Dear Prof. Kirk-Davidoff,

hope this letter finds you well. I am happy to submit the revised manuscript following the reviewers' suggestions.
To my understanding, I have reflected on all raised issues and incorporated them into the manuscript. As suggested, I have shifted the article's focus more toward the climate modeling subject, expanding on the pulse response examination given in Section 4 and adding the effects of climate uncertainty. Nevertheless, I kept some references to the climate economics audience since I believe the work is of value in that domain.

Three main findings are reiterated here:
1.) Developed a scheme to predict maximum carbon budget scenario-dependency from the impulse response function, applicable to the models of any complexity
2.) Quantified maximum possible carbon budget scenario-dependent deviations
3.) Connected the behaviour of pulse response with the carbon budget non-linearities, while deriving an alternative formula for the carbon budget equation based on T as a predictor.

Once again, thank you for giving me an opportunity to revise the article. I hope that the reviewers will find the revision worthy, and looking forward to their fresh input. Please find the point-by-point response to reviewers, in parts motivated by the answers in the discussion phase. The file ends with a list of changes. I skipped listing all the changes in text because that would be an overflow of text, as one can see by the amount of the changes tracked in the “track-changes” file.

Best regards,
Vito Avakumović

Point-by-point response:

Reviewer 1

General Comments

1) The first general comment argues that the deviation from the linearity is only observed in simple climate models and EMICs, while at the same time, the CMIP5 models in Tokarska et al. 2016 (Figure 3) show the linearity. To this, I have two arguments. In Tokarska et al. 2016, the CMIP5 models were forced with both 1% per year atmospheric concentration increase forcing (pure carbon dioxide emission forcing) and the extended RCP8.5 scenario (aerosols and other non-CO_2 gases). The review argues that Tokarska et al. 2016 clearly show the linearity between cumulative emissions and the temperature increase with solid lines. However, those are the results using RCP8.5 scenarios so they have other non-CO_2 effects included, whereas FAIR was forced only with CO_2 in my work. Additionally, it is questionable if we have a linear relationship in a solid line for BCC-CSM 1.1 (green) in the relevant domain (~2000 GtC). On the other hand, it is not clear from the figure if the relationship is linear (or non-concave) for the purely CO_2 experiment (dotted lines). To round up, I quote from the paper „*Figure 3a shows that the warming in the RCP 8.5-Ext simulations scaled by the ratio of CO2 to total forcing, for a given magnitude of cumulative emissions, is slightly higher than for the 1PCTCO2 simulations. One possible reason for this is the warming from non-CO2 greenhouse gases, which reduces the diagnosed cumulative emissions in the RCP 8.5-Ext* *simulations associated with the carbon-climate feedback*." A similar conclusion can be made by looking at the first figure that I attach below, which I refer to in my second argument.

With that in mind, I come to my second argument. Looking at the graph given in the summary for policymakers AR5 WG1 (IPCC, 2013) in the first figure below, one can detect a slightly concave relationship between cumulative emissions and temperature increase, for RCP scenarios and the stylized 1% per year concentration increase scenario, also conducted in CMIP5 models. The figure can be found below. I drew the green and white lines to visually emphasize the deviation from linearity. In my research, only CO_2 emissions are taken into account, so the grey plume in Figure SPM.10 is imperative in this case. I use the result from the old report since in the newest SPM WG1 (IPCC, 2021), the relationship between cumulative emissions and the temperature increase is shown only for SSP scenarios (Figure SPM 10.). Additionally, the exponential relationship between cumulative emissions and temperature derived in the paper only slightly deviates from linearity in the cumulative emissions regime that is inspected. This was not shown in the paper not to overflow the reader with plots and information, but it can be tested with the values provided in the text.

Albeit the relationship is nearly linear, the small and persistent difference in temperature output can make a large difference in economic assessments.

Moreover, if the FAIR model does produce a higher deviation from linearity than the ESMs, one of the points of deriving the new equation (Manuscript, Eq. 6) was to show that one can fairly well approximate a FAIR climate model with a single equation if the state-dependent TCRE is considered (derived from FAIR). This is a result independent of the linearity discussion.

Lastly, in the reivsed manuscript I have shown a way how the calibration of FaIR model effects the linearity of the carbon budget approach through the pulse response representation. Indeed, it is possible to calibrate FaIR to have a linear relationship (and also convex) between cumulative emissions and temperature increase.

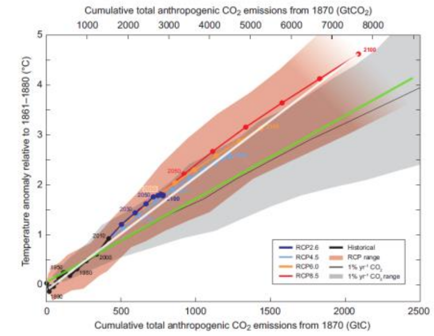

Figure SPM.10 | Global mean surface temperature increase as a function of cumulative total global CO_2 emissions from various lines of evidence. Multi-model results from a hierarchy of climate-carbon cycle models for each RCP until 2100 are shown with coloured lines and decadal means (dots). Some decadal means are labeled for clarity (e.g., 2050 indicating the decade 2040–2049). Model results over the historical period (1860 to 2010) are indicated in black. The coloured plume illustrates the multi-model spread over the four RCP scenarios and fades with the decreasing number of available models in RCP8.5. The multi-model mean and range simulated by CMIP5 models, forced by a CO_2 increase of 1% per year (1% yr^{-1} CO_2 simulations), is given by the thin black line and grey area. For a specific amount of cumulative CO_2 emissions, the 1% per year CO_2 simulations exhibit lower warming than those driven by RCPs, which include additional non-CO_2 forcings. Temperature values are given relative to the 1861–1880 base period, emissions relative to 1870. Decadal averages are connected by straight lines. For further technical details see the Technical Summary Supplementary Material. (Figure 12.45; TS TFE.8, Figure 1)

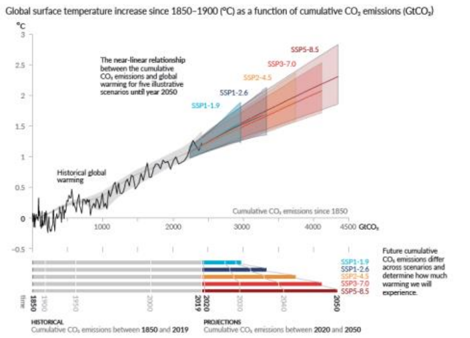

Figure SPM.10 | Near-linear relationship between cumulative CO_2 emissions and the increase in global surface temperature

Top panel: Historical data (thin black line) shows observed global surface temperature increase in °C since 1850–1900 as a function of historical cumulative carbon dioxide (CO_2) emissions in $GtCO_2$ from 1850 to 2019. The grey range with its central line shows a corresponding estimate of the historical human-caused surface warming (see Figure SPM.2). Coloured areas show the assessed *very likely* range of global surface temperature projections, and thick coloured central lines show the median estimate as a function of cumulative CO_2 emissions from 2020 until year 2050 for the set of illustrative scenarios (SSP1-1.9, SSP1-2.6, SSP2-4.5, SSP3-7.0, and SSP5-8.5; see Figure SPM.4). Projections use the cumulative CO_2 emissions of each respective scenario, and the projected global warming includes the contribution from all anthropogenic forcers. The relationship is illustrated over the domain of cumulative CO_2 emissions for which there is *high confidence* that the transient climate response to cumulative CO_2 emissions (TCRE) remains constant, and for the time period from 1850 to 2050 over which global CO_2 emissions remain net positive under all illustrative scenarios, as there is *limited evidence* supporting the quantitative application of TCRE to estimate temperature evolution under net negative CO_2 emissions.

Bottom panel: Historical and projected cumulative CO_2 emissions in $GtCO_2$ for the respective scenarios.

To conclude the point, the second reviewer pointed out the piece of literature that shows the non-linear carbon budget equation (Nicholls et al. 2020). Hence, previously published literature argues in favour of a possibility of non-linear relationship between cumulative emissions and temperature increase.

2.) On the second point, it was never argued that Green's function is an approximation of the real climate system, but rather an approximation of a model; in this case a FAIR model. Furthermore, the point of the paper was not to qualitatively detect the source of deviation from the real-world (natural) system, but to quantify what are the maximally possible deviations given the FAIR model, at the same

time testing whether FAIR (and its Green's approximation) can accurately capture path-independence. Hereby, I refer to the comments questioning both models' ability to capture the path independence. In my paper, the path dependence was tested by the optimization program given by Eq. 4. By using the optimization procedure, the full portfolio of all the possible emission pathways (under the given constraints) was tested, and not just „a simple series of idealized experiments using different rates of emissions“, as suggested in the review. Hence, using the optimisation program the pathways that generate the minimal and maximal temperature in a given year (y*) under the same cumulative emissions are generated. By subtracting those two, the maximal path-dependent deviation of the carbon budget is generated. Seeing it is small in magnitude, the conclusion is that the FAIR adheres to scenario independency of the carbon budget approach. Coming back to the Green's function that is derived from the FAIR model and running it through the same optimization program, it provides more or less same path-dependent deviations, confirming that it can accurately capture path-independence at least in context of FAIR model. Therefore, we use the shape of Green's function to provide the intuition behind the path-independency and ground the argumentation that it can be used to assess other simple climate models' adherence to carbon budget (explained above).
Additionally, the introduction of the Green's function provides a method to assess the carbon budget scenario-dependency of model of any complexity, not just simple climate models, which is one of the main novelties in the manuscript and offers a venue for further research.

3.) This point is addressed by providing all of the model runs and codes in open access online. My main concern in this validation request was that it is not specified which runs should be used to validate the GAMS translation. The logical choice would be the pulse response run. However, the inspection of the pulse responses was exactly the reason why I used GAMS and I do not see a way to do it in FaIR. Clarification is the following. To produce a pulse response one needs to set atmospheric carbon dioxide concentration to a fixed value and then generate the emissions that keep that level constant. GAMS makes this easy because in it, one only needs to change the role of emissions into a free variable (from being an input variable), and change the role of atmospheric concentration from a free variable to an input variable, and run the code normally afterward. In python this is not possible since one needs to have an explicit expression of how concentration depends on emissions in a functional form. FaIR, however, is constructed such that the concentration is implicitly dependent on emissions.

4.) This point was addressed in the revised manuscript in lines (260-265). Transient path-dependence is not independent of ZEC because the effect of ZEC cancels out when subtracting the max. and min. generated temperatures. Regardless, the implications of the work on ZEC are being addressed in the discussion in the revised manuscript (lines 620-628).

Specific comments

All of the specific comments given by referee 1 are now reflected upon in the revised text.

Reviewer 2

General comments

The reviewer captured the points of the paper correctly, with one correction on the sentence: „*They note that neither Green's formalism nor their new equation can capture scenario-dependence*". This is not quite the case. One can see that in the manuscript in Figures 1 and 2, right panels, that for various setups, Green's equation can indeed capture the scenario dependency of the full FaIR to a very high extent, especially in the mitigation scenarios (lower total F_{tot}). Hence, one innovation the paper presents is that once a pulse response of a climate model was generated, Green's equation can predict the order of magnitude of the maximum possible scenario dependence for such a budget. Hereby, Green's method is not restricted to a "simple" climate model. One can, in theory, use the pulse response generated by ESM and feed it into Green's equation, run the min/max optimization (Eq 4), and get the deviation while avoiding running the full-fledged (!) ESM in the optimization scheme (which would likely be impossible due to computational costs). In essence, the manuscript suggests that Green's equation is a practical tool to test the carbon budget scenario-dependency for any model.

Major concerns

Missing key literature

There was an oversight from my side when scanning the literature, resulting in claiming the novelty in the state-dependent carbon budget equation. This mistake has been undone, and I have contextualized my work with the work from Nicholls et al. (2020). Moreover, referee's 2 comment had helped me to expand my work and gain new insight, which was incorporated in the revised manuscript. In my answer to the reviewer in the discussion phase, I have extensively compared my work in context of Nicholls et al. (2020) work. In essence, the method provided in the manuscript gives an alternative way to derive the non-linear carbon budget equation. Starting from the pulse response function, we approximate it using T as a leading variable (of a thermodynamic system) using a first order Taylor expansion, and then with integration arrive at the equation. In newly introduced Sect 4.4., I show how different calibrations of FaIR bring about different pulse response behaviour. Following the method given in the original manuscript, one can derive carbon budget equations with different levels of non-linearities (not restricted to convex relationship only) and even the linear equation, depending on the model calibration. This fits to claims from previous literature but seen from the fresh perspective (pulse representation)

Writing

The newly revised manuscript has been given proof-read by the native english speaker, so hopefully the text reads better. Also, I followed the referee's advice and removed much of the text that could have been seen as redundant or "slow", and made the language more active.

Key insights get lost

Following the referee's advice, I have revised the whole manuscript in order to expand on the analysis in Section 4.

Journal fit

The revised manuscript now focuses more one climate modelling tools, while still referring to the implications that the work has for the climate economic audience.

Minor concerns

Clarity of methods presentation

The revised manuscript still has a reference to the optimization in the abstract, but I have put it in a context. I think it is important to keep it, since it is the key point that differs how the scenario-dependent effects are examined in the manuscript, compared to the previous literature.

The open-budget and net-zero budget are renamed to fit more in the literature lingua. As pointed above, I have now made clear distinction between scenario-dependent effects explored in the manuscript, and ZEC.

Lack of exploration of climate uncertainty

The revised manuscript now introduces the effects of the climate uncertainty on the pulse response representation, and consequently on carbon budget deviations.

Validation of model and code

I have referred to this in the reply to the first reviewer (above).

Claims about FaIR being 'best'

I have taken the reviewers advice and modified the statements in Sect 2.1.

Full-fledged

The full-fledged noun has been removed and replaced with full model, so there is a distinction between the Green's and the model itself.

Lower cumulative emissions

I have introduced lower cumulative emissions that would be in line with 1.5 °C in the subsection that briefly investigates the effects of including negative emissions into the portfolio.

Leftover temperature

T_left is prescribed only to show that Green's model can replicate the overall temperature increase. In context of testing scenario-dependent effects, it is not needed.

Moreover, running the Green's model over the historical period would not yield into correct temperature diagnosis, because the pulse response changes with changing climatic conditions (its magnitude decreases with higher background temperatures as shown in the revised manuscript Fig. 1). Hence, using the current-day pulse response as Green's function backwards in time would yield in underestimation of temperature, the same way it yields in overestimation later in the run with higher background temperatures (Fig. 5, top row).

Claims about time evolution of T_d

T_d declining to zero is indeed the feature of FaIR, which is itself a good emulator of more complex models. This is why I expanded the analysis with inclusion of the one-box model which has a drastically different pulse response behaviour. Inspecting those two separately solidifies the conclusions from Sect 4. that claim that the pulse response can be used as an indicator of carbon budget deviations.

Claims about impact of optimisation year

I have backed up this claim with the examples given in the newly introduced supplement.

Why can't you just use the simple climate model

Technically, you can. My original point was to show that one can simplify the model even further, making it even easier to implement. More importantly, simple climate models in their present form are not fit for analytical inspection because solving them explicitly takes a lot of effort and provides cumbersome expressions. Hence, the idea was to give the audience a simple, analytically tractable expression that connects temperature and emissions.

Nevertheless, the focus of the revised manuscript is completely shifted so this discussion is not a vital part of it anymore.

Figures

I have taken referee's advice and plotted the relative deviation of the linear and newly introduced non-linear carbon budget equation from FaIR temperature diagnosis. It can be seen in the revised manuscript in Figure 8, bottom row.

List of changes

- Overall, profoundly changed the text in all sections with drastic text cuts and newly added text
- Emphasized the key points that seemed to be obscured to some extent in the original manuscript
- Expanded Sect 4., making its findings a new central point of the manuscript
- Pulse response representation as a focal point of the revised manuscript
- Included another simple climate model in the analysis
    - Confirms the findings in the light of the pulse response
    - Tests the deviations in the context of structural model uncertainty
    - Solidifies the pulse response (in the role of Green's function) as the viable carbon budget deviations indicator
- Included the effects of negative emissions on the carbon budget scenario-dependent deviations
    - On top
- Included the emission pathways that generated min. and max. temperature in Sect 3, and the resulting temperature pathways (not restricted to the optimization year only)
- Investigated the implications of climate uncertainty
    - Argued how different calibrations of FaIR produce different pulse response representation, from which both non-linearities and scenario-dependency effects of the carbon budget equations can be inspected
- Incorporated the findings from Nicholls et al. (2020), and contextualized it to the findings of the manuscript
- Put ZEC in the context of the manuscript and the calculated deviations
- Connected ZEC with the pulse response, offering a venue for further research
- Added the relative deviation of the newly introduced non-linear equation generated temperatures (and linear equation generated ones), from FaIR-generated temperatures
- Included the plots of temperature leftover terms corresponding to emission cessation in four different years
- Generally fixed and changed the nomenclature
    - Changed the lambda in Eq. 1 to uppercase, and renamed it from "perfect budget", to simply "linear" carbon equation
    - FaIR
    - Transient budget instead of open budget, removed S1 and S2 as defining the two cases
    - Historical scenario -> idealized (RCP6.0) scenario
- Fixed the ordering of the figures so they appear by order of appearing in the text
- Generally, fixed the visuals of the figures
- Changed the discussion
    - Removed the repetitive parts and offered a few venues for further research
- Fixed the typos

References

Dietz, S., van der Ploeg, F., Rezai, A., and Venmans, F.: Are Economists Getting Climate Dynamics Right and Does It Matter?, Journal of the Association of Environmental and Resource Economists, 8, 895–921, https://doi.org/10.1086/713977, publisher: The University of Chicago Press, 2021.

IPCC, 2013: Summary for Policymakers. In: Climate Change 2013: The Physical Science Basis. Contribution of Working Group I to the Fifth Assessment Report of the Intergovernmental Panel on Climate Change [Stocker, T.F., D. Qin, G.-K. Plattner, M. Tignor, S.K. Allen, J. Boschung, A. Nauels, Y. Xia, V. Bex and P.M. Midgley (eds.)]. Cambridge University Press, Cambridge, United Kingdom and New York, NY, USA.

IPCC, 2021: Summary for Policymakers. In: Climate Change 2021: The Physical Science Basis. Contribution of Working Group I to the Sixth Assessment Report of the Intergovernmental Panel on Climate Change [Masson-Delmotte, V., P. Zhai, A. Pirani, S.L. Connors, C. Péan, S. Berger, N. Caud, Y. Chen, L. Goldfarb, M.I. Gomis, M. Huang, K. Leitzell, E. Lonnoy, J.B.R. Matthews, T.K. Maycock, T. Waterfield, O. Yelekçi, R. Yu, and B. Zhou (eds.)]. Cambridge University Press, Cambridge, United Kingdom and New York, NY, USA, pp. 3−32

Meinshausen, M., Raper, S. C. B., and Wigley, T. M. L.: Emulating coupled atmosphere-ocean and carbon cycle models with a simpler model, MAGICC6 – Part 1: Model description and calibration, Atmos. Chem. Phys., 11, 1417–1456, https://doi.org/10.5194/acp-11-1417-2011, 2011.

Nicholls, Z. R. J., Gieseke, R., Lewis, J., Nauels, A., & Meinshausen, M. (2020). Implications of non-linearities between cumulative CO2 emissions and CO2-induced warming for assessing the remaining carbon budget. *Environmental Research Letters*, *15*(7), 074017.

Tokarska, K. B., N. P. Gillett, A. J.Weaver, V. K. Arora, and M. Eby, 2016: The climate response to five trillion tonnes of carbon. Nature Climate Change, doi:DOI: 10.1038/NCLIMATE3036.