Review of Klose, Donges, Feudel, and Winkelmann*: "Rate–induced tipping cascades arising from interactions between the Greenland Ice Sheet and the Atlantic Meridional Overturning Circulation." (*Earth System Dynamics*, Paper:* esd-2023-20*)*

The manuscript of Klose and others presents the results of a conceptional model system where a global ocean interacts with the Greenland Ice Sheet (GrIS). They allow tipping cascades of the Atlantic Meridional Overturning Circulation (AMOC) stability by freshwater released into the North Atlantic. Besides a hosing flux, the meltwater contribution of disintegrating GrIS disturbs the stability of the AMOC and, via a potential feedback loop, controls Greenland's meltwater contribution. The authors access overshot and rate-induced tipping cascades in this highly idealized system.

The first conceptional model renders the global ocean by hydrographic properties in five boxes: Two dedicated Atlantic surface ocean regions (North Atlantic: N, Tropical Atlantic: T) beside surface boxes for the Southern Ocean (S) and the Indo-Pacific Ocean (IP). Those surface boxes communicate with a global bottom ocean box (B). In this model, the density difference between the Northern Atlantic and Southern Ocean drives the AMOC, which controls the North Atlantic temperature and determines the temporal salt flux between different boxes (Eq. 6 – 10).

The other conceptional model represents the Greenland Ice Sheet by a flowline model solving the shallow ice approximation. It has a (half) width of 1000 km, which equals approximately its actual latitudinal extent. The ice loss and gain are exclusively described by the surface mass balance (SMB), where the Lapse rate effect constitutes the melt elevation feedback.

The coupling between these two models is unidirectional or bidirectional. The unidirectional coupling considers only the meltwater flux of a shrinking GrIS into the Atlantic Ocean. This meltwater flux plus an additional freshwater hosing flux decreases the salinity in the Atlantic. In the bidirectional setup, the North Atlantic temperature feeds back on Greenland's surface mass balance.

The authors detect overshoot and rate-induced cascading tipping in their model system with a focus on the GrIS and the AMOC. These cascades are analyzed in the uni- and bidirectional setups where the bidirectional coupling considers the feedback loop between the North Atlantic temperature and the ice loss. This thermal coupling stabilizes the AMOC and the ice sheet because an enhanced meltwater flux reduces, via the density difference, the AMOC strength, lowering the North Atlantic temperatures, which, ultimately, damps additional melting.

This study is highly relevant since it addresses outstanding questions about the AMOC stability while considering the interaction between the AMOC and the Greenland Ice Sheet. At the same time, Greenland's meltwater release grows in a warming world. The authors highlight that it is not sufficient to consider a "fixed" threshold (overshoot tipping) beyond which the AMOC breaks down or GrIS disintegrates. They also underscore that changing rates could drive components in the coupled system beyond stable conditions (rate-induced tipping). In addition, a stabilizing feedback, such as reduced North Atlantic warming due to a weakened AMOC, may not be strong enough to offset the disintegration of the GrIS. Here, the reduced model complexity allows scanning the phase space for numerous tipping cascades. Although, these are not necessarily representative of the natural world.

In general, it was a pleasure to read the well-structured manuscript. The figures are of high quality, necessary, and informative.

I recommend the publication of the manuscript after minor corrections.

General comments

Although the manuscript is well organized and generally well written, section 4.2, including its subsections, could be better written. The language of section 4.2 is less clean than the remaining section. Therefore, I suggest revising this whole section.

Furthermore, the authors presented additional lengthy information in brackets, which may disturb the reader. I suggest the authors integrate this information into the general text or drop it if applicable.

In Figures three and four, small schematic icon-like figures of the remaining ice-thickness across Greenland seem to indicate remaining ice. If this is the case, please state more clearly where this pattern comes from because I do not see how the applied flowline ice sheet model can provide this pattern.

Occupationally, terms/variables are introduced, which are defined later in the text. In some cases, this needs to be clarified. For example, the text refers (Page 9, Line 232) to Figure 2, where the variable r_{a0} appears in the caption, while it is later introduced on Page 9, Line 246.

Specific comments

Main document

Page 3, Line 79: Add comma in "... for a limited forcing, given that the …"

Page 4, Line 112: Please introduce the not SI-unit Sverdrup, for example, by a footnote or additional text.

Page 2, Line 36 – 37; Page 4, Line 107 – 198; Page 4, Line 109 – 111, Page 4, Line 118, Page 5, Line 1124 – 125: Missing citation (Madsen et al. 2022).

Page 5, Line 132: Do you mean the citation "(compare e.g. Lohmann and Ditlevsen, 2021)?

Page 5, Line 151: You may consider adding additional information: "(… continent by the ocean without floating ice shelves in Oerlemans (1981))."

Page 6, Line 156: "Thereby, the surface mass balance a_s that is the difference between mass gain and mass loss is reduced and …"

Page 6, Equation 3: Is the variable $\widetilde{a_O}$ a spatial dependent variable? If so, please indicate or mention it. In addition, does this linear equation consider an increased vulnerability by a more than linear increase of the ablation zone for lowering height?

Page 6, Line 165: You may add at the end: "Furthermore, the surface mass balance equals in our setup the total mass balance."

Page 7, Equation 11: I can not find the definition of A_i in the text. Please add it, even if this information is available in the supplement's table. Furthermore, does a hosing of $H = 0.2\ Sv$, as shown in Figure 2b, correspond to an additional freshwater flux of $A_i \cdot H$? If so, wouldn't it correspond to a hosing freshwater flux of 0.014 Sv and 0.1504 Sv into the North Atlantic and Tropical Atlantic box, respectively? Since the manuscript addresses the impact of freshwater on the AMOC stability and the conditions at and around Greenland, it is surprising that the freshwater flux may be ten times larger in the tropics than in the North Atlantic. Anyhow, please clarify this point.

Page 8, Line 217: Does the surface mass balance on the ground correspond to the surface mass bal-

ance at the sea-level?

Page 8, Line 218 – 219: You may rephrase “For a declining overturning strength q of the AMOC with $H > H_{\mathrm{ref}}$, the temperature T_N in the North Atlantic box declines as well according to Eq. (4)” if applicable, e.g., “For active hosing ($H > H_{\mathrm{ref}}$), the AMOC overturning strength q declines as well as the temperature T_N in the North Atlantic box is driven by Eq. (4).”

Page 8, Line 219: You may rephrase “For $d_{\mathrm{oa}} = 0$, we recover a …” with “For $d_{\mathrm{oa}} = 0$, we obtain a …”

Page 8, Line 220: I find the wording “independent ice sheet” confusing and misleading. Please change it.

Page 9, Line 219 – 223: You may simplify these sentences: "A unidirectional coupling is obtained by $d_{\mathrm{oa}} = 0$, where Greenland is not exposed to any changes in the North Atlantic (Eq. 12). In addition, the freshwater flux by the Greenland Ice Sheet resulting from its mass loss (Bamber et al., 2012, 2018; Trusel et al., 2018) is added as F_{GIS} to the combined freshwater into the surface North Atlantic box as:”

Page 9, Line 229: Wouldn’t it be correct to state “($F_{\mathrm{GIS}} > 0\,\mathrm{Sv}$)”?

Page 9, Line 231: You may write "… freshwater flux into the Atlantic Ocean, which increases the …" or?

Page, Line 245 – 247: The sentence “More specifically, the surface …. is reached (Fig. 1(b))” is not clear. Please rephrase.

Page 9, Line 245 – 247: You may change the order in this sentence: “More specifically, with a ramping r_{ao}, the ground surface mass balance a_0 decreases linearly and, once crossed the deglaction threshold a_{0dgc}, the ice sheet stability is not sustainable.” or “More specifically, with a ramping r_{ao}, the ground surface mass balance a_0 decreases linearly, and the ice sheet becomes unstable, once crossed the deglaction threshold a_{0dgc} is crossed.

Page 10, Line 256: You may rephrase "By decreasing the surface mass balance at the ground level associated … ."

Page 10, Line 256: You may rephrase to “… threshold and eventually disintegrates completely … .”

Page 10, Line 257: You may replace the beginning of the sentence: "In the following, AMOC hosing H is kept constant.”

Page 10, Line 260 – 265: You may rewrite it: “The freshwater volume loss resulting from the forced deglaciation of Greenland corresponds to a time-varying GIS freshwater flux F_{GIS} into the North Atlantic. This time-dependent GIS freshwater flux first increases as the GIS disintegrates. Consequently, the AMOC grows, potentially overshooting its threshold (Ritchie et al., 2021), but eventually returns to $F_{\mathrm{GIS}} = 0\,\mathrm{Sv}$ under otherwise constant hosing (Fig. 2(a), the AMOC trajectory approximately follows the black lines)".

Page 10, Line 267: Please consider replacing "observed" with "detected" when describing the results

of simulations since model results are not measured and turned into observed properties. Therefore, please rephrase "… to a freshwater flux as detected in previous hosing experiments … ."

Page 10, Line 269 – 272: You may rephrase " In particular, the AMOC may transition to its 'off'–state in response to the Greenland Ice Sheet disintegration, which is accompanied by a temporary overshoot of the GIS freshwater flux threshold, resulting in an overshoot cascade (Fig. 2(c)). The increasing GIS freshwater flux puts the AMOC from the 'on' to the 'off'-state, while the AMOC does not recover after the decline of the GIS freshwater flux."

Page 10, Line 272 – 273: The following might be more appropriate: "The surface mass balance decreases substantially… , which results in a complete deglaciation of Greenland."

Page 10, Line 276: I'm unsure, but shouldn't it be: "AMOC weakening without tipping, as commonly detected in hosing … "?

Page 10, Line 278 – 279: Here is an example of avoiding unnecessary brackets: "… within 1000 years driven by a faster and stronger … ."

Page 10, Line 281: What do you think about rephrasing: "...deglaciation of Greenland, the AMOC leaves the stable 'on'-state. Rate-induced transition … ."?

Page 10, Line 284: I suggest replacing the text with "the AMOC to the changing freshwater flux by … ."

Page 10, Line 284: Please add a comma after the preposition: "Here, it is assumed … ."

Page 10, Line 285: Enclose the example by commas: " disturbances, e.g. in initial box salinities, are always .. ."

Page 10, Line 287: Add a comma for the subordinate clause at the end: "decline as studied, e.g. as scenario-dependent … ."

Page 12, Line 291 – 293: The end of the sentence is unclear; please improve the text.

Page 12, Line 294 – 295: Please extend the text to read: "of the tipping element drivers in our model."

Page 12, Line 309: add missing comma around "thus": " lower hosing values and, thus, for the AMOC … ."

Page 12, Line 312: I'm unsure, but I guess a comma is missing: "with a slow ice sheet decline, a high hosing determining the fixed … ."

Page 12, Line 314: The sentence needs to be clarified, or?

Page 12, Line 315 – 316: I would like to suggest: "an overshoot cascade changes by variations of Greenland Ice Sheet's melting patterns. More … "

Page 13, Line 340 – 342: I suggest: Here, …. of an AMOC weakening, and it may be …. surface mass balance, for a warming … ."

Page 13, Line 344: Do you mean "distinct tipping thresholds" or "different tipping thresholds"?

Page 13, Line 349: Please replace "observed" with "detected."

Page 15, Line 356 – 358: The sentence "With the AMOC tipping …. from the AMOC overturning strength" is unclear to me.

Page 15, Line 378: You may replace "ice sheet melting time" with "ice sheet disintegration time"?

Page 17, Line 389: I guess a comma is missing: “is accelerating (Shepherd et al., 2020), and its … .”

Page 17, Line 398: Since you are apparently using the British syntax predominately, replace "e.g.," with "e.g.".

Page 17, Line 404 – 406: Unclear sentence. Please rephrase.

Figure 1: Please increase the size of the hardly seen green points, which is stated in the text (Page 9, Line 235): "subcritical Hopf bifurcation at F_{GIS} Hopf (indicated by green points in Fig. 2(a))."

Figure 1, caption: The introduced variable r_{a0} has to be defined. Please find a way to introduce it and/or refer to the text.

Figure 3, caption: I would like to suggest the following modification to the figure caption:” Shown is the AMOC overturning strength, also taking … “ (drop “now”); “… declining from pink (100 %) to grey (0 %) as indicated by the right colorbar.”; “indicate the AMOC in its ‘on’-state, see bottom colorbar).”

Figure 3: Please define the green arrows in the caption and drop them.

Supplement Material

Table S1: Please add missing units, e.g., “psu” for S_0 .

Table S1, caption: You may also define the unit Sverdrup in the caption.

Table S2: What are the missing units of the listed salinity contents? Please add.

Table S3: Since the hosing flux H has the unit “Sv” in your figures (e.g., 2b), and the combined fresh-water flux according to equation (11) shall result in “Sv” as well, the unit of the parameters A_i are dubious. Please check.

Bibliography

Madsen, M. S., S. Yang, G. Aðalgeirsdóttir, S. H. Svendsen, C. B. Rodehacke, and I M Ringgaard. 2022. “The Role of an Interactive Greenland Ice Sheet in the Coupled Climate-Ice Sheet Model EC-Earth-PISM.” *Climate Dynamics*, no. 0123456789 (February). https://doi.org/10.1007/s00382-022-06184-6.