Supplementary Material: Rate–induced tipping cascades arising from interactions between the Greenland Ice Sheet and the Atlantic Meridional Overturning Circulation

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S1 Parameters for the AMOC box model and flowline model of the Greenland Ice Sheet

Table S1. Parameters of the AMOC box model (Eq. (4)–(11)) and the flowline model of the Greenland Ice Sheet (Eq. (1)-(3)). The AMOC box model parameters were determined by a calibration of the AMOC box model to a quasi–equilibrated ocean obtained from the coarse–resolution AOGCM FAMOUS$_B$ (Smith, 2012) under preindustrial atmospheric CO_2 concentration conditions (Wood et al., 2019). For details on the calibration procedure it is referred to Wood et al. (2019).

AMOC box model			**Flowline model of the Greenland Ice Sheet**		
α	0.12	$\mathrm{kg\,m^{-3}\,{}^{\circ}C^{-1}}$	A	10^{-16}	$\mathrm{a^{-1}\,Pa^{-3}}$
β	0.79	$\mathrm{kg\,m^{-3}\,psu^{-1}}$	n	3	
S_0	35		ρ	916.7	$\mathrm{kg\,m^{-3}}$
T_S	4.773	$^{\circ}\mathrm{C}$	L	600	km
T_0	2.650	$^{\circ}\mathrm{C}$	g	9.81	$\mathrm{m\,s^{-2}}$
η	74.492	Sv	Γ	5	$\mathrm{{}^{\circ}C\,km^{-1}}$
K_N	5.456	Sv	γ	4.4	$\mathrm{cm\,a^{-1}\,{}^{\circ}C^{-1}}$
K_S	5.447	Sv			
K_{IP}	96.817	Sv			
λ	2.79×10^7	$\mathrm{m^6\,kg^{-1}\,s^{-1}}$			
κ	0.39				
μ	5.5	$\mathrm{{}^{\circ}C^{-1}\,m^{-3}\,s} \times 10^{-8}$			
V_N	3.261×10^{16}	$\mathrm{m^3}$			
V_T	7.777×10^{16}	$\mathrm{m^3}$			
V_S	8.897×10^{16}	$\mathrm{m^3}$			
V_{IP}	22.02×10^{16}	$\mathrm{m^3}$			
V_B	86.490×10^{16}	$\mathrm{m^3}$			

Table S2. Initial conditions for the salinities of the AMOC box model (Eq. (4)–(11)) as given in Alkhayuon et al. (2019) determining the constant total salt content C.

Parameter	
S_N	0.034912
S_T	0.035435
S_S	0.034427
S_{IP}	0.034668
S_B	0.034538

Table S3. Surface freshwater fluxes (Eq. (11)) applied to the AMOC box model (Eq. (4)–(11)) following Wood et al. (2019).

Parameter	Preindustrial FAMOUS$_B$	
F_{N_0}	0.384	Sv
F_{S_0}	1.078	Sv
F_{T_0}	-0.723	Sv
F_{IP_0}	-0.739	Sv
A_N	0.070	Sv
A_S	-0.257	Sv
A_T	0.752	Sv
A_{IP}	-0.565	Sv

S2 Effects of varying coupling strength and rates of GIS surface mass balance decrease on emergent dynamics regimes

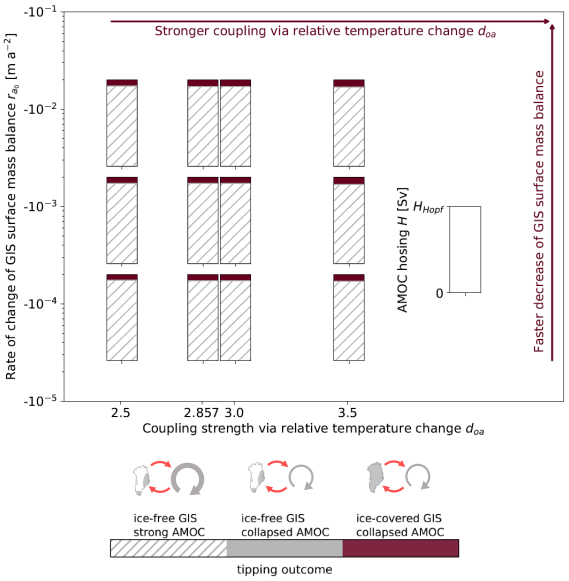

Figure S1. Emergent dynamics regimes of the Greenland Ice Sheet and the Atlantic Meridional Overturning Circulation for bidirectional coupling. Tipping outcomes in response to a limited linear decrease of the Greenland Ice Sheet surface mass balance on the ground with a ramping rate r_{a_0} (varied along outer vertical axis) for varying coupling via relative temperature change d_{oa} (varied along the outer horizontal axis). The AMOC hosing (vertical axis of bars) is kept constant between $H = 0$ Sv and the AMOC hosing threshold H_{Hopf}. The respective tipping outcome is indicated by the colouring (grey: GIS deglaciation, pink: no GIS deglaciation; stripes additionally indicate the AMOC in its 'on'–state).

References

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