Supplemental Material to: Abrupt climate change as rate-dependent cascading tipping point

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S1. ESTIMATION OF EARLY-WARNING SIGNALS FROM TIME SERIES

From the trend in a time series it is hard to infer whether an abrupt transition is imminent, and what type of transition this might be. Instead, most early-warning signals aim to extract generic features in the fluctuations around a trend that occur as a tipping point is approached. We consider several early-warning indicators leading up to the tipping points by estimating statistical properties of the fluctuations in a sliding window. The trends encountered here are due to the system dynamics trying to catch up with the moving equilibria during a parameter shift, and are nonlinear. Thus, to separate the fluctuations from the trend, a nonlinear detrending is necessary. We do this by subtracting a fit with a cubic function to the time series in the sliding window. While higher-order polynomials could more accurately detrend the signal, they would also remove more of the variability around the trend. As a result, the only free parameter is the sliding window size.

Choosing the optimal window size is done by two trade-offs. First, a significant early-warning signal needs to be achieved. Here, there is a trade-off between low uncertainty of the estimator (large window) and sufficient temporal resolution to detect the changes in the fluctuations before the transition (small window). The required temporal resolution depends on how fast the tipping point is approached. If it is approached fast, there is only a short time frame during which changes in the fluctuations occur. Second, there is a trade-off between removing the non-linear trend as precisely as possible (small window) and preserving as much of the variability used to detect the early-warning signal as possible (large window). If the window is chosen too large, there remains a residual trend, which leads to artifacts in the statistical indicators, depending on the noise level. This effect is shown in Fig. S1. Considering these trade-offs, we use a window size of 150 years for the simulations with the coupled model, and 200 years for simulations with the Stommel model. In the latter case there is a slightly smoother trend since no rapid transition of the sea ice is involved. The results are not sensitive to the precise values.

We note that the choice of the detrending method and sliding window size should also depend on the noise level and the rate of the parameter shift. However, for our purposes these two factors are tightly constrained. The rate of the parameter shift is chosen fast enough to obtain a dynamical regime with rate-induced transitions, but slow enough so that it is possible to consider early-warning indicators. The noise levels are constrained because we aim for a regime where there is significant tipping variability and delays, but not too many noise-induced transitions (see Sec. IIIB).

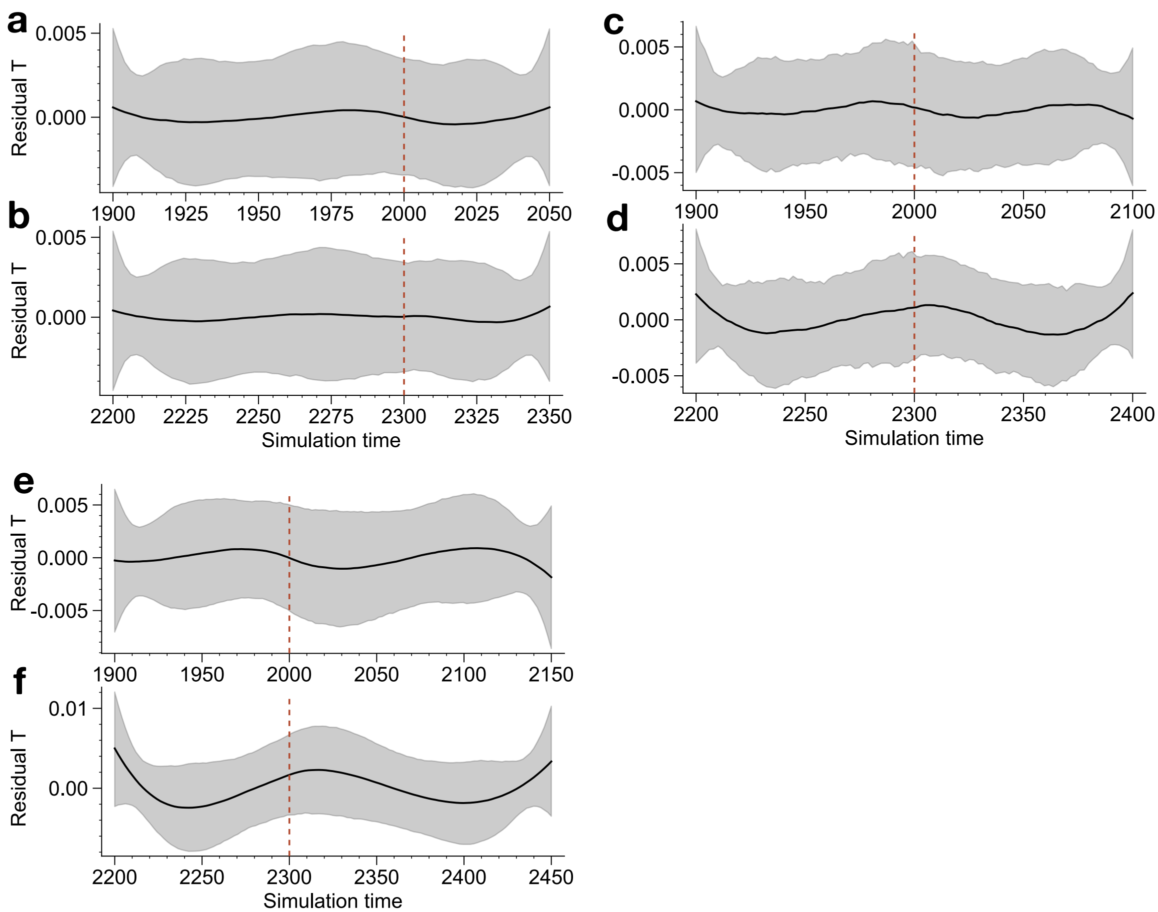

FIG. S1. Residuals after detrending with a cubic function of simulations with the Stommel model ($\sigma_T = \sigma_S = 0.2$), where η_1 is ramped from $\eta_1 = 2.65$ to $\eta_1 = 3.00$ within 300 years. The mean residuals are shown as the black line, and the gray shading illustrates the region in between the 5- and 95-percentile. The detrending is shown for a window of 150 years (**a**-**b**), 200 years (**c**-**d**), and 250 years (**e**-**f**). Panels **a**, **c** and **e** show time windows around the start of the parameter shift (red dashed line), whereas panels **b**, **d** and **f** show time windows around the end of the parameter shift (red dashed line). In **e** and **f** the average residuals show the remaining trends due to the imperfect fit of a cubic function to the non-linear trend of the model variables, which are as large as the residual fluctuations (shading).

S2. JACOBIAN ESTIMATED FROM TIME SERIES IN THE STOMMEL MODEL

In this paper we propose an early warning signal for rate-induced tipping based on estimating the Jacobian from noisy time series. In Fig. S2 we show that using the method presented in the Appendix A, the Jacobian in the vicinity of the fixed points as well as the saddle of the Stommel model can be inferred correctly with only a small quantitative bias. From simulations where the parameter η_1 is shifted from $\eta_1 = 2.65$ to $\eta_1 = 3.0$ within 300 years, we extract the part of the time series where the system is in the vicinity of the saddle (see Fig. 13), and detrend with a cubic function. Here only realizations are chosen where the systems stays in the vicinity of the saddle for at least 1000 years. For each realization, we also choose segments of the same length before and after the parameter shift to estimate the Jacobian around the 'off' attractor at $\eta_1 = 2.65$ (black) and the 'on' attractor at $\eta_1 = 3.0$, respectively. This gives rise to the three distributions of each Jacobian element around the saddle (orange), 'off' attractor (black), and 'on' attractor (blue) in each panel of the figure.

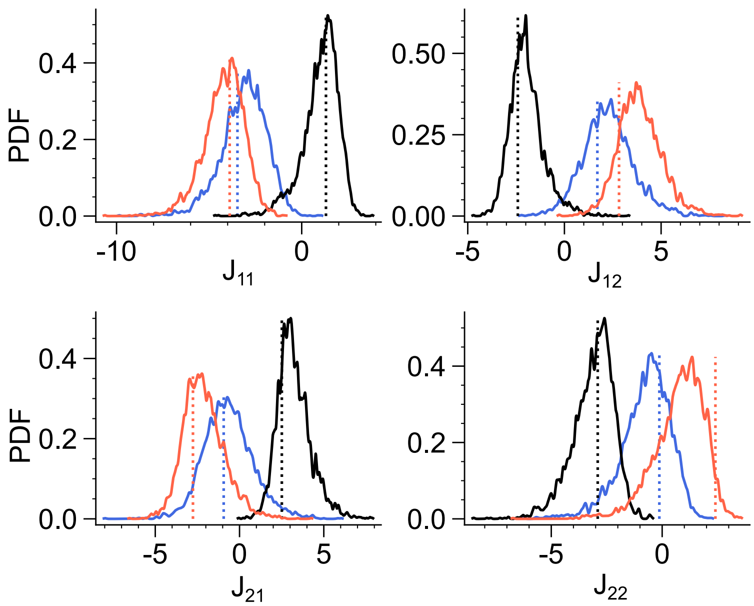

FIG. S2. Distributions of estimates of the Jacobian elements in the Stommel model ($\sigma_T = \sigma_S = 0.2$) from an ensemble of simulations where η_1 is ramped from $\eta_1 = 2.65$ to $\eta_1 = 3.0$ within 300 years. The different distributions represent the Jacobian elements around the 'off' attractor at $\eta_1 = 2.65$ (black), the 'on' attractor at $\eta_1 = 3.0$ (blue) and the saddle (red, see main text for more information). Only realizations have been chosen where the system spent at least 1000 years close to the saddle. The dashed lines correspond to the true values at the corresponding fixed points.