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Incomplete similarity of the ice-climate system

Mikhail Y. Verbitsky

Gen5 Group, LLC, Newton, MA, USA
UCLouvain, Earth and Life Institute, Louvain-la-Neuve, Belgium
Correspondence: Mikhaïl Verbitsky (verbitskys@gmail.com)

Abstract. Reconstruction and explanation of past climate evolution using proxy records is the essence of paleoclimatology. In this study, we use dimensional analysis and concepts of similarity to recognize theoretical limits of such forensic inquiries. Specifically, we demonstrate that incomplete similarity in the dynamical ice-climate system implies the absence of physical similarity in conglomerate similarity parameters. It means that major events of the past such as, for example, the middle-Pleistocene transition could have been produced by different physical processes, and, therefore, the task of disambiguation of the historical paleo-records may be fundamentally difficult, if not impossible. It also means that any future scenario may not have a unique cause and, in this sense, the orbital time-scale future may be to some extent insensitive to specific physical circumstances.

Introduction

Interpretation of most prominent events of climate history such as the middle-Pleistocene transition (Ruddiman et al., 1986, Lisiecki and Raymo, 2005, Clark et al., 2021) has been an inspiration for several generations of climate modelers (see for a review Saltzman, 2002, Clark, et al., 2006, Tziperman et al., 2006, Crucifix, 2013, Mitsui and Aihara, 2014, Paillard, 2015, Ashwin and Ditlevsen, 2015). While specific physical mechanisms invoked to explain changing glacial rhythmicity vary, they all include slow changes of ocean-atmosphere governing parameters (e.g., Saltzman and Maasch, 1991; Saltzman and Verbitsky, 1993; Raymo, 1997; Paillard and Parrenin, 2004) or glaciation parameters (Clark and Pollard, 1998). On a more general level, all these theories in fact assume slow changes in the intensities of positive (such as, for example, long-term variations in carbon dioxide concentration, e.g., Saltzman and Verbitsky, 1993) or negative (for example, regolith erosion, e.g., Clark and Pollard, 1998) system feedbacks. Though all physical phenomena invoked are, indeed, real and may be plausible, the following question still remains unanswered: Is it possible to disambiguate the past and elevate a single "correct" theory?

This is the classical attribution challenge that has been successfully addressed in the context of another well-known problem of geophysics: the causality of the observed global warming. For this purpose, the most comprehensive space-resolving models have been employed to reproduce observed time-series under different conditions and to prove (or discredit) a candidate physical phenomenon (e.g., Stocker, 2014). Certainly, these models cannot be employed on super-long orbital time scales due to computational constrains. In search for an alternative, we turn here to dimensional analysis. Historically, dimensional analysis and concepts of similarity have been used for studying physical phenomena, complementing even the most sophisticated computational tools and providing physical insight in situations where physical interpretation of the higher-complexity modeling results may be difficult. Here, on orbital timescales, when we retreat from physics-abundant space-resolving models to more conceptual dynamical models, dimensional analysis may be promoted from a supporting to a more prominent, prophetic, role.

In our study, we will use definitions of physical similarity and complete and incomplete similarity as they have been articulated by G. I. Barenblatt (2003). Suppose we have a physical phenomenon that is governed by *n* physical parameters, *k* parameters of which are parameters with independent dimensions. Then, according to the π-theorem (Buckingham, 1914), the phenomenon can be described by *n-k* adimensional similarity parameters $\pi_1, \pi_2, \dots, \pi_{n-k}$. We would have a case of a *physical similarity* if two physical phenomena have identical adimensional similarity parameters $\pi_1, \pi_2, \dots, \pi_{n-k}$. A *complete similarity* in a specific adimensional similarity parameter π_i happens when this parameter asymptotically disappears from the system. An *incomplete similarity* may be observed when adimensional similarity parameters cannot be neglected but they form governing conglomerate groups $\Pi_i = (\pi_1^{m_i})(\pi_2^{n_i}) \dots (\pi_{n-k}^{q_i})$ thus reducing the number of effective governing parameters ($i = 1,2, \dots, l; l < n - k$). While adimensional similarity parameters $\pi_1, \pi_2, \dots, \pi_{n-k}$ can be found using simple rules of dimensional analysis, there are no specific algorithms that can help us in finding their effective conglomerate groups Π_i. In our previous study (Verbitsky et al., 2018, VCV18 thereafter, Verbitsky and Crucifix, 2020), we have experimentally discovered the property of incomplete similarity in a dynamical ice-climate system. Specifically, we observed that most of the system dynamics in the orbital domain, i.e., the amplitude and the period of the system response to the astronomical

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forcing, may be attributed to the conglomerate (of several π_i) *V*-number, representing the ratio of the system's positive-to-negative feedback amplitudes. Importantly, *the incomplete similarity implies the absence of the physical similarity in conglomerate similarity parameters.* Indeed, since the same Π_i can be produced by multiple combinations of adimensional similarity parameters π_i, different physical processes can be invoked for the observed variability interpretation. Likewise, different physics may produce the same scenario of future climate evolution. Therefore, a discovery of incomplete similarity in the ice-climate system (in other words, the existence of the governing conglomerate similarity parameters) speaks to the fundamental difficulty for disambiguation of historical records. We will now illustrate this proposal with a few numerical experiments.

Method

For our experiments we employ the VCV18 dynamical model of the ice-climate system. It has been derived from the scaled mass- and heat-balance equations of the non-Newtonian ice flow (equations 1 and 2, correspondingly), and combined with a positive-feedback equation of the global climate temperature (equation 3):

$$\frac{dS}{dt} = \frac{4}{5}\zeta^{-1}S^{3/4}(a - \varepsilon F_S - \kappa\omega - c\theta) \quad (1)$$

$$\frac{d\theta}{dt} = \zeta^{-1}S^{-1/4}(a - \varepsilon F_S - \kappa\omega)\{\alpha\omega + \beta[S - S_0] - \theta\} \quad (2)$$

$$\frac{d\omega}{dt} = -\gamma[S - S_0] - \frac{\omega}{\tau} \quad (3)$$

Here, S (m^2) is the area of the glaciation, θ (^{o}C) is the basal ice-sheet temperature, and ω (^{o}C) is the global temperature of the ocean-atmosphere (rest of the climate) system. The parameter ζ (m$^{1/2}$) is a constant profile factor; a (m s^{-1}) is the snow precipitation rate; F_S is normalized external forcing, specifically, mid-July insolation at 65°N (Berger and Loutre, 1991) of the amplitude ε (m s^{-1}); κ (m s^{-1} ^{o}C^{-1}), c (m s^{-1} ^{o}C^{-1}), α (adimensional), β (^{o}C m^{-2}) and γ (^{o}C m^{-2} s^{-1}) are sensitivity coefficients; S_0 (m^2) is a reference glaciation area; and τ (s) is the timescale for ω.

We will now focus on the most remarkable feature of the dynamical system behavior - a period P of its response to the astronomical forcing. Indeed, it is the change of the climate variability from the predominant period P = 40 kyr to the main periods of P = 80-120 kyr that makes the middle-Pleistocene transition so extraordinary. Though the amplitude increase was considered, until recently, to be a necessary attribute of this transition, its presence in the paleo-records is now questioned (Clark et al, 2021). We demonstrated earlier (Verbitsky and Crucifix, 2020) that the period of the system (1) – (3) response to the obliquity forcing of period T is governed by only two dimensionless parameters:

$$P = T\Psi(\pi_1, \Pi_1) \quad (4)$$

i.e., $\pi_1 = \varepsilon/a$, the ratio of the astronomical forcing amplitude to terrestrial ice sheet mass influx, and a conglomerate parameter, the *V*-number,

$$\Pi_1 = V = \frac{\gamma\tau}{\beta c}(\alpha c + \kappa) \quad (5)$$

that represents the ratio of positive feedback amplitude to the amplitude of the negative feedback. Specifically, when $V \sim 0.75$ and $\varepsilon/a \sim 1$, the system exhibits the obliquity-period doubling. When the positive feedback and the obliquity forcing are less articulated, the system responds with the 40-kyr period. Thus, slow changes of the *V*-number and the ε/a ratio produce a change in the ice-climate behavior similar to the middle-Pleistocene transition. Since the physical interpretation of the governing parameters incorporated in the conglomerate *V*-number is very straightforward, we may observe a similar (in terms of the period-*P* bifurcation) system response to changes of a completely different physical nature. For example, parameter β defines intensity of the negative feedback and is formed as a result of interplay between vertical ice advection, internal friction, and geothermal heat flux (VCV18). Increased Peclet number of the growing ice sheet diminishes the role of the geothermal heat flux and may reduce parameter β thus increasing the *V*-number. The same period-*P* bifurcation can also be caused, for example, by slow changes in the parameter γ that defines the intensity of the positive feedback and incorporates effects of the albedo change or other atmospheric feedbacks. This feedback is applied directly to the ice sheet mass balance ($\gamma\tau\kappa$) and it is also advectively transported to the basal boundary layer eventually affecting ice mass balance as a component of basal sliding ($\gamma\tau\alpha c$).

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We solve the non-idealized system (1) – (3) for two cases we have just described. In both cases we invoke a global cooling trend. In our first experiment (Fig. 1a), this trend is translated into the increase of the Peclet number, reduction of β, weakening of the ice sheet negative feedback, and increase of the *V*-number from $V = 0.5$ to $V = 0.75$. The increased continentality of the climate is accounted by the ε/a ratio increase from $\varepsilon/a = 0.3$ to $\varepsilon/a = 1.7$. In the second experiment (Fig. 1b), the *V*-number also evolves from $V = 0.5$ to $V = 0.75$, but this time it is achieved by increased intensity of the positive feedback (γ). The millennial forcing is added to εF_S as a single sinusoid of $T_m = 5$ kyr period and $\varepsilon_m = 2\varepsilon$ amplitude.

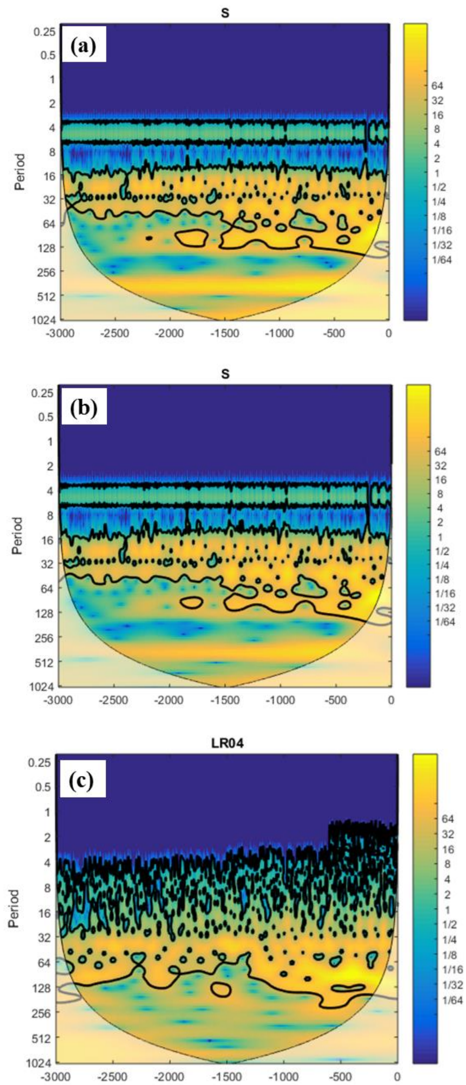

Fig. 1 Ice-climate system response to a cooling trend presented as an evolution of wavelet spectra over 3 Myr for calculated ice-sheet glaciation area S (10^6 km^2) – panels (a) and (b), and for the Lisiecki and Raymo (2005) benthic $\delta^{18}O$ record, panel (c). The *V*-number evolves from $V = 0.5$ to $V = 0.75$ due to weakening of the negative feedback (a) and due to intensified positive feedback (b). The color scale shows the continuous Morlet wavelet amplitude, the thick line indicates the peaks with 95 % confidence, and the shaded area indicates the cone of influence for wavelet transform.

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Though the time-series produced in these two cases are obviously non-identical, we can observe that different physical phenomena may produce the same changes in the conglomerate *V*-number and the same large-scale effect, i.e., the period-doubling bifurcation at about 1 Myr ago.

We do not attempt here to fully reproduce paleo-records such as the Lisiecki and Raymo (2005) or Clark et al. (2021), and a discussion of whether a period doubling should be accompanied by the amplitude increase is outside of the current paper's scope. We will just remark that the amplitude of the system response S' is the function of not just the period *P* but also of the *ε/a* ratio (Verbitsky and Crucifix, 2020),

$$S' = \varepsilon^2 P^2 \Phi(\varepsilon/a, V) \qquad (6)$$

and less articulated continentality of colder climates may explain a diminished amplitude contrasts as it has been recently advocated by Clark et al (2021).

Indeed, in the above experiments, we used mid-July insolation at 65°N (Berger and Loutre, 1991) for the last 3 million years as an astronomical forcing. Apart from that, these examples may also serve as an illustration of some future scenarios of the climate system behavior under post-industrial atmospheric carbon dioxide concentration reduction as implied by Ridgwell and Hargreaves (2007). Again, regardless of the physical nature of the underlying dynamical system, it exhibits 40-kyr rhythmicity of the first 1.5 million years of its evolution and consequent obliquity-period doubling.

Conclusions

The idea of the current presentation is simple but its implication may be far-reaching: If ice-climate system has a property of incomplete similarity, then we may be limited in our ability to disambiguate historical records and different physical processes may produce same future scenarios. The latter is intriguing because since B. Saltzman (1962) and E. Lorenz (1963) had discovered a hydrodynamic system's sensitivity to initial conditions, the concept of deterministic chaos became a dominant concept of weather and climate theory. Our findings suggest that if we consider super-long orbital time scales and, instead of time series, focus on their more generalized attributes such as the period of the system response to the astronomical forcing, we may observe that the behavior of these attributes may be, to some extent, insensitive to the physical nature of the governing processes.

But is incomplete similarity of the global, orbital-scale, climate system real? So far, this property has been found only in our VCV18 low-order dynamical model, and although this model has been explicitly derived from the conservation laws, the incomplete similarity of the ice-climate system will remain hypothetical until it is supported by empirical data. We speculate, though, that existing historical records may provide some support to this concept. To evaluate the feasibility of a diagnostic approach, let us entertain a simple scaling exercise. Suppose that an empirical time series, such as $\delta^{18}O$ record, is created by a parent system (other than the VCV18) which is controlled by *n* physical parameters (*k* of them having independent dimensions). If we choose the period of the astronomical forcing *T* to be among parameters with independent dimensions, then in accordance with the π-theorem we have:

$$P = T\Psi(\pi_1, \pi_2, \dots, \pi_{n-k}) \qquad (7)$$

The wavelet spectrum of the late Pleistocene $\delta^{18}O$ variability in response to the precession (~20-kyr period) and obliquity (~40-kyr period) forcing shows the dominance of 40-kyr and 80-kyr periods (Fig. 1c). If we are willing to accept it as a hint of $\Psi = 2$ for $T = 20$ kyr and for $T = 40$ kyr, then, since some of the similarity parameters $\pi_1, \pi_2, \dots, \pi_{n-k}$ depend on *T*, the period-*T* independence of Ψ may only happen when $\pi_1, \pi_2, \dots, \pi_{n-k}$ make conglomerate *T*-independent groups. In other words, *period independence of the Ψ function may be a signature of climate system incomplete similarity*. Indeed, the diagnostics of the Ψ function may require much more sophisticated instruments than our *ad hoc* reasoning, and the records will likely not explicitly reveal what the conglomerate similarity groups look like; nevertheless, their mere existence would corroborate the idea of this paper.

Competing interests: The author declares that he has no conflict of interest.

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