really up to the author and editor how far they want to push the paper's quality/readability. I would tidy these things up because I found the writing issues super distracting, but ultimately it's up to you and the editor.

As said previously, well done on a great paper.

Technical corrections

- lines 3-5: I had a really hard time understanding the sentence. Is the following change an accurate reflection of what you meant? "In this paper, the deviations of the carbon budget and the strict linear relationship implied by the carbon budget are examined through the lens of the function of the temperature response to an emission pulse (i.e, pulse response), and its relation to TCRE" -> "In this paper, the deviations of the carbon budget from the strict linear relationship implied by the TCRE is examined through the lens of impulse response and its relationship with a non-linear TCRE."
- line 5: "deviations" -> "deviation"
- line 6: "The former stems from the scenario choice, the emission pathway, under the fixed cumulative emissions" -> "The former stems from the scenario choice, i.e. the specific emission pathway for a given level of cumulative emissions"
- lines 8-9: "This paper shows how the pulse response in role of a Green's function gives a unifying perspective on both scenario and state-dependence" -> "This paper shows how the pulse response, viewed as a Green's function, gives a unifying perspective on both scenario and state-dependence"
- line 10: "independency" -> "independence"
- line 10-11: "under the given constraints" -> "for a given set of constraints"
- line 12-13: The sentence as written is very confusing. Does the following change clarify the intended meaning? "Moreover, using pulse response as a Green's function in the optimization program, the" -> "Moreover, using the pulse response, the"
- line 13: "full" -> "reduced-complexity". ESM people would rightly have a fit if we called FaIR a full model.
- line 19-21: "Green's function approach is eligible to diagnose both models' carbon budget scenario-dependency, leaving investigation with other and

more complex models to future work.” -> “The Green’s function approach can be used to diagnose both models’ carbon budget scenario-dependency, paving the way for future investigations and applications with other and more complex models.”

- line 63: “with the changing” -> “with changing”
- lines 64-65: “Moreover, this paper shows that state-dependency of TCRE leads to a non-linear carbon budget equation, as the non-constant TCRE leads to breakdown of linearity given by Eq. 1.” This sentence goes in circles (non-linear TCRE leads to non-linearity because it breaks the linear assumption). I suggest deleting it.
- line 67: “and not from the climate conditions of the system” -> “and not from the initial climate conditions of the system”. If you have a different pathway, then you have different climate conditions along the way to reaching the same cumulative emissions so I don’t think it is correct to imply that the climate conditions are entirely the same.
- line 68: “utilizing the high-complexity” -> “utilizing high-complexity”
- line 87: “of Green’s function” -> “of a Green’s function equation” ? The phrase “in the context of Green’s function” doesn’t make any sense to me.
- line 91: “independency” -> “independence” (again, probably worth doing a search for independency throughout, independency is an anarchic term for independence)
- line 92: ‘full model’ -> “FaIR” or whatever other model you mean. I wouldn’t use the term ‘full model’ anywhere in your paper because it is unclear and the idea of a ‘full model’ doesn’t really make sense (all models have limitations, so what makes one model full while another is not).
- line 99: as above re ‘full model’ (and, as above, I strongly suggest removing the phrase ‘full model’ from the manuscript entirely)
- line 108: “left out” -> “left”
- line 113: “in context” -> “in the context”
- line 114: “implications on” -> “implications for”
- line 117: as above re ‘full model’
- line 123: as above re ‘full model’
- line 125: “deviation” -> “the deviation”
- line 132: “changing pulse” -> “pulse” (changing is already implied by other words in the sentence)
- line 167: “the Green’s” -> “Green’s”
- line 253: “45 years” -> “45 years farther” (or further)

- line 265: “previous” -> “the previous”
- lines 303-304: I didn’t understand this sentence: “This way, assuming any function for the state dependency is avoided; rather, it is deducted from mapping $v(T)$ (Fig. 7, right graph)”
- line 305: “due to negative” -> “due to the negative”
- lines 307-308: “due to an an increasing probability of a climate system bifurcation thresholds” What does this mean and what reference do you have for this?
- line 321: “shown in next” -> “shown in the next”
- line 359: ‘full model’ (as above)
- line 361: ‘full model’ (as above)
- line 366: refer to the analysis you have which shows this (Appendix A)
- line 415: “exhibiting the same” -> “exhibiting deviations of the same”
- line 416: “especially precise for lower cumulative emissions” -> “with the Green’s function approach being especially close to FaIR for lower cumulative emissions”
- line 427: “Namely, in” -> “Namely,”
- lines 429 - 431: There are some missing words in this sentence so it doesn’t really make sense
- line 490: ‘had’ -> ‘has’
- line 495: ‘full model’
- line 515: ‘linearities’ -> ‘non-linearities’ ?
- line 518: ‘linear’ -> ‘linear (when viewed at the same time point)’ or something like this? The pulse isn’t constant so there must be some time element assumption in here no?
- line 540: “with FaIR” -> “with FaIR or some other model that captures the scenario- and state-dependent effects discussed here”
- line 662: ‘full model’ (as above)