DRAFT Response to Review #1

We appreciate the detailed review and constructive comments of the reviewer. Herein we provide responses point-by-point and explain how we will revise the paper to take them into account.

Page 1:
The first three paragraphs summarize the key issues. The last part of the first paragraph describes the difference between stationary and non-stationary trending series as follows:

“the mean value of a trend stationary time series changes over time because the time series increases/decreases by the same quantity period-after-period. Conversely, the mean value of a time series that contains a unit root changes over time due to accumulation of random changes.”

Since our choice of terminology differs from the reviewer’s it is worth elaborating on this point. The above statement is only true for the unit root case if the random changes have non-zero mean because a unit root process driven by mean zero innovations has a constant expected value (zero), so there is no trend. If the innovations have non-zero mean, then the process does have a linear time trend with mean zero unit root innovations around the deterministic trend. The slope of the linear time trend is often called the ‘drift’ when the unit root process is the special case of a random walk (serially uncorrelated mean zero innovations).

What is important to keep in mind in the analysis of time series processes that exhibit a steady increase is that the increase cannot be explained by the stochastic fluctuations of a unit root process. Instead it is represented by the linear trend component. For cointegration analysis it is the relationship between the stochastic fluctuations among unit root processes that matters. The steady increase represented by the linear trend is a separate component the identification of which is not directly related to determining whether the stochastic components are I(0) or I(1) or whether there is cointegration. Unfortunately, the time series econometrics literature has applied the label 'stochastic trend' to mean zero unit root processes which suggests such a series can systematically increase or decrease. But its mean is constant at zero so its motions should not be called a “trend”. Doing so can lead to spurious conclusions when using trend tests that assume the stochastic component is I(0), which is where the label ‘stochastic trend’ originated. A unit root process only systematically increases or decreases if its mean is increasing or decreasing over time. The simplest example is a random walk with drift which is the sum of a linear trend and a mean zero unit root process with serially uncorrelated innovations. When the innovations are serially correlated, the label random walk with drift no longer applies, but there is no agreed-upon alternative. In such a case we use “trend nonstationary” because this label makes it clear that the series has a deterministic trend (the drift from the first differenced representation) and I(1) fluctuations (but not necessarily random walk) around the trend. We avoid the label stochastic trend because that implies a mean zero I(1) process that is not trending.

Paragraphs 4&5: critique of our forcings data
We use 2 forcings data sets, the CMIP5 input series (shown in Fig 3) and the model-generated temperature counterfactuals from C22. The referee asks why we didn’t use the longer time series available from the David Stern website. One reason is that these data only go up to 2011. Another is
that they predate a decade’s worth of research on the forcing strength of aerosol emissions, which have led to downward revisions of IPCC consensus estimates.

The referee notes that our aerosol series does not show a large magnitude of net forcing. Figure 3 shows each forcing with the same vertical axis for comparison. Apparent flatness of the aerosol forcing compared to the GHG forcing reflects the fact that the CMIP5 series has a smaller range of values than previous estimates. Since the IPCC AR4 in 2007, the scientific literature has revised the strength of aerosol cooling downwards (or, since the forcing estimate is negative, revised the central estimate upward). The IPCC AR5 WG1 noted this change in the Working Group I report of the AR5 (IPCC 2013 p. 574).

Subsequently there have been numerous papers confirming the reduced aerosol forcing strength with particular focus on aerosol-cloud interactions turning out to be weaker than previously thought and weaker than is typically represented in many climate models. Detailed reviews are provided in Lewis and Curry (2018 see p. 6055) and in Lewis (2022) SI Section 5.2.4. Recent reconstructions of pre-industrial wildfire-related aerosol emissions have likewise reduced the implied net aerosol forcing estimates (Hamilton et al. 2018, Liu et al. 2021).

Therefore it would not make sense for us to use pre-CMIP5 aerosol forcing estimates. Even the CMIP5 aerosol forcings may be too large in absolute magnitude but, as we explain, there is no single, convenient series available in the CMIP6 archive.

Other points:
- We didn’t linearize the forcings. They are centered on 0 but otherwise used as-is
- We find evidence that the forcings are I(1) or I(2) which is the same as Kaufmann and Stern so we don’t see any disagreement on that point
- Our cointegration analysis uses the Cummins et al (2022) model-generated counterfactual temperature simulations rather than the forcing series themselves, based on the argument in C22 that this is a valid substitution. The specific discussion point in that section requires that we use the C22 data, and once again it is a more recent data series and would be more likely to reflect the post-2010 revision in aerosol forcing estimates.
different orders of integration. The property of the summed series will depend on which component is dominant. In Appendix B we show how a large amount of I(0) noise overlaid on a smooth I(1) series can cause a unit root test to be biased towards rejecting the I(1) null, and if this is the case, de-noising in the form of averaging out the I(0) noise process should move the test results towards non-rejection. Our empirical results provide evidence for this effect in the model data case, since the de-noised model data moves towards the non-rejection region, although we still reject the unit root null against a broken trend alternative. No such effect is observed with the observational data. Regarding the forcings, we agree that they are I(1) or I(2).

We are aware that our test scores on model data contradict previous findings, such as Kaufmann et al. (2013). It should be noted that we are using different models, different tests and a different time interval so obtaining the same results is not guaranteed. We agree however that we did an inadequate job of positioning our work in the context of previous findings and explaining why our results differ in some cases. We will remedy this by including a more extensive discussion of relevant prior studies, including Gay-Garcia et al. (2009), Kaufmann et al. (2010) & (2013), Mills (2010) and Estrada and Perron (2019).

**Paragraph 3: contrast with earlier literature**

The reviewer is correct that we did not adequately discuss the earlier literature on whether temperatures are trend-with-break+I(0) errors or nonstationary. Specifically the findings of Gay-Garcia et al. (2009) and the subsequent debate with Kaufmann et al. (2010) and the comment of Mills (2010) are relevant, as is the recent work of Estrada and Perron (2019). The latter, for instance, argues that both temperatures and forcings are trend-with-break+I(0), whereas most climate econometrics authors follow Kaufmann et al. (2010) in arguing that both are I(1). We find that temperatures are trend-with-break+I(0) while anthropogenic forcings are I(1) or I(2) and propose that this is the puzzle needing to be explained, if it is assumed that temperatures inherit the stationarity properties of anthropogenic forcings. Our theoretical and empirical results propose one possible explanation.

The reviewer refers to the in-sample test used in K10, in which they found that in the NH (but not in the SH) a cointegrating model based on I(1) temperatures achieves a better in-sample fit than the trend-with-break+I(0) errors. Such a test is more pertinent in forecasting applications, but that is not what we are doing. We are using the trend+break specification as an approximation to the steady increase in temperatures and forcings for the purposes of studying whether the random fluctuations are better characterized as I(0) or I(1). This would be a standard preliminary analysis before building a forecasting model that relates temperatures to forcings and would be informative as to whether or not a cointegrating model is reasonable. It’s not surprising that the cointegration model used by Kaufmann et al (2010) gives a better fit than the trend+break model. The trend+break fitted model in their Figure 1 does not model any dynamics in the random component (for instance lagged temperatures) and does not include covariates that can help explain temperatures. The cointegration model has both and would be expected to give a better fit regardless of whether or not there is cointegration in the stochastic components.

**Paragraph 4: contrast with S&K 2000**

The analysis in Stern and Kaufmann (2000) applies a different technique (structural time series analysis) on a different data set and answers different questions. We don’t think it is necessary, in principle, for every study on the topic to use the same methods, as long as the methods used in a study are appropriate for the questions being asked and the data being used, which is the case in
our study. However the substantive question raised by S&K2000, namely whether the NH and SH behave differently and the absence of a property (e.g. nonstationarity) at the global level may mask its presence in the hemispheres in a form that cancels out in the aggregate, is valid. We will examine this question by re-doing our analysis on the NH alone to see if different results emerge. We are in the process of assembling this data set.

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Paragraph 1: use of C22 data

Here again the main question being posed can be addressed by redoing our analysis on NH data, which we will do.

The reviewer critiques our use of climate model-generated data for the cointegration analysis in Section 3.5. But these are the data used in Cummins et al. 2022 and it is necessary for us to use them. It is valid to point out that this method presupposes the validity of climate models, but this is a weakness of many attribution methods in climatology. We find evidence of cointegration of the model series with each other but not with temperatures. This finding is useful for reconciling our results with the stationarity of the error terms in the C22 regression model, which they interpreted to mean successful signal detection but without having established that the observed temperatures themselves were I(1). The reviewer makes a rather strong assertion in claiming that any failure to find cointegration must mean that the models that generated the data are invalid, since the data must be cointegrated. This amounts to assuming the conclusion. Furthermore, if the models are indeed invalid, the problem may be that they overstate the coupling between forcings and temperature, in which case stationarity of the observed temperatures would be expected.

Paragraph 2: critique of Dergiades result

This is a fair comment. The paragraph will be removed. However we need to address the Dergiades result and we will do so in a more formal way. The Mann (equivalently Wahl and Ammann) reconstruction is available back to 1000 AD. We can replicate the Dergiades result on the post-1700 segment showing that the unit root test wanders into the non-rejection region. However it spends long intervals in the non-rejection region in the pre-1600 interval as well, even though the underlying hypothesis (that non-rejection is due to the emerging dominance of anthropogenic forcings) rules this out. This raises the question of whether the proxy reconstruction is suitable for the purpose, and here it is valid to point out that the reconstruction is a splice of many segments composed of differing proxy rosters, and the discontinuities may cause problems for the types of tests being used, which is a point raised by Pretis and Hendry (2013) in a similar context.

Paragraph 3: terminology

The reviewer is correct that a random walk with drift is the conventional label for a time series process that is the sum of a linear deterministic trend and a random walk stochastic process. But, a random walk is a special case of a unit root process in which the innovations are serially uncorrelated. This is why we prefer the label trend-nonstationary because it does not imply the random walk special case. The ‘trend’ part of the label refers to the deterministic trend and the ‘nonstationary’ part of the label refers to the mean zero unit root stochastic component which may be serially correlated.
Paragraph 4: no need to ask about conflicting results
It's not clear what is the objection to asking the question, since obtaining conflicting test results is very common. We ask the question to motivate the discussion of the required steps in constructing a valid unit root test and discriminating among conflicting inferences especially with respect to choice of lag length for the ADF regressions.

Sources:


IPCC Fourth Assessment Report (2007), Working Group I

IPCC Fifth Assessment Report (2013), Working Group II


Liu, Pengfei et al. (2021) Improved estimates of preindustrial biomass burning reduce the magnitude of aerosol climate forcing in the Southern Hemisphere. Science Advances 7(22) DOI: 10.1126/sciadv.abc1379 https://www.science.org/doi/10.1126/sciadv.abc1379

