Response to Anonymous Referee #2,

Dear Referee 2, I am grateful for your comments and suggestions that will help me to improve the manuscript.

Summary: Overall, the goal of this manuscript is to demonstrate that a reduced order model of ice sheets exhibits incomplete similarity. I will be honest that I found this study to be hard to follow. I apologize to the authors in advance, if misunderstood what they did or said. Based on the difficult I had following the approach, I might not be the right person to review this manuscript. Nonetheless, my comments are below.
Answer: Thank you for your efforts. I am the one who should apologize for bringing to your attention a manuscript that is not explicable on its own. Yes, the goal of the paper, as you correctly observed, is "to demonstrate that a reduced-order model of ice sheets (*and climate* – MV) exhibits incomplete similarity". But this is only part of the goal. Most importantly, I wanted to demonstrate that, because of incomplete similarity, different combinations of similarity parameters (physically unsimilar processes) may lead to the same outcome.

I will explain this in my detailed answers below.

Comment: The basic system of equation is presented early on. It would make the manuscript much more accessible to provide an expanded description of the model and the physical interpretation of the parameters. For example, the parameter "a" is described as a snow precipitation rate. But the snowfall rate depends on the climate. Glacial cycles are known to be drier than interglacial cycles. And does the snowfall rate also include the melt rate? Or is that specified separately? Clearly, the melt rate has to depend on climate doesn't it? And then there are a host of "sensitivity coefficients". What do these physically represent and how would I measure them?
Answer: The model used in this study has been extensively described in Verbitsky et al, 2018 (VCV18 thereafter), and all questions you are raising above have been addressed there. I agree with you though that, for convenience of our readers, additional model description would be helpful.
Action: Additional model description will be provided

Comment: My next question arises from the assertion of incomplete similarity and description of what this means. Now I am vaguely familiar with similarity and incomplete similarity. The author's first assertion is that the period of the system only depends on two nondimensional numbers. Here it would be helpful to provide estimates of the physical magnitudes of each of the parameters based on whatever observations are available and to provide a physical interpretation for the "V-number" and why this controls the period. But I think my biggest question is I cannot follow the connection between the period doubling and incomplete similarity. The typical definition of complete similarity is usually that the similarity function becomes independent of some non-dimensional group in the limit that the non-dimensional number tends to zero or infinity. By contrast, the definition of incomplete similarity is that the similarity function does not become independent of the non-dimensional group as the group tends to zero or infinity. Instead, you end up with a scaling law where the scaling function becomes proportional to the non-dimensional group to some power. As the authors note, it is not usually possible to determine the scaling power by dimensional analysis alone. In the exposition (line 94), neither of the parameters tend to zero or infinity. The one parameter is 1 and the other is 0.75, neither of which can be considered large or small compared to one. So then the question: what does this have to do with incomplete similarity? When I have done calculations to determine incomplete similarity the goal has usually been to determine the scaling exponent, but I am uncertain if the authors even tried to find the

scaling exponent. I apologize to the authors if I misunderstood the analysis or their approach. Maybe I'm coming at it from the wrong direction.
Answer: Your understanding of complete and incomplete similarity is indeed correct, but, yes, in this study we approached incomplete similarity from a different direction. Finding of incomplete similarity was not our initial goal. We have been motivated to find physics responsible for period doubling bifurcation, and when we figured out that it is defined by the ratio of positive and negative feedbacks in the system (the *V*-number), we then first become suspicious that incomplete similarity may be involved.

Indeed, the VCV18 system has 11 governing parameters, 3 of them are parameters with independent dimensions. It means that the VCV18 behavior can be fully described by 8 dimensionless similarity parameters $\pi_1 - \pi_8$:

$$\pi_1 = \frac{\varepsilon}{a}, \pi_2 = \alpha, \pi_3 = \kappa\gamma\varepsilon T^3, \pi_4 = c\gamma\varepsilon T^3, \pi_5 = \frac{T}{\tau}, \pi_6 = \frac{\gamma T}{\beta}, \pi_7 = \frac{S_0}{\varepsilon^2 T^2}, \pi_8 = \frac{\varsigma}{\varepsilon^{1/2} T^{1/2}}$$

At the same time, we experimentally established that the period of the system response depends on smaller number of parameters, namely:

$P = T\Psi(\pi_1, \Pi_1)$
where

$$\Pi_1 = V = \left(\pi_2 + \frac{\pi_3}{\pi_4}\right)\frac{\pi_6}{\pi_5} = \frac{\gamma\tau}{\beta c}(\alpha c + \kappa)$$

It is the moment when we finally realized that we are dealing with incomplete similarity. This insight provides us with additional powerful vision: different combinations of π_i may produce the same *V*-number, i.e., physically unsimilar processes (formed by not identical π_i) may cause the same outcome. To my knowledge this proposition is novel.
Action: We will include above reasoning in the paper.

Comment: I also did not understand the figures provided. The x-axis and colorer aren't labeled and the y-axis doesn't have units. What are we supposed to see here?
Answer: The horizontal axis is time (kyr before present), the vertical axis is the period of the system response (kyr), the color scale shows the continuous Morlet wavelet amplitude.
Action: The figure legend will be updated.

Comment: There is another subtle issue with the analysis which is that it is always difficult to determine if the behavior of a mathematical model is a feature of the simplifications of the mathematical model or is common to the more nuanced physics that is more representative of the physical system. Here it is unclear if the authors are claiming that their simplified model obeys incomplete similarity or if the general ice-climate system obeys incomplete similarity.
Answer: In this regard, the author is very explicit - see Conclusions (lines 146-149): "But is incomplete similarity of the global, orbital-scale, climate system real? So far, **this property has been found only in our VCV18 low-order dynamical model**, and although this model has been explicitly derived from the conservation laws, the **incomplete similarity of the ice-climate system will remain hypothetical until it is supported by empirical data**." Further, the author frames the answer to this question as a challenge for future research.
Action: If the Abstract contributed to such misinterpretation, its language will be revisited, otherwise no action is required.