

Dear Referee 1,

Following your recommendations, we suggest to precede our main hypothesis with the following paragraph (new text is marked red). We hope it will satisfactorily explain our postulate and at the same time connect our reasoning with the existing research.

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**2.1 The main hypothesis and its implication.** In the series of papers (Verbitsky et al, 2018, Verbitsky and Crucifix, 2020, 2021, and Verbitsky, 2021) we investigated dynamics of the ice-climate system on the orbital timescales. The model that has been analyzed consists of scaled conservation equations of the non-Newtonian ice flow combined with an energy-balance equation of the global temperature:

$$\frac{dS}{dt} = \frac{4}{5} \zeta^{-1} S^{3/4} (a - \varepsilon F_S - \kappa \omega - c \theta) \quad (1)$$

$$\frac{d\theta}{dt} = \zeta^{-1} S^{-1/4} (a - \varepsilon F_S - \kappa \omega) \{ \alpha \omega + \beta [S - S_0] - \theta \} \quad (2)$$

$$\frac{d\omega}{dt} = -\gamma [S - S_0] - \frac{\omega}{\tau} \quad (3)$$

Here  $S$  (m<sup>2</sup>) is the glaciation area,  $\theta$  (°C) is the basal ice sheet temperature, and  $\omega$  (°C) is the global climate temperature. The profile factor  $\zeta$  (m<sup>1/2</sup>) is assumed to be a constant;  $a$  (m/s) is snow precipitation rate;  $F_S$  is an adimensional, normalized external forcing;  $\varepsilon$  (m/s) is the external forcing amplitude;  $\kappa$  (m s<sup>-1</sup> °C<sup>-1</sup>) and  $c$  (m s<sup>-1</sup> °C<sup>-1</sup>) are sensitivity coefficients describing ice mass balance response to  $\omega$  and  $\theta$ ; the adimensional coefficient  $\alpha$  defines basal temperature response to  $\omega$  changes,  $\beta$  (°C/m<sup>2</sup>) and  $\gamma$  (°C m<sup>-2</sup> s<sup>-1</sup>) define the sensitivity of basal temperature  $\theta$  and global temperature  $\omega$ , respectively to the changes of the ice sheet area  $S$ ,  $S_0$  (m<sup>2</sup>) is a reference glaciation area, and  $\tau$  (s) is relaxation timescale for  $\omega$ .

In the system (1) – (3), the ice sheet area makes a positive feedback for the global temperature and the ice sheet basal temperature is a delayed negative feedback for ice dynamics. The most remarkable property of this dynamical system is that on astronomical time scales its dynamics is largely defined by the  $V$ -number representing a ratio of positive-to-negative feedback magnitudes.

The behavior of the above system is fully described by eight adimensional similarity parameters

$$\pi_1 = \frac{\varepsilon}{a}, \pi_2 = \alpha, \pi_3 = \kappa \gamma \varepsilon T^3, \pi_4 = c \gamma \varepsilon T^3, \pi_5 = \frac{T}{\tau}, \pi_6 = \frac{\gamma T}{\beta}, \pi_7 = \frac{S_0}{\varepsilon^2 T^2}, \pi_8 = \frac{\zeta}{\varepsilon^{1/2} T^{1/2}}$$

where  $T$  is the period of the external forcing.

If we replace astronomical timescale  $T$  with a faster timescale (let say 1-10 years) and adjust other model parameters in such a way that numerical values of similarity parameters remain the same, the dynamics of this fast system will be identical to the behavior of the system (1) – (3) on the orbital time scales.

This reasoning brings us to the intriguing observation. Specifically, if the contemporary climate can be described by the energy-balance equation, like equation (3), and its positive feedback mechanism (that is not, indeed, the land ice any longer) is controlled by a delayed negative feedback (like, for example, it may be the case for the methane (e.g., Dean et al, 2018)), then climate's behavior may be largely governed by magnitudes of its positive and negative feedbacks.

Accordingly, our main hypothesis is as general as the following: The climate system response to anthropogenic radiative forcing depends on both internal dynamics and external forcing. Further, the internal dynamics of the climate system is defined by magnitudes of its positive and negative feedbacks. Thus our main hypothesis postulates that the global temperature response is a function of time, of feedback timescales, and of the external forcing intensity and shape:

$$T = \varphi(\tau_p, \tau_n, \varepsilon, t, \lambda) \quad (4)$$

#### Additional references

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Verbitsky, M. Y. and Crucifix, M.: ESD Ideas: The Peclet number is a cornerstone of the orbital and millennial Pleistocene variability, *Earth Syst. Dynam.*, 12, 63–67, <https://doi.org/10.5194/esd-12-63-2021>, 2021.

Verbitsky, M. Y., Crucifix, M., and Volobuev, D. M.: A theory of Pleistocene glacial rhythmicity, *Earth Syst. Dynam.*, 9, 1025–1043, <https://doi.org/10.5194/esd-9-1025-2018>, 2018.