Supplementary Information for “Origins and suppression of bifurcation phenomena in lower-order monsoon models”

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1 Generic 6-d system from QTCM equations

The standard case of Boos and Storelvmo [2016], with stratification included, along with the assumptions of no nonlinear advection in momentum equation, neglect of rotation and zonal velocity components, zonal symmetry etc. is derived from the partial differential equations (PDEs) of QTCM (see Fig.S1). The equations are reduced to a set of ordinary differential equations using finite differences (FD), and the resulting equations are given separately for the non-zero precipitation case and zero precipitation case. The boundary conditions are then substituted, and the resulting equations are presented separately for the two cases. This is followed by further applications of the assumptions. The assumptions for the standard case are neglect of zonal and barotropic velocity components, neglect of rotation, neglect of nonlinear advection in the momentum equations, neglect of time derivatives and zonal symmetry. These assumptions yield a momentum equation for baroclinic meridional velocity which permits a thermally direct flow as assumed by Boos and Storelvmo [2016]. The resultant PDEs are as

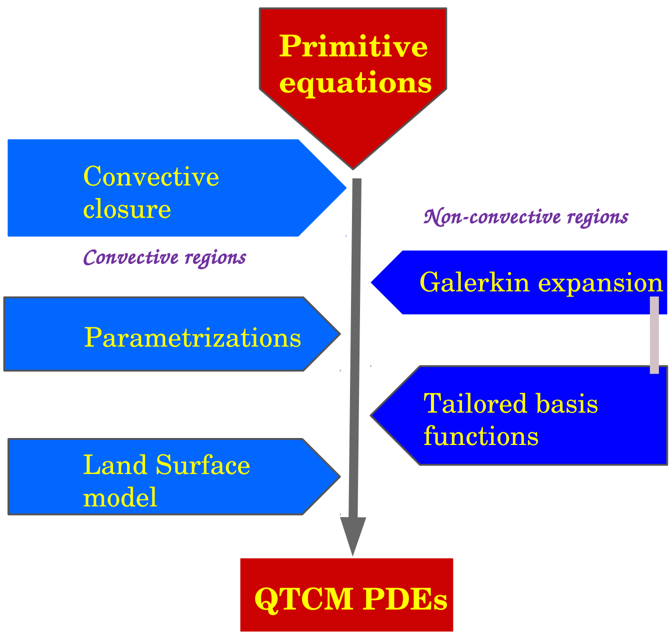

Figure S1: Flowchart of operations within QTCM framework.

follows:

<u>QTCM PDEs:</u>

baroclinic zonal velocity equation

$$\frac{\partial u_1}{\partial t} + \frac{\partial T1}{\partial x}\kappa + \frac{\partial v_1}{\partial x}v_0 + \epsilon_{01}u_0 + \epsilon_1 u_1 - fv_1 - K_H(\frac{\partial^2 u_1}{\partial x^2} + \frac{\partial^2 u_1}{\partial y^2}) + \frac{\partial u0}{\partial x}u_1 + \frac{\partial u_1}{\partial x}u_0 + \frac{\partial v0}{\partial x}v_1 + (\frac{\partial u_1}{\partial x}\langle v_1^3 \rangle u_1)/\langle v_1^2 \rangle - (\frac{\partial u_1}{\partial x}\langle v_1 \Omega \partial_p v_1 \rangle u_1)/\langle v_1^2 \rangle - (\frac{\partial v_1}{\partial y}\langle v_1 \Omega \partial_p v_1 \rangle u_1)/\langle v_1^2 \rangle + (\frac{\partial v_1}{\partial x}\langle v_1^3 \rangle v_1)/\langle v_1^2 \rangle = 0$$

baroclinic meridional velocity equation

$$\frac{\partial v_1}{\partial t} + \frac{\partial T_1}{\partial y}\kappa - fu_1 + \epsilon_{01}v_0 + \epsilon_1 v_1 - K_H(\frac{\partial^2 v_1}{\partial x^2} + \frac{\partial^2 v_1}{\partial y^2}) + \frac{\partial u_0}{\partial y}u_1 + \frac{\partial u_1}{\partial y}u_0 + \frac{\partial v_0}{\partial y}v_1 + \frac{\partial v_1}{\partial y}v_0 + (\frac{\partial u_1}{\partial y}\langle v_1^3 \rangle u_1)/\langle v_1^2 \rangle + (\frac{\partial v_1}{\partial y}\langle v_1^3 \rangle v_1)/\langle v_1^2 \rangle - (\frac{\partial u_1}{\partial x}\langle v_1 \Omega \partial_p v_1 \rangle v_1)/\langle v_1^2 \rangle - (\frac{\partial v_1}{\partial y}\langle v_1 \Omega \partial_p v_1 \rangle v_1)/\langle v_1^2 \rangle = 0$$

barotropic zonal velocity equation

$$\frac{\partial \phi_0}{\partial x} + \frac{\partial u_0}{\partial t} + \frac{\partial u_0}{\partial x}u_0 + \frac{\partial v_0}{\partial x}v_0 + \epsilon_0 u_0 + \epsilon_{10}u_1 - fv_0 - K_H(\frac{\partial^2 u_0}{\partial x^2} + \frac{\partial^2 u0}{\partial y^2}) + \langle v_1^2 \rangle(\frac{\partial u_1}{\partial x}u_1 + \frac{\partial v1}{\partial y}u_1) + \langle v_1^2 \rangle(\frac{\partial u_1}{\partial x}u_1 + \frac{\partial v_1}{\partial x}v_1) = 0 \tag{1}$$

barotropic meridional velocity equation

$$\frac{\partial \phi_0}{\partial y} + \frac{\partial v_0}{\partial t} + \frac{\partial u_0}{\partial y}v_0 + \frac{\partial v_0}{\partial y}v_0 + fu_0 + \epsilon_0 v_0 + \epsilon_{10}v_1 - K_H(\frac{\partial^2 v_0}{\partial x^2} + \frac{\partial^2 v_0}{\partial y^2}) + \langle v_1^2 \rangle(\frac{\partial u_1}{\partial y}u_1 + \frac{\partial v_1}{\partial y}v_1) + \langle v_1^2 \rangle(\frac{\partial u_1}{\partial x}v_1 + \frac{\partial v_1}{\partial y}v_1) = 0$$

Thermodynamic equation

$$M_{s1}\frac{\partial u_1}{\partial x} - P + M_{s1}\frac{\partial v_1}{\partial y} + \hat{a}_1\frac{\partial T_1}{\partial t} - (g(H+R))/p_t - K_H\hat{a}_1\frac{\partial^2 T_1}{\partial x^2} - K_H\hat{a}_1\frac{\partial^2 T_1}{\partial y^2} + \hat{a}_1\frac{\partial T_1}{\partial x}u_0 + \hat{a}_1\frac{\partial T_1}{\partial y}v_0 + \frac{\partial T1}{\partial x}\langle a_1 v_1 \rangle u_1 + \frac{\partial T1}{\partial y}\langle a_1 v_1 \rangle v_1 = 0$$

Moisture equation

$$P - M_{q1}\frac{\partial u_1}{\partial x} - M_{q_1}\frac{\partial v_1}{\partial y} + \hat{b}_1\frac{\partial q_1}{\partial t} - K_H\hat{b}_1\frac{\partial q_1}{\partial x^2} - K_H\hat{b}_1\frac{\partial^2 q_1}{\partial y^2} - (Eg)/p_t + \hat{b}_1\frac{\partial q_1}{\partial x}u_0 + \hat{b}_1\frac{\partial q_1}{\partial y}v_0 + \frac{\partial q_1}{\partial x}\langle b_1 v_1 \rangle u_1 + \frac{\partial q_1}{\partial y}\langle b_1 v_1 \rangle v_1 = 0$$

Application of finite differences for spatial derivatives in these PDEs yields the following sets of equation, one each for $P = 0$ and $P > 0$ cases:

Case of non-zero precipitation

Baroclinic zonal velocity equation: FD form

$$\dot{u}_{1A}/8 + \dot{u}_{1B}/8 + \dot{u}_{1C}/8 + \dot{u}_{1D}/8 + \epsilon_{01}u_{0C} + \epsilon_1 u_{1C} - f v_{1C} + (v_{0C}(v_{1B} - v_{1D}))/L + (\kappa(T_{1B} - T_{1D}))/L + (u_{1C}(u_{0B} - u_{0D}))/L + (u_{0C}(u_{1B} - u_{1D}))/L + (v_{1C}(v_{0B} - v_{0D}))/L + (\langle v_1^3\rangle u_{1C}(u_{1B} - u_{1D}))/(L\langle v_1^2\rangle) - (\langle v_1\Omega\partial_p v_1\rangle u_{1C}(u_{1B} - u_{1D}))/(L\langle v_1^2\rangle) - (\langle v_1\Omega\partial_p v_1\rangle u_{1C}(v_{1A} - v_{1C}))/(L\langle v_1^2\rangle) + (\langle v_1^3\rangle v_{1C}(v_{1B} - v_{1D}))/(L\langle v_1^2\rangle) = 0$$

Baroclinic meridional velocity equation: FD form

$$\dot{v}_{1A}/8 + \dot{v}_{1B}/8 + \dot{v}_{1C}/8 + \dot{v}_{1D}/8 - f u_{1C} + \epsilon_{01}v_{0C} + \epsilon_1 v_{1C} + (\kappa(T_{1A} - T_{1C}))/L + (u_{1C}(u_{0A} - u_{0C}))/L + (u_{0C}(u_{1A} - u_{1C}))/L + (v_{1C}(v_{0A} - v_{0C}))/L + (v_{0C}(v_{1A} - v_{1C}))/L + (\langle v_1^3\rangle u_{1C}(u_{1A} - u_{1C}))/(L\langle v_1^2\rangle) - (\langle v_1\Omega\partial_p v_1\rangle v_{1C}(u_{1B} - u_{1D}))/(L\langle v_1^2\rangle) + (\langle v_1^3\rangle v_{1C}(v_{1A} - v_{1C}))/(L\langle v_1^2\rangle) - (\langle v_1\Omega\partial_p v_1\rangle v_{1C}(v_{1A} - v_{1C}))/(L\langle v_1^2\rangle) = 0$$

Barotropic zonal velocity equation: FD form

$$\frac{\partial\phi_0}{\partial x} + \dot{u}_{0A}/8 + \dot{u}_{0B}/8 + \dot{u}_{0C}/8 + \dot{u}_{0D}/8 + \epsilon_0 u_{0C} + \epsilon_{10}u_{1C} - f v_{0C} + (u_{0C}(u_{0B} - u_{0D}))/L + (v_{0C}(v_{0B} - v_{0D}))/L + \langle v_1^2\rangle((u_{1C}(u_{1B} - u_{1D}))/L + (u_{1C}(v_{1A} - v_{1C}))/L) + \langle v_1^2\rangle((u_{1C}(u_{1B} - u_{1D}))/L + (v_{1C}(v_{1B} - v_{1D}))/L) = 0$$

Barotropic meridional velocity equation: FD form

$$\frac{\partial\phi_0}{\partial y} + \dot{v}_{0A}/8 + \dot{v}_{0B}/8 + \dot{v}_{0C}/8 + \dot{v}_{0D}/8 + f u_{0C} + \epsilon_0 v_{0C} + \epsilon_{10}v_{1C} + (v_{0C}(u_{0A} - u_{0C}))/L + (v_{0C}(v_{0A} - v_{0C}))/L + \langle v_1^2\rangle((u_{1C}(u_{1A} - u_{1C}))/L + (v_{1C}(v_{1A} - v_{1C}))/L) + \langle v_1^2\rangle((v_{1C}(u_{1B} - u_{1D}))/L + (v_{1C}(v_{1A} - v_{1C}))/L) = 0$$

Thermodynamic equation: FD form

$$\hat{a}_1(\dot{T}_{1A}/8 + \dot{T}_{1B}/8 + \dot{T}_{1C}/8 + \dot{T}_{1D}/8) - P - (g(H + R))/p_t + ((u_{1B} - u_{1D})(M_{sr} + M_{sp}T_1))/L + ((v_{1A} - v_{1C})(M_{sr} + M_{sp}T_1))/L + (\hat{a}_1 u_{0C}(T_{1B} - T_{1D}))/L + (\hat{a}_1 v_{0C}(T_{1A} - T_{1C}))/L + (\langle a_1 v_1\rangle u_{1C}(T_{1B} - T_{1D}))/L + (\langle a_1 v_1\rangle v_{1C}(T_{1A} - T_{1C}))/L = 0$$

Moisture equation: FD form

$$P + \hat{b}_1(\dot{q}_{1A}/8 + \dot{q}_{1B}/8 + \dot{q}_{1C}/8 + \dot{q}_{1D}/8) - ((u_{1B} - u_{1D})(M_{qr} + M_{qp}q_1))/L - ((v_{1A} - v_{1C})(M_{qr} + M_{qp}q_1))/L - (Eg)/p_t + (\hat{b}_1 u_{0C}(q_{1B} - q_{1D}))/L + (\hat{b}_1 v_{0C}(q_{1A} - q_{1C}))/L + (\langle b_1 v_1\rangle u_{1C}(q_{1B} - q_{1D}))/L + (\langle b_1 v_1\rangle v_{1C}(q_{1A} - q_{1C}))/L = 0 \tag{2}$$

Case of no precipitation

Baroclinic zonal velocity equation: FD form

$$\dot{u}_{1A}/8+\dot{u}_{1B}/8+\dot{u}_{1C}/8+\dot{u}_{1D}/8+\epsilon_{01}u_{0C}+\epsilon_1 u_{1C}-fv_{1C}+(v_{0C}(v_{1B}-v_{1D}))/L+$$
$$(\kappa(T_{1B}-T_{1D}))/L+(u_{1C}(u_{0B}-u_{0D}))/L+(u_{0C}(u_{1B}-u_{1D}))/L+(v_{1C}(v_{0B}-v_{0D}))/L+$$
$$((\langle v_1^3\rangle u_{1C}(u_{1B}-u_{1D}))/(L\langle v_1^2\rangle)-((\langle v_1\Omega\partial_p v_1\rangle u_{1C}(u_{1B}-u_{1D}))/(L\langle v_1^2\rangle)-$$
$$((\langle v_1\Omega\partial_p v_1\rangle u_{1C}(v_{1A}-v_{1C}))/(L\langle v_1^2\rangle)+$$
$$((\langle v_1^3\rangle v_{1C}(v_{1B}-v_{1D}))/(L\langle v_1^2\rangle)=0.$$

Baroclinic meridional velocity equation: FD form

$$\dot{v}_{1A}/8+\dot{v}_{1B}/8+\dot{v}_{1C}/8+\dot{v}_{1D}/8-fu_{1C}+\epsilon_{01}v_{0C}+\epsilon_1 v_{1C}+(\kappa(T_{1A}-T_{1C}))/L+$$
$$(u_{1C}(u_{0A}-u_{0C}))/L+(u_{0C}(u_{1A}-u_{1C}))/L+(v_{1C}(v_{0A}-v_{0C}))/L+(v_{0C}(v_{1A}-v_{1C}))/L+$$
$$((\langle v_1^3\rangle u_{1C}(u_{1A}-u_{1C}))/(L\langle v_1^2\rangle)-((\langle v_1\Omega\partial_p v_1\rangle v_{1C}(u_{1B}-u_{1D}))/(L\langle v_1^2\rangle)+$$
$$((\langle v_1^3\rangle v_{1C}(v_{1A}-v_{1C}))/(L\langle v_1^2\rangle)-((\langle v_1\Omega\partial_p v_1\rangle v_{1C}(v_{1A}-v_{1C}))/(L\langle v_1^2\rangle)=0.$$

Barotropic zonal velocity equation: FD form

$$\frac{\partial\phi_0}{\partial x}+\dot{u}_{0A}/8+\dot{u}_{0B}/8+\dot{u}_{0C}/8+\dot{u}_{0D}/8+\epsilon_0 u_{0C}+\epsilon_{10}u_{1C}-fv_{0C}+(u_{0C}(u_{0B}-u_{0D}))/L+$$
$$(v_{0C}(v_{0B}-v_{0D}))/L+\langle v_1^2\rangle((u_{1C}(u_{1B}-u_{1D}))/L+$$
$$(u_{1C}(v_{1A}-v_{1C}))/L)+\langle v_1^2\rangle((u_{1C}(u_{1B}-u_{1D}))/L+(v_{1C}(v_{1B}-v_{1D}))/L)=0.$$

Barotropic meridional velocity equation: FD form

$$\frac{\partial\phi_0}{\partial y}+\dot{v}_{0A}/8+\dot{v}_{0B}/8+\dot{v}_{0C}/8+\dot{v}_{0D}/8+fu_{0C}+\epsilon_0 v_{0C}+\epsilon_{10}v_{1C}+(v_{0C}(u_{0A}-u_{0C}))/L+$$
$$(v_{0C}(v_{0A}-v_{0C}))/L+\langle v_1^2\rangle((u_{1C}(u_{1A}-u_{1C}))/L+$$
$$(v_{1C}(v_{1A}-v_{1C}))/L)+\langle v_1^2\rangle((v_{1C}(u_{1B}-u_{1D}))/L+(v_{1C}(v_{1A}-v_{1C}))/L)=0.$$

Thermodynamic equation: FD form

$$\hat{a}_1(\dot{T}_{1A}/8+\dot{T}_{1B}/8+\dot{T}_{1C}/8+\dot{T}_{1D}/8)-P_{np}-(g(H+R))/p_t+$$
$$((u_{1B}-u_{1D})(M_{sr}+M_{sp}T_1))/L+((v_{1A}-v_{1C})(M_{sr}+M_{sp}T_1))/L+(\hat{a}_1u_{0C}(T_{1B}-T_{1D}))/L+$$
$$(\hat{a}_1v_{0C}(T_{1A}-T_{1C}))/L+(\langle a_1v_1\rangle u_{1C}(T_{1B}-T_{1D}))/L+(\langle a_1v_1\rangle v_{1C}(T_{1A}-T_{1C}))/L=0.$$

Moisture equation: FD form

$$P_{np}+\hat{b}_1(\dot{q}_{1A}/8+\dot{q}_{1B}/8+\dot{q}_{1C}/8+\dot{q}_{1D}/8)-((u_{1B}-u_{1D})(M_{qr}+M_{qp}q_1))/L-$$
$$((v_{1A}-v_{1C})(M_{qr}+M_{qp}q_1))/L-(Eg)/p_t+\hat{b}_1u_{0C}(q_{1B}-q_{1D}))/L+$$
$$(\hat{b}_1v_{0C}(q_{1A}-q_{1C}))/L+(\langle b_1v_1\rangle u_{1C}(q_{1B}-q_{1D}))/L+(\langle b_1v_1\rangle v_{1C}(q_{1A}-q_{1C}))/L=0. \quad (3)$$

These FD forms are general simplification of QTCM PDEs into ODEs. The resultant sets of simplified ODEs for the $P=0$ case and $P>0$ case, after application of boundary conditions (land-ocean-ocean-ocean) are as follows:

Non zero precipitation Case

Baroclinic zonal velocity equation: FD form BCs substituted

$(3\dot{u}_{1s})/8 + \epsilon_{01}u_{0s} + \epsilon_1 u_{1s} - f v_{1s} + (\langle v_1 \Omega \partial_p v_1 \rangle u_{1s} v_{1s})/(L\langle v_1^2 \rangle) = 0$

Baroclinic meridional velocity equation: FD form BCs substituted

$(3\dot{v}_{1s})/8 - f u_{1s} + \epsilon_{01}v_{0s} + \epsilon_1 v_{1s} + (T_{1L}\kappa)/L - (T_{1s}\kappa)/L - (2u_{0s}u_{1s})/L - (2v_{0s}v_{1s})/L -$
$(\langle v_1^3 \rangle u_{1s}^2)/(L\langle v_1^2 \rangle) - (\langle v_1^3 \rangle v_{1s}^2)/(L\langle v_1^2 \rangle) + (\langle v_1 \Omega \partial_p v_1 \rangle v_{1s}^2)/(L\langle v_1^2 \rangle) = 0$

Barotropic zonal velocity equation: FD form BCs substituted

$\frac{\partial \phi_0}{\partial x} + (3\dot{u}_{0s})/8 + \epsilon_0 u_{0s} + \epsilon_{10}u_{1s} - f v_{0s} - (\langle v_1^2 \rangle u_{1s} v_{1s})/L = 0$

Barotropic meridional velocity equation: FD form BCs substituted (4)

$\frac{\partial \phi_0}{\partial y} + (3\dot{v}_{0s})/8 + f u_{0s} + \epsilon_0 v_{0s} + \epsilon_{10}v_{1s} - v_{0s}^2/L - (u_{0s}v_{0s})/L - (\langle v_1^2 \rangle u_{1s}^2)/L -$
$(2\langle v_1^2 \rangle v_{1s}^2)/L = 0$

Thermodynamic equation: FD form BCs substituted

$(\dot{T}_{1L}\hat{a}_1)/8 + (3\dot{T}_{1s}\hat{a}_1)/8 + T_{1L}/\tau_c - q_{1L}/\tau_c - (M_{sr}v_{1s})/L - (Hg)/p_t - (M_{sp}T_{1L}v_{1s})/L -$
$(Rg)/p_t +$
$(T_{1L}\hat{a}_1 v_{0s})/L - (T_{1s}\hat{a}_1 v_{0s})/L + (T_{1L}\langle a_1 v_1 \rangle v_{1s})/L - (T_{1s}\langle a_1 v_1 \rangle v_{1s})/L = 0.$

Moisture equation: FD form BCs substituted

$(\hat{b}_1 \dot{q}_{1L})/8 + (3\hat{b}_1 \dot{q}_{1s})/8 - T_{1L}/\tau_c + q_{1L}/\tau_c + (M_{qr}v_{1s})/L - (Eg)/p_t + (M_{qp}q_{1L}v_{1s})/L +$
$(\hat{b}_1 q_{1L} v_{0s})/L - (\hat{b}_1 q_{1s} v_{0s})/L + (\langle b_1 v_1 \rangle q_{1L} v_{1s})/L - (\langle b_1 v_1 \rangle q_{1s} v_{1s})/L = 0.$

Zero precipitation Case

Baroclinic zonal velocity equation: FD form BCs substituted

$(3\dot{u}_{1s})/8 + \epsilon_{01}u_{0s} + \epsilon_1 u_{1s} - f v_{1s} + (\langle v_1 \Omega \partial_p v_1 \rangle u_{1s} v_{1s})/(L\langle v_1^2 \rangle) = 0.$

Baroclinic meridional velocity equation: FD form BCs substituted

$(3\dot{v}_{1s})/8 - f u_{1s} + \epsilon_{01}v_{0s} + \epsilon_1 v_{1s} + (T_{1L}\kappa)/L - (T_{1s}\kappa)/L - (2u_{0s}u_{1s})/L - (2v_{0s}v_{1s})/L -$
$(\langle v_1^3 \rangle u_{1s}^2)/(L\langle v_1^2 \rangle) - (\langle v_1^3 \rangle v_{1s}^2)/(L\langle v_1^2 \rangle) + (\langle v_1 \Omega \partial_p v_1 \rangle v_{1s}^2)/(L\langle v_1^2 \rangle) = 0.$

Barotropic zonal velocity equation: FD form BCs substituted

$\frac{\partial \phi_0}{\partial x} + (3\dot{u}_{0s})/8 + \epsilon_0 u_{0s} + \epsilon_{10}u_{1s} - f v_{0s} - (\langle v_1^2 \rangle u_{1s} v_{1s})/L = 0.$

Barotropic meridional velocity equation: FD form BCs substituted (5)

$\frac{\partial \phi_0}{\partial y} + (3\dot{v}_{0s})/8 + f u_{0s} + \epsilon_0 v_{0s} + \epsilon_{10}v_{1s} - v_{0s}^2/L - (u_{0s}v_{0s})/L - (\langle v_1^2 \rangle u_{1s}^2)/L -$
$(2\langle v_1^2 \rangle v_{1s}^2)/L = 0.$

Thermodynamic equation: FD form BCs substituted

$(\dot{T}_{1L}\hat{a}_1)/8 + (3\dot{T}_{1s}\hat{a}_1)/8 - (M_{sr}v_{1s})/L - (Hg)/p_t - (M_{sp}T_{1L}v_{1s})/L - (Rg)/p_t +$
$(T_{1L}\hat{a}_1 v_{0s})/L - (T_{1s}\hat{a}_1 v_{0s})/L + (T_{1L}\langle a_1 v_1 \rangle v_{1s})/L - (T_{1s}\langle a_1 v_1 \rangle v_{1s})/L = 0.$

Moisture equation: FD form BCs substituted

$(\hat{b}_1 \dot{q}_{1L})/8 + (3\hat{b}_1 \dot{q}_{1s})/8 + (M_{qr}v_{1s})/L - (Eg)/p_t + (M_{qp}q_{1L}v_{1s})/L + (\hat{b}_1 q_{1L} v_{0s})/L -$
$(\hat{b}_1 q_{1s} v_{0s})/L + (\langle b_1 v_1 \rangle q_{1L} v_{1s})/L - (\langle b_1 v_1 \rangle q_{1s} v_{1s})/L = 0.$

Since the present study deals exclusively with the equilibria in the system, these equations can be simplified further by ignoring the time derivatives, and be simplified into one algebraic equation each for the $P = 0$ case and $P > 0$. These are as follows:

Simplified equations after assumptions:
Non zero precipitation Case
Baroclinic meridional velocity equation: FD form with BCs & assumptions
$\epsilon_1 v_{1s} + (T_{1L}\kappa)/L - (T_{1s}\kappa)/L = 0.$
Thermodynamic velocity equation: FD form with BCs & assumptions
$T_{1L}/\tau_c - q_{1L}/\tau_c - (M_{sr}v_{1s})/L - (Hg)/p_t - (M_{sp}T_{1L}v_{1s})/L - (Rg)/p_t +$
$(T_{1L}\langle a_1 v_1\rangle v_{1s})/L - (T_{1s}\langle a_1 v_1\rangle v_{1s})/L = 0.$ (6)
Moisture velocity equation: FD form with BCs & assumptions
$q_{1L}/\tau_c - T_{1L}/\tau_c + (M_{qr}v_{1s})/L - (Eg)/p_t + (M_{qp}q_{1L}v_{1s})/L + (\langle b_1 v_1\rangle q_{1L}v_{1s})/L -$
$(\langle b_1 v_1\rangle q_{1s}v_{1s})/L = 0.$

Zero precipitation Case
Baroclinic meridional velocity equation: FD form with BCs & assumptions
$\epsilon_1 v_{1s} + (T_{1L}\kappa)/L - (T_{1s}\kappa)/L = 0.$
Thermodynamic velocity equation: FD form with BCs & assumptions
$(1T_{1L}\langle a_1 v_1\rangle v_{1s})/L - (Hg)/p_t - (M_{sp}T_{1L}v_{1s})/L - (Rg)/p_t -$ (7)
$(M_{sr}v_{1s})/L - (T_{1s}\langle a_1 v_1\rangle v_{1s})/L = 0.$
Moisture velocity equation: FD form with BCs & assumptions
$(M_{qr}v_{1s})/L - (Eg)/p_t + (M_{qp}q_{1L}v_{1s})/L + (\langle b_1 v_1\rangle q_{1L}v_{1s})/L - (\langle b_1 v_1\rangle q_{1s}v_{1s})/L = 0.$

The final algebraic equation to yield the equilibria can be expressed in terms of one of the three unknown variables. Here, it is presented in terms of the meridional velocity component, for the two cases of precipitation (see Eqns. 8). Note that the equation for the case of zero precipitation is innately quadratic equation. The equation for the non-zero precipitation case is cubic. However, note the the cubic equation, when $M_{qp} = -\langle b_1 v_1\rangle$, becomes quadratic too. This explains the emergence of quadratic solutions, or saddle-nodes for the standard case with stratification. The suppression of stratification too doesn't change the quadratic nature, only changing the coefficients of the terms. This explains why quantitative but not qualitative changes in the bifurcation structure occurs for the suppressed stratification case. Note, however, that the physical relevance or irrelevance of the equilibria lead to the final interpretation being far different.

Final 1-d equations:
P not zero: cubic equation
$(LM_{qp}M_{sp}\epsilon_1 p_t\tau_c - LM_{qp}\epsilon_1\langle a_1 v_1\rangle p_t\tau_c + LM_{sp}\epsilon_1\langle b_1 v_1\rangle p_t\tau_c - L\epsilon_1\langle a_1 v_1\rangle\langle b_1 v_1\rangle p_t\tau_c)v_{1s}^3 +$
$(L^2M_{sp}\epsilon_1 p_t - L^2M_{qp}\epsilon_1 p_t - L^2\epsilon_1\langle a_1 v_1\rangle p_t - L^2\epsilon_1\langle b_1 v_1\rangle p_t - M_{qp}M_{sr}\kappa p_t\tau_c - M_{sr}\kappa\langle b_1 v_1\rangle p_t\tau_c -$
$M_{qp}M_{sp}T_{1s}\kappa p_t\tau_c - M_{sp}T_{1s}\kappa\langle b_1 v_1\rangle p_t\tau_c)v_{1s}^2 + (LM_{qr}\kappa p_t - LM_{sr}\kappa p_t + LM_{qp}T_{1s}\kappa p_t -$ (8)
$LM_{sp}T_{1s}\kappa p_t + LT_{1s}\kappa\langle b_1 v_1\rangle p_t - L\kappa\langle b_1 v_1\rangle p_t q_{1s} - HLM_{qp}g\kappa\tau_c - HLg\kappa\langle b_1 v_1\rangle\tau_c -$
$LRg\kappa\langle b_1 v_1\rangle\tau_c - LM_{qp}Rg\kappa\tau_c)v_{1s} - (EL^2g\kappa + HL^2g\kappa + L^2Rg\kappa) = 0.$
P=0: quadratic equation
$((M_{sp}\epsilon_1)/\kappa - (\epsilon_1\langle a_1 v_1\rangle)/\kappa)v_{1s}^2 + (-M_{sr}/L - (M_{sp}T_{1s})/L)v_{1s} - ((Hg)/p_t + (Rg)/p_t) = 0.$

2 Effect of advection coefficient sign on stratification change

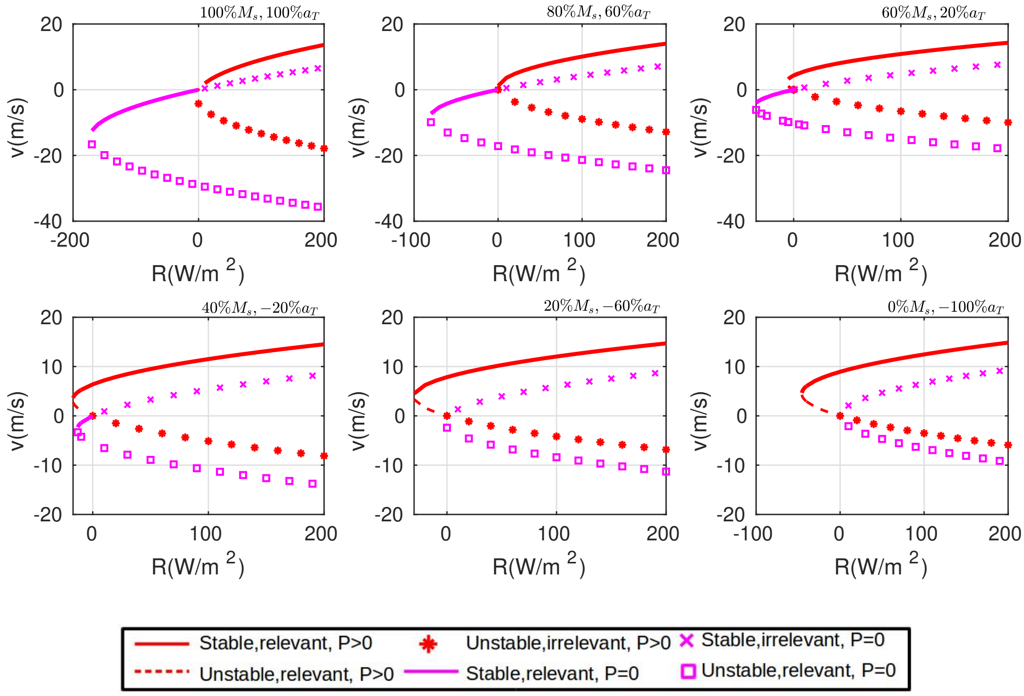

Figure S2: Standard Case of Boos & Storelvmo (2016) with stratification being present. The advection parameter is varied linearly as the stratification parameter is reduced linearly to zero.

It is to be noted that Boos and Storelvmo [2016] change the sign of a_T, the advection coefficient in thermodynamic equation, as dry thermal stratification M_s is changed abruptly from 100% of its standard value to 0. For a gradual change in M_s as in this study, the M_s threshold at which this sign change would be implemented has to be determined. It is noticed that the need to reverse the sign of a_T follows from change in the dominant balance in the thermodynamic equation as M_s is changed. Correspondingly, it is found that a_T needs to be reversed once the stratification term loses dominance, which is seen to happen in the standard case at approximately $75\%M_s$ (This critical percentage seems to depend on the ratio between M_{sp} and $a_T = \langle a_1 v_1 \rangle$). For smaller M_s, it is the advection term which is dominant and it primarily balances the forcing and convective heating. Hence, the sign of a_T is reversed in this study for cases with stratification parameter less than $75\%M_s$.

Figure 2 shows the effect of varying a_T gradually along with change in M_s. The drift of the saddle-node curves against each other, leading to a positive equilibria at bifurcation point for $P > 0$, is preserved. Thus, in this case too, reduction in M_s changes the final solution from a non-bifurcatory one to bifurcatory one. Such a gradual change of a_T avoids the abrupt change in dynamics caused by changing the sign of a_T at around $75\%M_s$. A more robust method to vary a_T suitably when M_s is varied, is beyond the scope of present study.

3 Pitchfork in a cubic equation

Figure S3 shows the effect of change in the stratification sensitivity parameter M_{qp} on the pitchfork bifurcation. For a reduction of M_{qp} (alternatively, an increase of a_q), the saddle-node like structures are preserved for both $P = 0$ and $P > 0$ cases and the bistable region

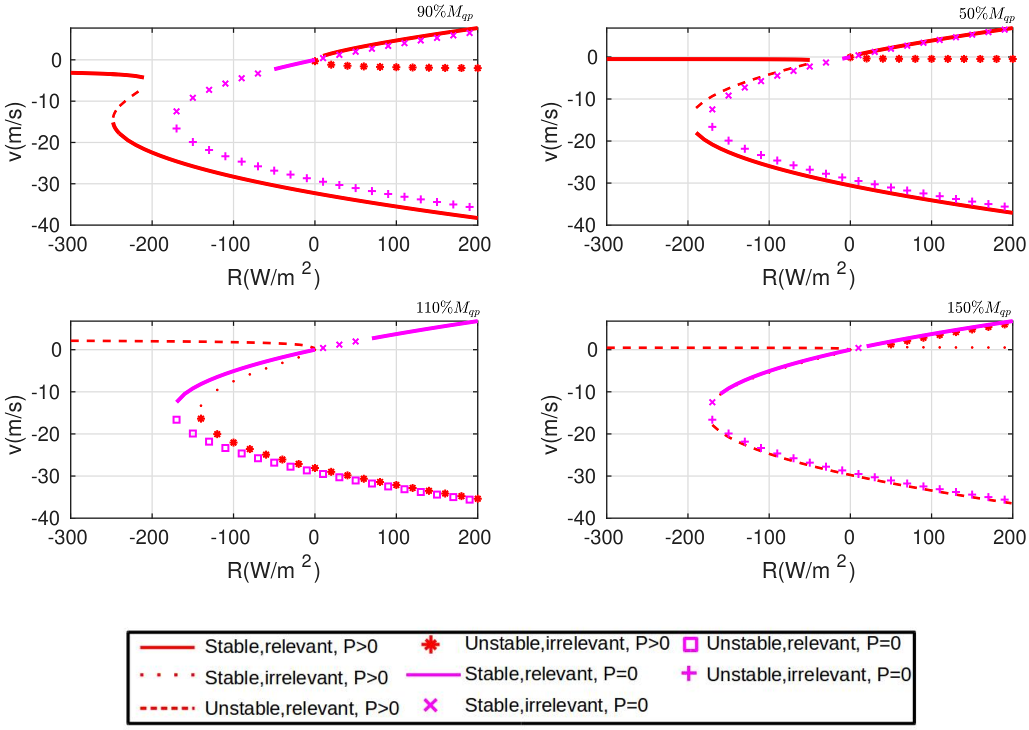

Figure S3: Standard Case of Boos & Storelvmo (2016) with stratification being present. Here the value of M_qp is varied from its standard value.

for the other arm of pitchfork widens. Eventually, for 0%M_{qp}, the bifurcation diagram looks like a regular supercritical pitchfork with the quadratic like arm shifted towards more negative values. The case of increase of M_{qp} (or decrease in a_q), in Fig. S3, shows a change of the pitchfork (the red curves, since pitchfork corresponds to the $P > 0$ case) into a subcritical one. Further increase in M_{qp} causes the pitchfork to again resemble one with the quadratic arm shifted towards more negative values.

References

William R. Boos and Trude Storelvmo. Near-linear response of mean monsoon strength to a broad range of radiative forcings. *Proceedings of the National Academy of Sciences*, 113(6):1510–1515, 2016. ISSN 0027-8424. doi: 10.1073/pnas.1517143113. URL `https://www.pnas.org/content/113/6/1510`.