Reply to Referee 1

First of all, thank you very much for reviewing our manuscript in detail and giving us very useful feedback. In what follows, we respond to your comments and questions, point by point, and propose several changes to the manuscript. We consider that these changes will substantially improve the quality and clarity of our manuscript.

In order to improve the readability of our replies we applied a color/type coding to discriminate our replies from the referee's comments. We have attached our replies as a pdf document since color coding is not available in the browser based text editor.

Color/type coding:
Comment by the referee.
Reply from the authors.

Page 1, lines 12-13:
I think it's better to briefly describe the influence of obliquity than to name-drop 'vibration-enhanced synchronization' in the abstract.

Yes, we will briefly describe the influence of the obliquity in the revised manuscript: The obliquity variations help the glacial cycles synchronize to ~100-kyr eccentricity cycles more tightly. This effect of obliquity variations is similar to noise-induced phenomena such as stochastic resonance as well as noise-enhanced synchronization. Hence we term it as vibration-enhanced synchronization.

Introduction:
The 100-kyr problem is described in some detail, but the 41-kyr problem is not mentioned at all. I find this odd, as the study is essentially about both problems.

Yes, we will add a description in the revised manuscript.

Page 3, lines 48-49:
It would, if (and only if) it is in fact linked to eccentricity. If it is for example related to 2/3 obliquity cycles, or 4/5 precession cycles, it wouldn't have to involve damping of the longer eccentricity period.

We agree. The frequency locking to ~100-kyr cycles (2~3 obliquity or 4~5 precession cycles) implies the lack of 400-kyr power. However, if it is not frequency locking (for example, if it is linear response), the prominence of 100-kyr power does not necessarily imply the lack of 400-kyr power. In our study, we indeed elucidate that the phenomenon is synchronization (frequency locking). We rephrase the corresponding sentence so that the point is clear.

Page 3, lines 62-63:
Similar time scale, and amplitude as well?

Yes, amplitude as well, although in real phenomena, the amplitude of oscillations often increases because the forcing is often not small.

Page 4, line 91:
I think you can remove this sentence, as a proper scientific discussion naturally involves giving caveats.

Thank you. We follow your suggestion.

Page 5, lines 121-122:
Remove the brackets.

We will re-write the sentence without brackets.

Page 5, lines 124-125: 'the CLIMBER-2 is simulated'
The grammar is incorrect.

Thank you. We correct the sentence as follows. "In each, CLIMBER-2 runs for a fixed astronomical configuration …"

Page 5, lines 126-127:
Not sure if this information is really needed in a non-technical paper.

We understand your point. We will briefly mention the computation cost just in the Supplementary material.

Page 6, lines 155-156:
It is further evidence that changes in the internal dynamics of the Earth system are necessary to explain the MPT in CLIMBER-2.

Thank you for suggesting a proper sentence, restricting the conclusion in CLIMBER-2. We will adopt it.

Page 6, line 157:
'is' should be 'are'

Corrected.

Fig. S7:
Add a cyan line to panel B.

We will add the cyan line in the revised paper.

Page 9, line 172: 'Among others'
Can this be changed to 'chiefly' (or a similar word)?

Yes, we can change it to "Chiefly".

Page 9, lines 190-191:
What is different about a termination? Why is glaciogenic dust deposition sustained during the deglaciation when it wasn't in the interstadials before?

The glaciogenic dust emissions and hence depositions are high when developed ice sheets retreat. The model assumes that the source of the glaciogenic dust emissions is the terrestrial sediments that are exported from the margin of ice sheets and left outside the ice sheet. The exportation of the sediments by an ice sheet is high if the ice sheet spreads over the area covered by thick terrestrial sediments and if most of the ice sheet base over this area is at the pressure melting point enabling fast sliding of the ice sheet (Ganopolski and Calov CP 2011; Ganopolski et al. CP 2010). Also the transported sediments are likely to be exposed to the air when the ice sheet retreats rather than advances.

Page 10, line 196:
A spurious imbalance?

It reflects a long transient dynamics, where the carbon fluxes are still slightly imbalanced. We suppose that the system will eventually achieve a more regular limit cycle behavior without a drift. However, it takes at least more than one million years. Therefore we consider that the oscillatory behavior with the subtle drift is not spurious but an essential character underlying the modeled ice age cycles.

Page 10, line 206: 'oblquity'
Typo.

Corrected

Page 12, line 225: 'two sets of sensitivity experiments'
Add 'additional'

Added.

Figs. 7 and 10:
I found myself drawing lines at x=1.0 (true conditions). Perhaps these can be included in the figures?

Thank you for this suggestion. However the figures are already busy. So we would like to keep these figures as they are.

Page 14, lines 264-269:
I must admit this part is lost on me. I wonder if it is really necessary to compare this mechanism to others, without any further explanation or discussion.

Thank you for pointing out this point. We consider that some link to noise-induced synchronization is useful for introducing our new term "vibration-induced synchronization". In the revision, we will move those comparisons to the Discussion Section or will enlarge the explanation.

Figure 11:
My compliments on this figure, it summarizes the paper perfectly.

Summary and discussion:
In general, I prefer separate discussion and summary sections. The latter can be quite short, just a paragraph. This works better when I just want to check the conclusions of a paper again.

Thank you for this advice. We will consider if separating the summary and discussion fits to this manuscript.

Page 18, line 285: 'suitable'
Perhaps change to 'small', 'specific', or 'limited'.

Thank you. We will change it to 'limited'.

Page 19, line 301:
What in particular is improved in CLIMBER-X, that makes this model more reliable?

CLIMBER-X is improved from CLIMBER-2 in several respects: (1) Resolution. The atmosphere component of CLIMBER-X is a statistical-dynamical model similar to that of CLIMBER-2 but has a substantially higher resolution (5°x5°). The 3-D frictional-geostrophic balance model GOLDSTEIN is employed in CLIMBER-X (Willeit et al. GMD 2022) instead of the 3-basin zonally averaged model in CLIMBER-2. (2) Most parameterizations are improved with available high-quality data. (3) Improved and more detailed carbon cycle processes on land and in the ocean, where the state-of-the-art HAMOCC6 model is employed (Willeit et al. GMD 2023). Mainly those three aspects make CLIMBER-X better to simulate the present and past climate fields and the historical evolution and distribution of carbon contents (Willeit et al. GMD 2022, 2023).

Page 19, lines 303-313:
IcIES-MIROC is climatically forced using a matrix interpolation method with pre-run climate simulations. That's an important difference to CLIMBER-2, which is a fully coupled model. This difference should be mentioned, as it could (in part) explain the difference in results.

Thank you for this valuable comment. We will mention the difference in discussion.

Page 19, lines 312-313:
This is true for the post-MPT period mostly.

Yes, this is true mostly. We write "simulated pre-MPT glacial cycles" for accuracy.

Page 19, line 319: 'Introduction'
Should be 'the introduction'.

Corrected.

Page 19, line 320-322:
What kind of model do Le Treut and Ghil (1983) use? Please briefly explain.

We will add an explanation in the revision. Their model is a simplified physical model consisting of three ordinary differential equations for ice sheet extent, bedrock deflection, and temperature determined by energy balance.

Page 19, lines 326-329:
Reading this, I can't help but wondering what happens if the astronomical forcing as a whole (so obliquity and eccentricity/precession combined) is decreased. Maybe as an idea for a next study, as in principle this article describes enough experiments as it is.

Thank you for this question. It is answered by the experiments corresponding to the diagonal line in Fig. 7. So far we can only assume that the synchronization to the astronomical forcing is lost at some point of forcing amplitude as it decreases. As suggested, we will consider this as an idea for a follow-up study.

Figure B1:
Perhaps, you could include a phase wheel of obliquity and precession during terminations, like Figure 4 (top panels) in Watanabe et al. (2023).

Thank you for this suggestion. The phase wheel can provide more detailed/quantitative information for the timings of terminations. However, we believe that Figure B1 is enough for showing that the timings of terminations are constrained more tightly by precession peaks rather than obliquity peaks. So we would like to keep Figure B1 as it is.