

Lotka’s Wheel and the long arm of history: how does the distant past determine today’s global rate of energy consumption?

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Abstract. Global economic production, or the world GDP, has risen steadily relative to world primary energy demands, suggesting that technological change is driving a gradual decoupling of society from its resource needs and associated pollution. Here we show that in each of the 50 years following 1970 for which reliable data are available, one Exajoule of world energy was consumed to sustain each 5.50 ± 0.21 trillion constant 2019 US dollars of running cumulative production summed over human history, a scaling that notably does not apply to yearly production or physical capital. The half-century for which this fixed ratio holds covers two thirds of historical growth in energy demands, so the implication is that society is not in fact decoupling from resource needs. Rather, assuming persistence of the scaling, it can be expected that future environmental impacts will be more strongly guided by past activities, or inertia, than is generally permitted within economic and climate modeling prescriptions that allow for policy to spur more rapid change.

10 1 Introduction

Alfred J. Lotka regarded the “life-struggle” as a competition for available energy whereby the role of any physical system, subject to external constraints, is to maximize the energy flux through it. “The influence of man, as the most successful species in the competitive struggle, seems to have been to accelerate the circulation of matter through the life cycle, both by ‘enlarging the wheel’, and by causing it to “spin faster”(Lotka, 1922).

15 While physical constraints may seem an inevitability for any societal action, their consideration nonetheless remains a fringe view relative to the traditional economic treatments most widely used to guide economic and climate policy (Tol, 2018; Nordhaus, 2017). There, “production functions” consider resource extraction as just one sector of the economy, no more significant than the services sector, for instance. Such an approach permits consideration of whether, through technological change and efficiency gains, human prosperity can continue to grow while limiting adverse impacts from resource depletion
20 and environmental degradation through waste production (Victor, 2010; Deutch, 2017).

Here we extend our past work that has described a new macroeconomic quantity – historically cumulative production – which can be shown to have had a quantifiable constant relationship with primary energy resource demands (Garrett, 2011, 2012; Garrett et al., 2020). By using a longer available data set, the relationship is demonstrated to hold over a half-century of global growth. Other, more traditional economic quantities such as the world GDP, or economic capital, are not found to exhibit

25 this scaling law, so the result suggests that time series for historically cumulative production may be used to provide a “top-down” guide for facilitating long-run dynamic predictions of future interactions between society, natural resource depletion and discovery, and climate change.

2 Results

To avoid complications associated with the details of trade, or interactions between economic sectors, this study is focused
 30 only on global quantities, as described in the Materials and Methods below. Annual energy demands can be expressed as an instantaneous quantity E with units of power (e.g., Terawatts) or a yearly quantity E_i with units of energy (e.g., Exajoules). For example, $E_{2019} = 609$ means that humanity in 2019 was powered by 609 Exajoules or at a rate of 19.3 Terawatts. Annual economic production (Gross Domestic Product) or output is defined monetarily as the sum of tallied financial exchanges made to acquire final goods and services within a given year. After adjusting for inflation, we denote this quantity as Y_i , expressed
 35 in units of constant 2019 USD, or as an instantaneous rate Y as 2019 USD per year.

Given that humanity’s billions emerged from the past, the magnitude of civilization’s annual energy demands might be thought to be tied to an economic quantity that is not a rate – as for Y – but rather has accumulated through time and has units of currency. The first candidate we consider for such an accumulated quantity is economic capital K_i , one of the primary factors of production. The second is a time integral of production, not just over one year – as is done in calculation of Y_i –
 40 but over the entirety of history, what we term the world historically cumulative production $W_i = \sum_{j=1}^i Y_j$, or expressed in continuous form as

$$W(t) = \int_0^t Y(t') dt' \tag{1}$$

The contribution of depreciation to W is addressed later.

Time series for Y_i , K_i , W_i and E_i are shown in Figure 1 covering a 50 year period between 1970 and 2019. Global energy
 45 consumption E increased by a factor of 2.8, production Y increased by a factor of 4.5, and economic capital K increased by a factor of 7.9. The ratio $y = Y/E$, sometimes termed the energy productivity, has trended steadily upward. Defining growth rates in quantity X as $R_X = (1/X)dX/dt = d \ln X/dt$, a least-squares fit to the data gives $R_y = 1.00\%$ per year. Meanwhile, the ratio $k = K/E$ grew at rate $R_k = 1.96\%$ per year, nearly twice as fast as y , or a doubling time of 35 years. The economy appears to be becoming rapidly less energy intensive, suggesting that technological innovation is enabling more to be done
 50 with less (Sorrell, 2014).

It would seem natural to infer that technology is allowing the global economy to undergo a long-term decoupling from resource constraints. However, a comparison between W_i and E_i suggests otherwise. Relative to Y_i and K_i , cumulative production W_i increased more slowly, by a factor of 2.7, similar to the factor found for E_i . Expressed (for simplicity) as a

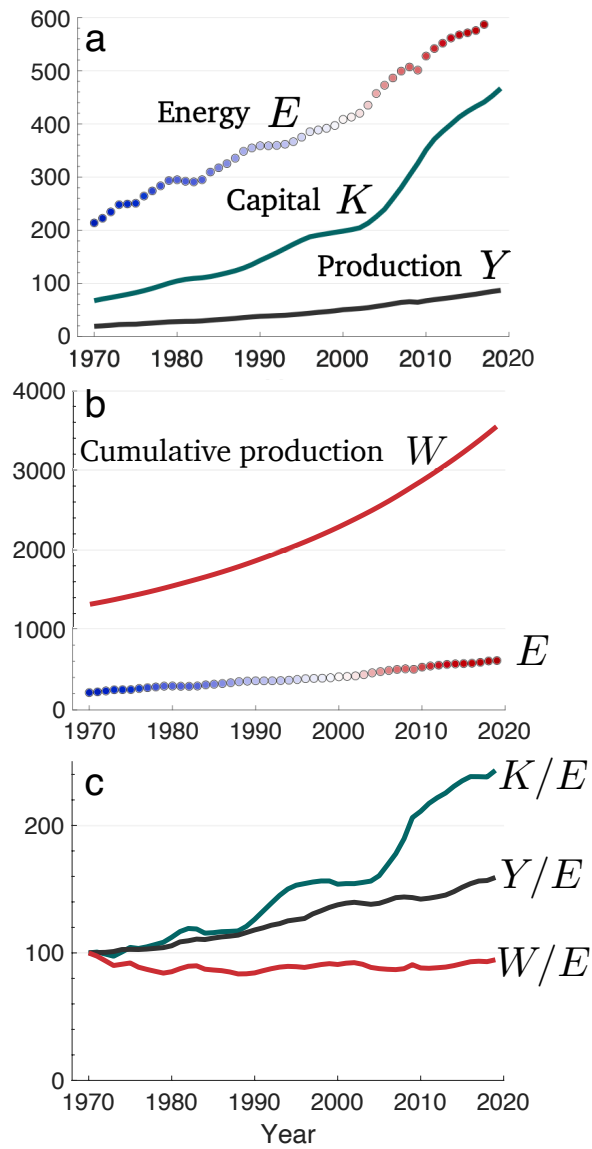


Figure 1. a: Time series for the period 1970 to 2019 of global yearly annual primary energy consumption E_i in Exajoules, the world annual GDP Y_i in currency per year, and the total value of physical capital capital stock K_i in units of currency. b: Energy and historically cumulative production W_i in currency. c: The ratio of economic values to annual energy consumption, setting the ratio in 1970 to 100. All currency units are in trillion 2019 USD

continuous function, the ratio $w = W/E$ has fluctuated to some degree, but the average tendency has been $R_w = -0.02\%$ per
 55 year, far less than either Y/E or K/E . The average value of the ratio is:

$$w = \frac{W}{E} = 5.50 \pm 0.21 \quad (2)$$

in units of trillion 2019 USD of cumulative production per Exajoule of energy consumed each year.

Thus, the relationship between W and E does not appear to be one only of correlation between two quantities, as for example
 has been noted for E and Y (Jarvis, 2018). Instead the two quantities have maintained a linear scaling over the half century
 60 period for which widely published data are available. A least-squares fit to the logarithms of W and E yields the relationship
 $W = 5.47E^{1.00}$. Calculated instead as a linear fit, the relevant expression is $W = 5.67E - 66$. Note the intercept of the fit where
 $E = 0$ is equivalent to $W = -66$ trillion 2019 USD. This value is just -1.9% of the 2019 value for W_i of 3547 trillion 2019
 USD, and so sufficiently small as to plausibly approximate the origin. By contrast, the linear fit for world GDP and energy
 is $Y = 0.17E - 21$. So, while Y and E are correlated, they do not scale since the intercept corresponding with zero energy
 65 demands is $Y = -21$ trillion 2019 USD, or -25% of the 2019 value.

3 Discussion

A quantity identified here as the historically cumulative global production W appears to be an economic expression of the
 rotational power of Lotka's Wheel, that is the capacity to drive the collective to-and-from of civilization's circulations through
 the relationship $W = wE$, where w is nearly a constant. Certainly, an objection might be raised that the past 50 years is too
 70 short relative to the time span of humanity to draw meaningful conclusions about the relationship of cumulative production to
 energy demands. Measured in units of years this may be true. However, the period covers roughly two-thirds of humanity's
 growth in its energy consumptive demands, or 1.5 doublings in E , during which clearly a great deal changed in humanity's
 social and technological makeup.

With respect to an inflation-adjusted production relation, taking the first derivative of Eq. 1, and assuming $W = wE$ for
 75 constant w , yields

$$Y = \frac{dW}{dt} = w \frac{dE}{dt} \quad (3)$$

Expressing economic production as related to a change in energy demands, that is its derivative with respect to time, does
 differ significantly from prior approaches that most usually employ production functions ignoring the role of energy altogether.
 In the few studies where production functions do appeal to energetic demands, the functional dependence is to some non-
 80 unity exponent of E (Ayres et al., 2003; Keen et al., 2019), which is physically and dimensionally nonsensical. While it is
 certainly possible to obtain through fitting a non-unity exponent relating two quantities, it cannot be presumed the fit expresses
 something fundamental about the system unless the appropriate units for physical quantities are maintained.

Note that the quantity W is highly smoothed because it is a summation, or integration, over history and the global economy.
 Thus, even given a strong apparent relationship of E to W , variability in E cannot as easily be related to economic production

85 evaluated on scales of years. Nonetheless, calculated as a running decadal mean, the average ratio of production to change in energy consumption is

$$\hat{w} = \frac{Y}{dE/dt} = 5.9 \pm 2.2 \quad (4)$$

in units of trillion 2019 USD per Exajoule consumed each year, a very similar value to Eq. 2 although more noisy being a differential. Implicitly, our collective societal assessment of the final inflation-adjusted value of goods and services Y appears to correspond with “enlarging the wheel” or enabling it to “spin faster”, that is the technological innovation of a larger human system that is newly consumptive over and beyond the scenario where energy consumption rates stay constant. Current energy demands sustain the wheel’s rotation against dissipation. With an excess of available energy, an effective phase change is enabled to convert raw materials to newly created components of civilization, and to accelerate societal circulations so that they include them. In fact there is some suggestion of a near equal division between changes in size and speed as two independent modes of variability. A linear scaling has been noted between the magnitude of a city’s population and how fast its inhabitants walk (Bettencourt et al., 2007). More globally, over the 50 year period considered, world population – as a measure of size – increased at an average rate of 1.46% per year. Meanwhile, per capita world GDP – as a plausible metric for speed – increased at the nearly equivalent rate of 1.55% per year.

At some level, the empirical nature of Eq. 2 stands on its own. Nonetheless, its simplicity may come across as counter-intuitive, especially considering that W is not directly tied to any current economic transaction, only to the past. By way of explanation, consider the circulations within our bodies, brains, and machines, and our activities such as housework, transport to and from work and the grocery store, and even conversation among family and friends, that all of these require energy in some form. While each one of them may indirectly involve a financial transaction at some prior stage, for cleaning products, gasoline, or food, no financially quantifiable purchase is in fact made at the point at which the energy is consumed.

A possible counterargument is that historically distant production cannot linger to contribute to current energy demands. Fig trees grown for the enjoyment of Ancient Greeks would seemingly have nothing to do with the power consumption of internet servers today. In traditional economic accounting, current capital is formed through economic production Y after subtracting both depreciation at rate δ and consumption C of goods and services, that is $dK/dt = (Y - C) - \delta K$. Expressing consumption as $C = cY$ and adopting a simplified production function of form $Y = \beta K$ where $\beta = Y/K$ is the production efficiency (or the inverse of the capital-to-output ratio), it follows that

$$\frac{dK}{dt} = (\beta - \delta')K, \quad (5)$$

so that the capital exponential growth rate is $R_K = (\beta - \delta')$ where $\delta' = \delta + c\beta$ and consumption itself can be viewed as a form of depreciation of very short-lived capital. From data for Y_i and K_i , the value of β over the past 50 years has steadily declined at an average rate of 0.95% per year but the average value is approximately 0.24 or 24%. So, considering how capital grew at an average annual rate of 4.0% over the same period, the implication is that the annual rate δ' at which capital has been devalued is approximately $24\% - 4\% = 20\%$, that is capital value in traditional accounting has a halving time of just 3.5 years. Well-known concerns may be raised about any comparison of rates of capital formation with capital valuation, and with how

valuations of varied capital stocks should be aggregated (Samuelson, 1966; Sraffa, 1975). Nonetheless, the benefits of past productivity clearly persist for much longer. We may no longer use the personal computers of the 1980s, but current devices
120 are derived from that seminal transformation. Ancient Greek fig trees died over 2000 years ago, but important aspects of the culture of fig-eating Ancient Greeks have lasted to today.

The crux of this valuation problem appears to be that the long-distant contributions of past civilizations to politics, science, athletics, architecture, and language are ignored in traditional accounting because they cannot be monetized on an open market, even though without them the bulk of our modern infrastructure for wealth-generation would disappear. As noted by Piketty,
125 “All wealth creation depends on the social division of labor and on the intellectual capital accumulated over the entire course of human history,” continuing “the total value of public and private capital, evaluated in terms of market prices for national accounting purposes, constitutes only a tiny part of what humanity actually values - namely, the part that the community had chosen (rightly or wrongly) to exploit through economic transactions in the marketplace” (Piketty, 2020). Capital K , as valued by current markets, is an order of magnitude smaller than the more abstract quantity of historically cumulative production
130 W . The scaling between W and E then suggests that energy is required not just to sustain that which we believe potentially available to be sold today, but also the unspoken utility of that which has previously been produced. Civilization was not built in a day.

There are important analogs in the biological and physical world that may provide a guide. The energy of a wheel’s rotation is the product of its mass and the square of its radius and rotational frequency, all quantities that increase through a prior history
135 of material and energetic increments. In a cloud, a snow crystal grows through the diffusion of vapor molecules. Current vapor consumption depends on the reach of the crystal branches into the surrounding vapor field, insofar as the branches have built upon a prior accumulation of condensed vapor residing within the unexposed crystal interior (Lamb and Verlinde, 2011). The leaves of a deciduous tree enable photosynthesis that fuels fluid circulations through the exterior sapwood. Leaves die seasonally as the sapwood turns into heartwood that, while not actively connected to a larger rejuvenated leaf crown in the
140 following year, structurally supports it (Shinozaki et al., 1964; Oohata and Shinozaki, 1979). Inevitably there are loss processes, such as friction for a wheel, moments of evaporation or breakup for a snow crystal, or disease and predation for a tree. But, in all cases past consumption is the primary determinant of the system’s current energetic demands.

Conclusions

We have identified a nearly constant relationship between world historically cumulative inflation-adjusted economic production
145 and current energy demands that has held for the past half-century, a period during which resource consumptive demands nearly tripled. The result suggests that humanity’s current metabolic needs are independent of the current economy as typically quantified financially by the GDP. Instead, cumulative production is better considered as an emergent property of a system that boot-strapped itself to its current state through past technological creation that has allowed more to be produced with less Haff (2014); Garrett et al. (2020). Whatever the explanation, the relationship’s persistence would appear to place substantial bounds
150 on humanity’s future interactions with its environment. For one, it implies that present sustenance cannot be decoupled from

past growth, implying a much greater role for inertia than has been broadly assumed, for example, in the integrated assessment models used to evaluate the coupling between humanity and climate (Nordhaus, 2017). Even if world GDP growth falls to zero from its recent levels close to 3% per year, long-term decadal-scale growth in resource demands and waste production would continue to accelerate. More worryingly, the result suggests that it is only by way of collapse of the previous growth that led to the wealth we enjoy today, effectively by shrinking Lotka’s Wheel, that our resource demands and waste production can decline. Eq. 1 offers no direct mathematical approach for such an event to occur, except perhaps through hyper-inflation, as this would lead to high values of the GDP deflator that in economic accounting yield values of the inflation-adjusted GDP much lower than the nominal GDP. Historically, hyper-inflation has been associated with periods of societal contraction (Zhang et al., 2007) suggesting a possible link of inflation to decay (Garrett, 2012).

On the topic of climate policy, a constant value of w implies that economic production can be decoupled from carbon dioxide emissions, but only provided a rapid switch to renewables or nuclear energy. All newly added energy production would need to be emissions free, which based on recent consumption growth rates works out to about 1 Gigawatt per day. Alternatively, or concurrently, some means would need to be devised for decoupling W from E by increasing the value of w . Given the value of w has varied little while society has changed tremendously over the last 50 years, it is difficult to conceive how this would be managed. That said, adjusting w upward could be seen as new target for mitigating future climate damages.

Appendix A: Methods

Yearly statistics for world primary energy E_i are available for both consumption and production from the Energy Information Administration (EIA) of the U.S. Department of Energy (DOE) for the period 1980 through 2018, and for consumption from British Petroleum for the years 1965 through 2019 (DOE, 2020; Bri, 2020). A yearly composite of E_i in units of Exajoules for the years 1970 to 2019 is created from the average of the three datasets while using single sources where only one is available. Economic production is tallied and averaged using World Bank and United Nations statistics for the years 1970 to 2019 (The World Bank, 2019; UNs, 2020) and expressed here in units of trillions of market exchange rate, inflation-adjusted “real” year 2019 dollars. Statistics for the aggregated capital stock of 180 countries K_i are available from the Penn World Tables (Feenstra et al., 2015).

The world historically cumulative production $W_i = \sum_{j=1}^i Y_j$ requires for its calculation yearly estimates of Y_j prior to 1970, for which we apply a cubic spline fit to the Maddison Database (Maddison, 2003) for years after 1 C.E. Adjustments are made to the Maddison dataset to account for the chosen inflation-adjusted year of the dataset and to convert from currency expressed in purchasing power parity dollars rather to market exchange units using as a basis the time period between 1970 and 1992 for which concurrent MER and PPP statistics are available. The value for cumulative production in 1 C.E. $W(1)$ is obtained by assuming that the population and W were growing equally fast at that time. Population data from 1.C.E and one century before and after show that global population was 170 million and growing at 0.059 % per year (United States Census Bureau, 2021). While there are inevitable uncertainties in the reconstruction of W as with any other, the yearly values of W since 1970 that are emphasized here cover two-thirds of total growth and so the calculations are more strongly weighted by recent data that is

presumably most accurate. Thus, calculation of W , most particularly the conclusion that w is nearly a constant, can be shown
185 to be relatively insensitive to uncertainty in the older statistics (Garrett et al., 2020).

Author contributions. T.J.G. and M.R.G conceived the study, T.J.G. analysed the results. All authors wrote and reviewed the manuscript.

Competing interests. The authors declare no competing interests.

Acknowledgements. This work was supported by the National Institute of Economic and Social Research and the Economic and Social
190 Research Council (ES/R00787X/1), whose views it does not represent. Review comments from Peter Haff contributed substantially to
framing of the arguments.

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