# Lotka’s Wheel and the long arm of history: how does the distant past determine today’s global rate of energy consumption?

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**Abstract.** Global economic production – the world GDP – ~~is rising steadily~~ has been steadily rising relative to world primary energy demands, ~~which suggests that technological change is driving~~ lending hope that technology can drive a gradual decoupling of society from its resource needs and associated environmental pollution. Here we present a contrasting argument, ~~showing~~ that in each of the 50 years following 1970 for which reliable data are available, one Exajoule of world energy was ~~consumed~~ required to sustain each $5.50 \pm 0.21$ trillion ~~constant 2019~~ year-2019 US dollars of ~~running cumulative production summed over human history, while no similar scaling applied to~~ historically cumulative inflation-adjusted economic production. No similar scaling was found to apply to the more familiar quantities of yearly ~~production~~ economic production, capital formation, or physical capital. The half-century ~~for which this fixed ratio has~~ during which the fixed ratio held covers two thirds of historical growth in energy demands, ~~implying that society may be fundamentally coupled to~~ so the implication is that society is not in fact decoupling from its resource needs. ~~Assuming the scaling continues to persist, it can be expected~~ Instead, if the scaling persists, the expectation should be that future environmental impacts will remain strongly ~~anchored~~ tethered to society’s unchangeable past~~, or by inertia, much more so than has generally been permitted within~~. Inertia will play a more dominant role in future societal trajectories than is generally permitted by economic and climate modeling and policy prescriptions that allow for ~~policy to spur~~ more rapid change.

## 1 Introduction

Alfred J. Lotka regarded the “life-struggle” as a competition for available energy~~whereby the role~~. The role in this struggle of any physical system, subject to external constraints, is to maximize the ~~energy flux~~ flow of energy through it. Lotka proposed, “The influence of man, as the most successful species in the competitive struggle, seems to have been to accelerate the circulation of matter through the life cycle, both by ‘enlarging the wheel’, and by causing it to “spin faster”~~(Lotka, 1922)~~... the physical quantity in question is of the dimensions of power". “In every instance considered, natural selection will so operate as to increase the total mass of the organic system, to increase the rate of circulation of matter through the system, and to increase the total energy flux through the system, *so long as there is presented an un-utilized residue of matter and available energy."* (Lotka, 1922) (our italics).

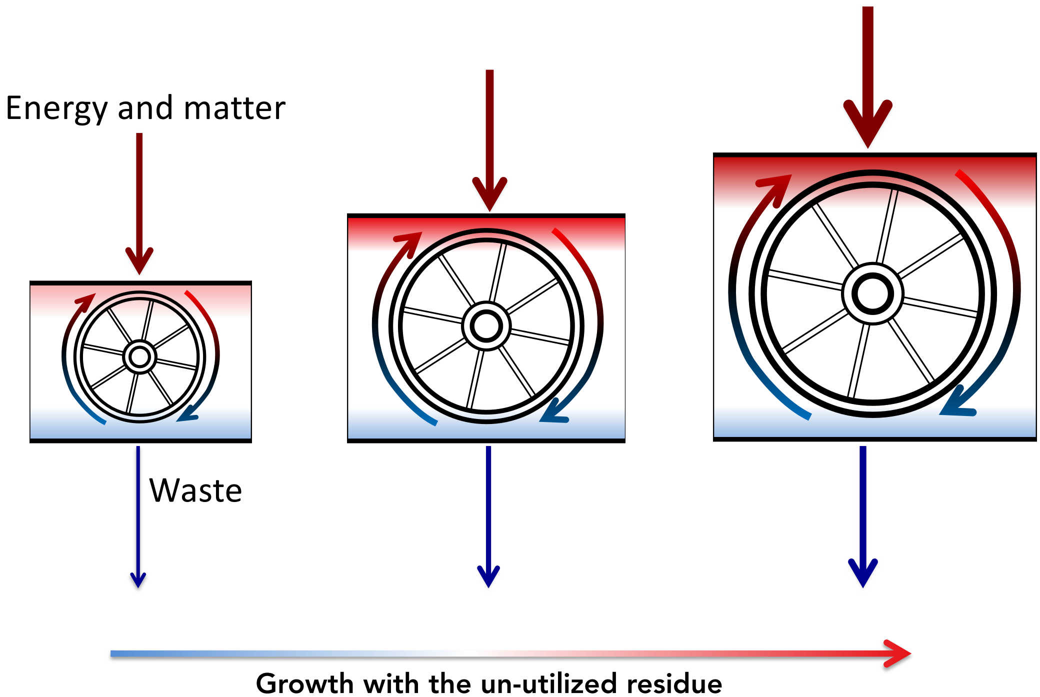


**Figure 1.** Representation of Lotka's view on the thermodynamic mechanisms governing system growth, involving a wheel that enlarges and accelerates using an "un-utilized residue" of energy and matter

~~In~~ Adopting Lotka's ~~view, physical constraints are an inevitability component of any societal action, yet their~~ perspective, illustrated in Fig. 1, the field of thermodynamics should be essential to any understanding or treatment of societal actions. Yet, even a century later, its consideration remains a fringe view in the economic treatments most widely used to guide economic and climate policy (Tol, 2018; Nordhaus, 2017)~~. There,~~ , where "production functions" ~~consider~~ treat resource extraction as just one sector of the economy, no more significant than~~the services sector~~, for instance~~. The approach permits consideration of technological change through efficiency gains as able to lift~~ , the services sector. An important consequence of these modeling frameworks is that they permit technological change and efficiency gains to be key mechanisms for simultaneously lifting human prosperity while limiting adverse impacts from resource depletion and environmental degradation through waste production (Victor, 2010; Deutch, 2017).

~~Here we extend~~ As a counterpoint to the traditional approach, in our past work ~~where~~ we described a new macroeconomic quantity – historically cumulative production – that we demonstrated to have had a quantifiable constant relationship with world primary energy resource demands~~(Garrett, 2011, 2012; Garrett et al., 2020). By using~~ , or civilization's collective power. A consequence of the relationship is that the inflation-adjusted GDP is more closely related to a surplus of energy – or Lotka's "un-utilized residue" – than to the rate of energy consumption itself (Garrett, 2011, 2012; Garrett et al., 2020). Here, we use a longer available data set ~~, the relationship is~~ than previously available to show that the relationship can be demonstrated to hold over a half-century of global growth . Other, more traditional economic quantities such as the world GDP, or economic capital, are not found to exhibit this scaling law, so the result suggests that time series for covering the period between 1970 and 2019. More generally, this new time series of historically cumulative production may be used to provide offers a "top-down" guide metric for facilitating long-run dynamic predictions of future interactions between society, natural resource depletion and discovery, and climate change.

## 2 Results

To avoid complications associated with the details of trade, interactions between economic sectors, or distinctions between energy types, this study is focused only on global quantities, as described in the Materials and Methods below. Annual primary energy sources, those that are available to drive civilization activities of whatever type, are consumed and ultimately dissipated as waste heat at a rate that can be expressed as an instantaneous quantity $E$ (e.g., Terawatts) or a yearly-averaged quantity $E_i$ with units of power (e.g., either Terawatts or Exajoules per year) (Garrett et al., 2020). For example, $E_{2019} = 609$ means that humanity over the course of 2019 was powered by 609 Exajoules or at a rate of 19.3 Terawatts. Annual economic production (Gross Domestic Product) or output is defined monetarily as the sum of tallied financial exchanges made to acquire final goods and services within a given year. After adjusting for inflation, we denote this quantity as $Y_i$, expressed in units of constant 2019 USD per year, effectively a yearly average of the instantaneous rate $Y$ in 2019 USD per year.

Given that humanity's billions emerged from the past, the magnitude of civilization's annual energy demands might be thought to be tied to an economic quantity that is not a rate – as for $Y$ – but rather has accumulated through time and has units of currency. The first candidate we consider for such an accumulated quantity is economic capital $K_i$, one of the primary factors of production. The second is a time integral of production, not just over one year – as is done in calculation of $Y_i$ – but over the entirety of history, what we term the world historically cumulative production $W_i = \sum_{j=1}^{i} Y_j$, or expressed in continuous form as

$$W(t) = \int_0^t Y(t')\,dt' \tag{1}$$

The contribution of depreciation to $W$ is addressed later.

Time series for $Y_i$, $K_i$, $W_i$ and $E_i$ are shown in Figure 2 covering a 50 year period between 1970 and 2019. Global energy consumption $E$ increased by a factor of 2.8, production $Y$ increased by a factor of 4.5, and economic capital $K$ increased by a factor of 7.9. A related quantity, the rate of capital formation, $dK/dt$, is not shown because it is implicit in the curve for $K$, however its value varied considerably. While the ratio $(dK/dt)/E$ increased by a factor of 1.5 between 1970 and 2019, the relative increase was 3.2 in 2009 and 0.34 in 1982. The By contrast, the ratio $y = Y/E$, sometimes termed the energy productivity, has trended steadily upward. Defining growth rates in quantity $X$ as $R_X = (1/X)dX/dt = d\ln X/dt$, a least-squares fit to the data gives $R_y = 1.00\%$ per year. Meanwhile, the ratio $k = K/E$ grew at rate $R_k = 1.96\%$ per year, nearly twice as fast as $y$, or a doubling time of 35 years. The economy appears to be picture is of an economy that is becoming rapidly less energy intensive, suggesting that technological innovation is enabling more to be done with less (Sorrell, 2014).

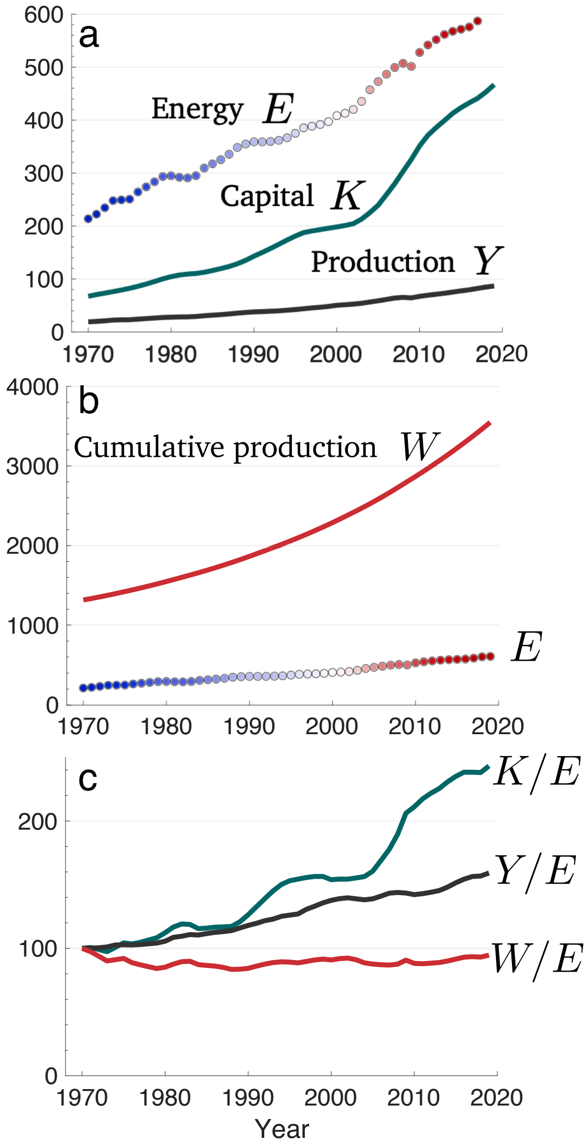


**Figure 2.** a: Time series for the period 1970 to 2019 of global yearly annual primary energy consumption $E_i$ in Exajoules per year, the world annual GDP $Y_i$ in yearly currency, and the total value of physical capital capital stock $K_i$ in units of currency. b: Energy in Exajoules per year and historically cumulative production $W_i$ in currency. c: The ratio of economic values to annual energy consumption, setting the ratio in 1970 to 100. All currency units are in trillion 2019 USD

It would be natural to infer from a history of increasing $Y/E$ that ~~technology is allowing the global economy to undergo a~~ the human acumen for invention has been a driving force behind a long-term decoupling of the global economy from resource constraints. However, a comparison between $W_i$ and $E_i$ suggests otherwise. Relative to $Y_i$ and $K_i$, cumulative production $W_i$ increased more slowly by a factor of 2.7~~, a similar increase as~~. This ratio is nearly identical to the factor of 2.8 increase found for $E_i$. Expressed (for simplicity) as a continuous function, the ratio $w = W/E$ has fluctuated to some degree, but the average tendency has been $R_w = -0.02\,\%$ per year, far less than the tendencies of either $Y/E$ or $K/E$. The average value~~of the ratio is~~:

$$w = \frac{W}{E} = 5.50 \pm 0.21 \tag{2}$$

in units of trillion 2019 USD of cumulative production per Exajoule of energy consumed each year.

~~This~~ As the ratio is nearly a constant, the relationship between $W$ and $E$ does not appear to be one ~~only~~ merely of correlation between two quantities, as for example has been noted for $E$ and $Y$ (Jarvis, 2018). Instead ~~the two quantities~~ $W$ and $E$ have maintained a linear scaling over the half century period for which widely published data are available. A least-squares fit to the logarithms of $W$ and $E$ yields the relationship $W = 5.47E^{1.00}$ Calculated instead as a linear fit, the relevant expression is $W = 5.67E - 66$. Note the intercept of the fit where $E = 0$ is equivalent to $W = -66$ trillion 2019 USD~~. This value~~, a value that is just -1.9% of the 2019 value for $W_i$ of 3547 trillion 2019 USD, and so sufficiently small as to plausibly approximate the origin. By contrast, the linear fit for world GDP and energy is $Y = 0.17E - 21$ with an intercept of $Y = -21$ trillion 2019 USD, or -25% of its 2019 value. So, while $Y$ and $E$ ~~are~~ may be correlated, they do not scale ~~: the intercept corresponding with zero energy demands is $Y = -21$ trillion 2019 USD, or -25% of the 2019 value~~as in the same manner as $W$ and $E$.

## 3 Discussion

We interpret the quantity identified here as the historically cumulative global production $W$ as an economic expression of the rotational power of Lotka's Wheel, that is the capacity to drive the collective to-and-from of civilization's circulations through the relationship $W = wE$, where $w$ is nearly a constant. Certainly, an objection might be raised that the past 50 years is too short relative to the time span of humanity to draw meaningful conclusions about the relationship of cumulative production to energy demands. Measured in units of years, this may be true. However, the last half-century covers a remarkable two-thirds of humanity's growth in its energy consumptive demands, or 1.5 doublings in $E$, during which a great deal changed in humanity's social and technological makeup.

With respect to an inflation-adjusted production relation, taking the first derivative of Eq. 1, and assuming $W = wE$ for constant $w$, yields

$$Y = \frac{dW}{dt} = w\frac{dE}{dt} \tag{3}$$

~~In the~~ The statement is that real economic production is related to the *rate of increase* in world primary energy consumption, consistent with Lotka's 1922 arguments about the un-utilized residue being required for power growth. It bears some discussion as it suggests that the real GDP emerges only when energy is available in excess of civilization's daily needs (Garrett et al., 2020). For the hypothetical limiting case of $dE/dt = 0$, the world ~~attains~~ would attain a sort of metabolic steady-state characterized by a balance between energetic and material inputs and outputs. ~~Energy consumption maintains a fixedrate, but also there is~~ While civilization's power would be fixed, the implication is that no *real* ~~economic production. Nominal production may remain~~world economic production could occur. Such a conclusion may seem peculiar, but it is important to note that it does not forbid positive nominal production, provided that it is completely eroded by the GDP deflator, or inflation. This ~~hypothetical~~ case of economic collapse, even if not a collapse of energy consumption, may not be survivable given how society is currently constructed. If so, ~~the~~ any point at which $dE/dt = 0$ is satisfied may only represent a temporary marker on a pathway to more complete thermodynamic collapse~~with the~~, one where the alternative steady-state condition of $E = 0$ ~~. A distinction must be made with the~~ is in fact reached, and civilization's power is zero. A quite different steady-state condition is one where $dY/dt = 0$, namely ~~one~~ of constant inflation-adjusted economic ~~*production*~~ production $Y$, ~~or~~ and zero GDP growth, ~~as this would imply~~ in which case there is continued expansion of energy demands at rate $Y/w$. In the constant GDP growth case preferred by governments, with fixed $d\ln Y/dt$, energy consumption accelerates.

~~Eq. 3 expresses economic production as proportional to an increase in energy demands, that is its derivative with respect to time, which differs~~ Even if it differs quite markedly from prior approaches, especially those that tend to ignore any explicit mention of the role of energy~~. Where energetic demands are considered,~~ , Eq. 3 is only a mathematical consequence of the empirically validated expression for constant $w$ given by Eq. 2. Certainly there are macroeconomic treatments where energy demands are also considered, however the production functions ~~are~~ tend to be highly complex, and they do not appeal foremost to the dimensions of the problem~~are not considered~~. Rather than starting with the constraint that the factors of economic production, of whatever combination, must tally dimensionally to units of currency per time, quantities such as dimensionless capital, labor, and useful work are set to non-integer exponents, or are themselves placed in exponents (Ayres et al., 2003; Ayres and Warr, 2009; Lindenberger and Kümmel, 2011; Keen et al., 2019). ~~While the~~ Such functions can be shown to reproduce past behaviors for specific nations, but it is only by way of specifying coefficients, or "output elasticities", that are themselves determined from past economic conditions, and that are allowed to vary according to the time period considered. ~~The production functions become moving targets , and therefore~~ Hence, the production functions are moving targets that cannot be presumed to express ~~something~~ anything fundamental about the long-run evolution of the economic system. As attributed by E. Fermi to J. von Neumman "with four parameters I can fit an elephant, and with five I can make him wiggle his trunk."

~~Here~~Eq. 3, by contrast, ~~Eq. 3~~ is simple, dimensionally reasoned, and assumes only that $w$ is a constant~~, so it~~. It can be readily refuted (or supported)~~with data. It~~ , here with decades of data from multiple sources. The approach does nonetheless have its limitations. Note that the quantity of cumulative inflation-adjusted economic production $W$ is highly smoothed because it is a summation, or integration, over history and the global economy. Thus, even given a strong ~~apparent~~ multi-decadal relationship of $E$ to $W$, year-to-year variability in $E$ cannot as easily be related to ~~national economic productionevaluated on scales of years~~yearly economic production, especially on national or sectoral scales much finer than the world as a whole. Nonetheless,

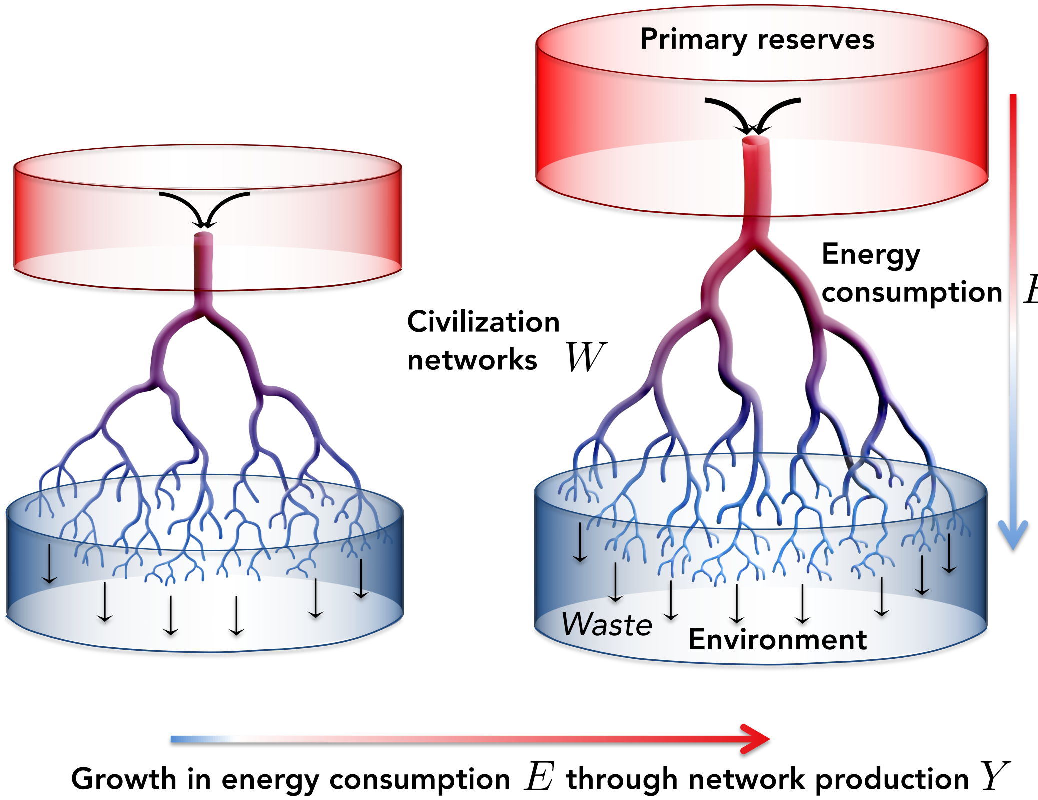


**Figure 3.** Elaboration on Lotka's Wheel. Civilization growth related to increases in its power at rate $dE/dt$ as tied to network production through the inflation-adjusted GDP $Y$. Current power $E$ is thus tied to the historically cumulative GDP through $W = \int_0^t Y(t)\,dt' = w\int_0^t (dE/dt)\,dt' = wE$.

calculated as a running decadal mean, the average ratio of global production to yearly changes in energy consumption is

$$\widehat{w} = \frac{Y}{dE/dt} = 5.9 \pm 2.2 \qquad (4)$$

in units of trillion 2019 USD per Exajoule consumed each year, ~~a very similar value to~~ which is very similar to that expressed for $w$ given by Eq. 2~~although more noisy being a differential . Implicitly~~, although the variability is higher given the comparison of $Y$ to a differential in $E$.

Thermodynamically, the collective societal assessment of the final inflation-adjusted value of goods and services $Y$ appears to correspond with "enlarging the wheel" or enabling it to "spin faster", that is the technological innovation of a larger human system, one that is newly consumptive of primary reserves over and beyond the scenario where energy consumption rates stay constant. Current energy demands sustain the wheel's rotation against dissipation. ~~So, it~~ It is only with an excess or "un-utilized residue" of available energy that an effective phase change ~~is enabled to convert raw materials via economic production to newly created components of civilization , and to accelerate societal circulations so that they include them~~ becomes possible whereby raw materials are converted through economic production into newly created civilization networks, and with increasing power societal movements are accelerated along them (Figure 3. In fact, there is some suggestion that ~~the division between~~ changes in size and speed ~~as~~ are two independent modes of variability ~~is nearly equal~~that are nearly equally divided. A linear scaling has been noted between the ~~magnitude~~ size of a city's population and how fast its inhabitants walk (Bettencourt et al., 2007). More globally, over the 50 year period considered, world population – as a measure of size – increased at an average rate of 1.46% per year. Meanwhile, per capita world GDP – as a plausible metric for speed – increased at the nearly equivalent rate of 1.55% per year.

At some level, the empirical nature of Eq. 2 stands on its own~~.~~, and so too its implications for economic production through Eq. 3. Nonetheless, its simplicity may come across as counter-intuitive, especially considering that $W$ is not directly tied to any current economic transaction, only to the past. By way of explanation, consider the circulations within our bodies, brains, and machines, and our activities such as housework, transport to and from work and the grocery store, and even conversation among family and friends, that all of these require current energy consumption in some form. Each one of these may involve a financial transaction at some prior stage, for cleaning products, gasoline, or food, ~~yet~~ but crucially no financially quantifiable purchase is made at the point at which the energy is consumed, only in the past.

A possible counterargument is that economic models already account for recent purchases, but that historically distant production and consumption cannot linger to contribute to ~~current energy demands , considering that fig~~ energy demands today. Fig trees grown for the enjoyment of Ancient Greeks would seemingly have nothing to do with the power consumption of internet servers today. ~~The way the problem is approached in~~

The effective lifetime of past production can be estimated within models that employ traditional economic accounting~~is to consider that current capital is~~. Capital is formed through economic production $Y$ after subtracting both depreciation at rate $\delta$ and consumption $C$ of goods and services, ~~that~~ the underlying equation is $dK/dt = (Y - C) - \delta K$. Expressing consumption as $C = cY$ and adopting a simplified production function of form $Y = \beta K$ where $\beta = Y/K$ is the production efficiency (or the inverse of the capital-to-output ratio), it follows that the rate of capital formation is

$$\frac{dK}{dt} = (\beta - \delta')K, \tag{5}$$

~~so that the capital~~ Dividing both sides by $K$, the exponential growth rate of capital is $R_K = (\beta - \delta')$ where $\delta' = \delta + c\beta$~~:~~. Purely mathematically speaking, consumption itself can be viewed as a form of depreciation of very short-lived capital ~~. From~~ at rate $c\beta$, in addition to depreciation at rate $\delta$.

To obtain the value of $\delta'$, from data for $Y_i$ and $K_i$, the value of ~~$\beta$~~ $\beta = Y/K$ over the past 50 years has steadily declined at an average rate of 0.95% per year ~~with~~ but it has had an average value of approximately ~~0.24~~ 0.24 or 24%. ~~Considering that~~ Meanwhile, capital grew at an average annual rate of 4.0%~~over the same period~~. So, the implication is that the annual rate ~~$\delta'$ at which capital has been devalued~~ $\delta' = R_K - \beta$ of capital devaluation due to combined consumption and depreciation is approximately $24\% - 4\% = 20\%$~~, that is capital value in traditional accounting has a halving time of just~~. Effectively, previously produced capital halves its value in 3.5 years.

Well-known concerns may be raised about any comparison of rates of capital formation with capital valuation, and with how valuations of varied capital stocks should be aggregated (Samuelson, 1966; Sraffa, 1975). Nonetheless, ~~the~~ whatever the uncertainties, the implication that capital halves its value in just 3.5 years seems quite peculiarly short. The benefits of past productivity clearly persist for much longer. We may no longer use the personal computers of the 1980s, but ~~current devices are derived from~~ we would not have current devices without that seminal transformation. Going back further, Ancient Greek fig trees died over 2000 years ago, but important aspects of the culture of fig-eating Ancient Greeks ~~have lasted~~ continue to today.

The crux of this valuation problem appears to be that the long-distant~~contributions of past civilizations~~ , or even fairly recent contributions of humanity to politics, science, athletics, architecture, and language are implicitly ignored in traditional ~~accounting because they~~ economic accounting. Perhaps this is simply because historically important innovations – such as controlled combustion, or the alphabet – cannot be monetized on ~~an~~ the open market, even though without them ~~the bulk of our~~ most of modern infrastructure for wealth-generation would ~~disappear. As noted by Piketty, "~~collapse. Like "dark-matter" in astronomy that cannot be seen but is known to be the bulk of our universe, there appears also to be a "dark-value" in economics.

T. Piketty describes the issue well: "All wealth creation depends on the social division of labor and on the intellectual capital accumulated over the entire course of human history," continuing "the total value of public and private capital, evaluated in terms of market prices for national accounting purposes, constitutes only a tiny part of what humanity actually values - namely, the part that the community had chosen (rightly or wrongly) to exploit through economic transactions in the marketplace" (Piketty, 2020). ~~Capital~~ Here, we showed that capital $K$, as valued by curent markets, is ~~an~~ a full order of magnitude smaller than the more abstract quantity of historically cumulative production $W$. The scaling between $W$ and $E$ then suggests that energy consumption is required not just to sustain that which we believe potentially available to be sold today, but also the unspoken ~~utility~~ "dark-value" of that which has previously been produced. Civilization was not built in ~~a day~~3.5 years.

There are important analogs in the biological and physical world that may provide a useful guide to economic growth theory. ~~The energy of a wheel's~~ For the analogy of Lotka's Wheel, the energy of rotation is the product of its mass and the square of its radius and rotational frequency, all quantities that increase through a prior history of material and energetic increments. In a cloud, a snow crystal grows through the diffusion of vapor molecules; current vapor consumption depends on the reach of the crystal branches into the surrounding vapor field, insofar as the branches have built upon a prior accumulation of condensed vapor residing within the unexposed crystal interior (Lamb and Verlinde, 2011). The leaves of a deciduous tree enable photosynthesis that fuels fluid circulations through the exterior sapwood; the leaves die seasonally as the sapwood turns into heartwood that, while not actively connected to a larger rejuvenated leaf crown in the following year, structurally supports it (Shinozaki et al., 1964; Oohata and Shinozaki, 1979). Systems may even undergo quite dramatic changes in character while maintaining at all stages a dependence on previously consumptive states, such as with the succession of species that occurs during development of new forest following a major disturbance (Oliver, 1980). Inevitably there are loss processes, such as friction for a wheel, moments of evaporation or breakup for a snow crystal, or disease and predation for a tree or forest. But, in all cases ~~even quite distant~~ historical past consumption is the primary determinant of the system's current energetic demands.

## Conclusions

We have identified a nearly constant relationship $w$ between world historically cumulative inflation-adjusted economic production $W$ and current energy demands $E$. The scaling $W = wE$ has held for the past half-century, a period during which resource consumptive demands nearly tripled, suggesting that humanity's current metabolic needs are best considered as emerging from past innovations that allowed for surplus Haff (2014); Garrett et al. (2020). The relationship's persistence appears to place substantial bounds on humanity's future interactions with its environment. It implies that present sustenance cannot be decoupled from past growth, ~~or that inertia plays~~ inertia playing a much greater role in societal trajectories than has been broadly assumed, ~~for example~~ especially in the integrated assessment models widely used to evaluate the coupling between humanity and climate (Nordhaus, 2017). ~~Even~~

Thus, even if world GDP growth falls to zero from its recent levels close to 3% per year, the scaling suggests that long-term decadal-scale growth in resource demands and waste production will continue to accelerate. ~~The scaling suggests that it~~ It is only by ~~way of collapse of previous growth that led to the~~ collapsing the historic acccumulation of wealth we enjoy today, effectively by shrinking and slowing Lotka's Wheel, will our resource demands and waste production decline. Eq. 1 ~~offers no direct mathematical approach for~~ does not directly indicate what such an event ~~to occur, except perhaps through~~ would look like, although it does suggest hyper-inflation. In economic accounting, ~~this would lead to high values of~~ the GDP deflator ~~so that~~ would be sufficiently large for the inflation-adjusted real GDP ~~would~~ to be much lower than the nominal GDP. Historically, hyper-inflation has been associated with periods of societal contraction (Zhang et al., 2007) ~~pointing to a possible link of inflation to decay~~ suggesting some link between current economic inflation and the fraying of previously built societal networks (Garrett, 2012).

On the topic of climate policy, ~~a constant value of~~ the constant value for $w$ ~~permits~~ described here does not forbid economic production to become decoupled from carbon dioxide emissions~~, but only provided a rapid~~. However, the switch from carbon fuels to renewables or nuclear energy would need to be extraordinarily rapid. Simply to stabilize carbon emissions, much less reduce them, any newly added energy production ~~would need to be emissions free, which based~~ must be carbon emissions free. Based on recent consumption growth rates, this works out to about 1 Gigawatt of non-carbon energy per day. Alternatively, or concurrently, some means would need to be devised for decoupling historically cumulative wealth $W$ from current energy consumption $E$, effectively by increasing the value of ~~$w$~~ $w = W/E$. Given the value of $w$ has varied so little over the last 50 years~~while society has~~, a period during which society changed tremendously, it is difficult to conceive how this would be managed. That said, adjusting $w$ upward could be seen as a new target for mitigating future climate damages.

## Appendix A: Methods

Yearly statistics for world primary energy $E_i$ are available for both consumption and production from the Energy Information Administration (EIA) of the U.S. Department of Energy (DOE) for the period 1980 through 2018, and for consumption from British Petroleum (BP) for the years 1965 through 2019 (DOE, 2020; Bri, 2020). A yearly composite of $E_i$ in units of Exajoules per year for the years 1970 to 2019 is created from the average of the three datasets while using single sources where only one is available. The difference between the values in the BP and EIA data sets is significant, $8.5 \pm 1.5\%$, but it is steady, and small relative to the $180\%$ increase in energy consumption over the 50-year time period considered here. Economic production is tallied and averaged using World Bank (WB) and United Nations (UN) statistics for the years 1970 to 2019 (The World Bank, 2019; UNs, 2020) and expressed here in units of trillions of market exchange rate, inflation-adjusted "real" year 2019 dollars. Statistics for the aggregated capital stock of 180 countries $K_i$ are available from the Penn World Tables (PWT) (Feenstra et al., 2015). Uncertainties in UN, WB and PWT economic values are not published. They are assumed here, as with the energy estimates, to be small compared to the many factor increase in their sizes.

The world historically cumulative production $W_i = \sum_{j=1}^{i} Y_j$ requires for its calculation yearly estimates of $Y_j$ prior to 1970, for which we apply a cubic spline fit to the Maddison Database (Maddison, 2003) for years after 1 C.E. ~~Adjustments are made to the Maddison dataset to account for the chosen inflation-adjusted year of the dataset~~ The dataset is adjusted for inflation and to convert from currency expressed in purchasing power parity dollars ~~rather~~ to market exchange units using as a basis for adjustment the time period between 1970 and 1992 for which concurrent MER and PPP statistics are available. The value for cumulative production in 1 C.E. $W(1)$ is obtained by assuming that ~~the population and~~ $W$ ~~were growing equally fast~~ was growing as fast as population at that time at rate $R_W = d\ln W/dt$ and that $Y(1) = R_W W(1)$. Population data from 1.C.E and one century before and after ~~show~~ suggest that global population was 170 million and growing at 0.059 % per year (United States Census Bureau, 2021). While there are inevitable uncertainties in the reconstruction of $W$ as with any other, the yearly values of $W$ since 1970 that are emphasized here cover two-thirds of total growth and so the calculations are more strongly weighted by recent data that is presumably most accurate. Thus, calculation of $W$, most particularly the conclusion that $w$ is nearly a constant, can be shown to be relatively insensitive to uncertainty in the older statistics (Garrett et al., 2020).

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*Competing interests.* The authors declare no competing interests.

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