I welcome the author's constructive comments. The new diagrams allow me to further specify my critique. But let me start with a simple question. Imagine an economy in which GDP ´shrinks every year with a constant rate, until the economy simply vanishes. According to your approach, this economy would still add a certain amount of GDP every year, hence grow in the absolute terms of cumulative GDP. Would you still stick to your argument? In fact, this economy would also consume less and less energy. In my view, this simple argument shows that your addition approach is deeply flawed.

It doesn't sound as if the reviewer is questioning the empirical justification of the identity described in the article, which is its main focus. The arguments that are made here follow directly from the identity that is described. However, we agree that interpretation of the result requires some careful consideration, hopefully without immediately concluding that the approach is flawed. We have rewritten/rearranged some of the text a little to address the reviewer's point.

We interpret the quantity identified here as the historically cumulative global production W as an economic expression of the rotational power of Lotka's Wheel, that is the capacity to drive the collective to-and-from of civilization's circulations through the relationship $W = wE$, where w is nearly a constant. Certainly, an objection might be raised that the past 50 years is too short relative to the time span of humanity to draw meaningful conclusions about the relationship of historically cumulative production to current energy demands. Measured in units of years, this may be true. However, the last half-century covers a remarkable two-thirds of humanity's total growth expressed in terms of energy consumption, or 1.5 doublings in E, during which a great deal changed in humanity's social and technological makeup.

Taking the first derivative of Eq. 1 yields an inflation-adjusted economic production relation. Assuming $W = wE$ for constant w, then

$$Y = \frac{dW}{dt} = w\frac{dE}{dt} \quad (1)$$

Real economic production is related to the rate of increase *in world primary energy consumption. The implication is that the real GDP is a tally of the instantaneous monetary exchanges that, directly or indirectly, increase civilization's ability to access more energy in the future. For the case that $dY/dt = 0$, namely that there is constant inflation-adjusted economic production Y or zero GDP growth, energy demands expand at rate Y/w. If there is GDP growth, as preferred by governments, and $d\ln Y/dt > 0$, then world energy consumption accelerates.*

Eq. assumes only that w is a constant, a result that can be readily refuted, or supported as it is here with decades of data from multiple sources. The approach does nonetheless have some important limitations, notably an inability to resolve short-term, fine-scale behaviors. The evolution of cumulative inflation-adjusted economic production W is highly smoothed because it is a summation, or integration, over history and the global economy. Even given a strong multi-decadal relationship of E to W, year-to-year variability in E, such as during recessions or pandemics, cannot be easily related to yearly economic production, especially on national or sectoral scales much smaller than the world as a whole. That said, calculated as a running decadal mean, the average ratio of global production to yearly changes in energy consumption is

$$\widehat{w} = \frac{Y}{dE/dt} = 5.9 \pm 2.2 \quad (2)$$

in units of trillion 2019 USD per Exajoule consumed each year, which is very similar to that expressed for w given by Eq. **??***, although the variability is higher given the comparison of Y to a differential in E.*

Nonetheless, Eq. can also be seen as being highly counterintuitive, as it suggests for the hypothetical limiting case of $dE/dt = 0$ – one where the world attains a sort of metabolic steady-state with energetic and material inputs and outputs in balance – that real *world economic production disappears, that is $Y = 0$. Such a result would seem highly peculiar viewed from any traditional economic perspective.*

It is important to note, however, that zero real, inflation-adjusted production does not forbid non-zero, positive nominal production. If there is a large difference between the nominal and real GDP, it appears in economic accounts as high values of the GDP deflator, or as hyper-inflation. Interpreted physically, even as current production continues to grow those civilization networks that dissipate energy, there is concurrent rapid fraying of networks constructed through past production, sufficient to offset any productive gains Garrett (2014).

Thus, given the severe economic constraints to society, a metabolic steady-state may only represent a temporary marker prior to a more complete collapse, thermodynamic as well as economic. Along this pathway, any external resources that become available to civilization would no longer be sufficient to become an un-utilized residue. Like a patient consumed by cancer, any growth would be more than offset by internal consumption – burning the furniture to heat the house, so to speak. Nominal production may remain, but it is fueled more by internal than external resources. Eventually, the point of complete collapse is reached, whereupon both civilization power and nominal production equal zero.

Let me turn to the diagram showing Lotka's wheel. This appears confusing to me. The arrows labelled 'waste' seem to contribute to the growth of the un-utilized residue. That would violate the Second Law, as 'waste' is ultimately entropy production, which by definition cannot be utilized for running the wheel. A correct diagram would show how the wheel contributes to harnessing more energy. Further, the diagram suggests that energy is simply speeding up the wheel. That does not match with Lotka's original account: What is missing is the key role of natural selection. Natural selection is about competing wheels, such that eventually those surpass others in competition that increase energy throughputs, in relative terms.

We have adjusted the diagram for clarity. The un-utlized residue is not waste, and it contributes as stated in the text to both speeding up and growing the wheel. The aspect of natural selection is not addressed explicitly in this study, although it is a topic for a future publication. It's not strictly important here as the treatment is of civilization as a whole, which is a) unique in the natural world in terms of its power, and b) doesn't compete with other civilizations.

Why is this important for assessing the authors' approach? Lotka considers the evolution of structures. The question is how far cumulative annual GDP reflects this. The authors refer to Bettencourt et al. and indeed, the paradigmatic example for the wheel, in the economic context, is urbanization: cities are Lotka's wheels. Urban growth is the key driver of GDP growth and wealth creation. But Bettencourt and many others have argued that the cause of this is precisely what is NOT covered in GDP, by definition, namely positive externalities of knowledge production. This is the gist of New Growth Theory modelling, and central to urban and regional economics. Indeed, positive externalities are what Piketty refers to as 'dark matter'. Knowledge grows, in this sense is a stock, and knowledge guides the designs of technological artefacts that make up Lotka's wheel.

Now, if by definition positive externalities are not covered by GDP data, how can one argue that GDP, added up through time, reflects the resulting stock of knowledge? The authors must present a convincing causal account for that, which is still missing. For me, this is simply a logical contradiction, given the definition and measurement of GDP. The authors seem to suggest that cumulative GDP directly reflects network growth in figure 2. But what is the causal process that would establish such a connection?

We nowhere argue that "the GDP, added up through time, reflects the resulting stock of knowledge". Certainly knowledge (presumably encapsulated in neuronal connections) is one component of civilization networks, but only among a very great many that include such material connections as fiber-optic cables. As for a causal process, this was discussed extensively in prior papers. There is a positive feedback governing the growth of an interface between civilization and its resources (Garrett, 2014, 2015). The past inflation-adjusted GDP built these networks, and these enable the current GDP, so there is a positive feedback leading to network growth. These dynamics are not the focus of this rather more simple paper which is focused on the constant described, although they are alluded to just prior to the conclusions where we have added the statement.

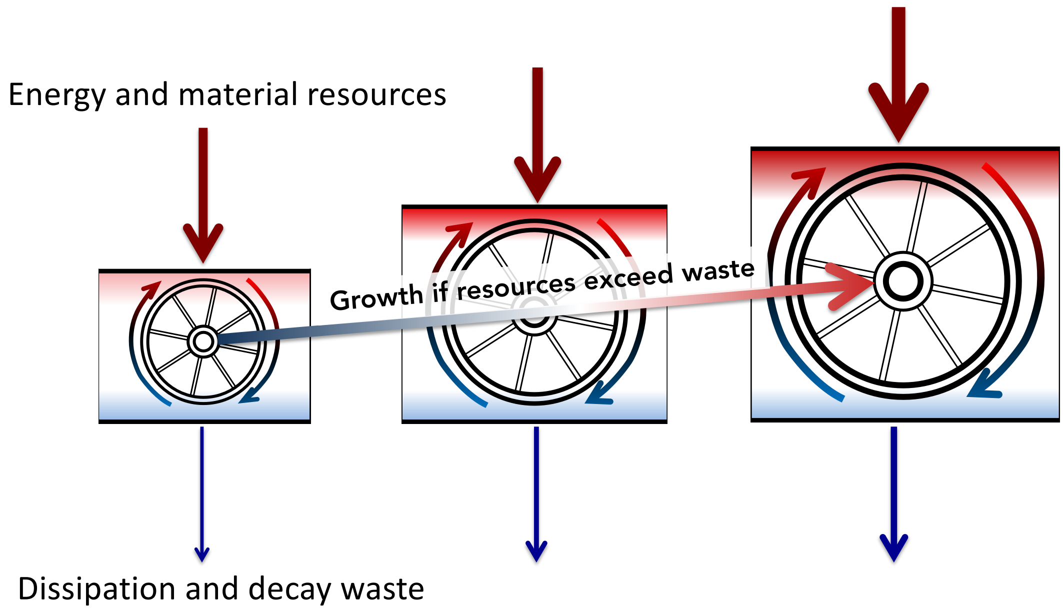

Figure 1. Representation of Lotka's view on the thermodynamic mechanisms governing system growth, involving a wheel that enlarges and accelerates using an "un-utilized residue" of energy and matter

In this fashion, the dynamics governing trajectories of civilization's energy consumption and GDP growth can be shown to be governed by resource discovery and environmental decay, and in a manner that accurately reflects its evolution between 1960 and 2010 (Garrett, 2014, 2015).

The authors do not need to teach an economics professor 1q‘ed. But how does energy consumption causally relate to value added as a measure? I agree that gross production in value terms may also fail to be a good choice, but for that reason I also referred to Material Flow Analysis or other Input-Output frameworks. Every intermediary stage of production is a movement in Lotka's wheel, isn't it? Why do you disregard this as irrelevant?

In no way does the paper treat any current action as irrelevant in terms of energy consumption, including the intermediate steps that factor into gross production. In fact the associated energy consumption is precisely the point. Any current action requires networks only made possible through prior (not current) production, specifically the inflation-adjusted real production that did add to total network value W, and hence to the then future capacity to consume energy E.

Lotka's wheel is not an accounting scheme which ultimately refers to subjective values, meaning preferences as expressed in demand for products priced on the marketplace and thereby becoming manifest in GDP measures. Lotka's wheel is a physical structure moving forward. My flute example is a mere didactic story that illustrates that claiming connection between value added and energy requires a causal account about underlying matterenergy facts. Teaching music could be regarded as extremely valuable, as compared to building fancy cars, in different societies. That would result in very different wheels, wouldn't you agree?

Science does not need to result in universal consensus. Hence, I do not expect you to give up your approach, but to add a convincing story about the underlying physical mechanisms. Perhaps one argument could be that in the past, society preferred the fancy cars, and not the music, hence creating value added resulted in a high powered wheel, through time. That would explain the correlation?

We believe the reviewer misinterprets the implications of the results. Major cities have symphony orchestras composed of musicians who typically aren't paid that much, and who use human energy to exercise their craft. Meanwhile, the value of their contribution to society is immense – even to those who do not attend performances. Suppose that New York lacked any symphony or opera. It would presumably lose a quantity of real estate value far greater than the amount of money required to simply sustain the orchestra, because the city would be seen generally as being in decline, less culturally vibrant and to many an unattractive place to move. Symphonies are subsidized for good reason. New York without any orchestra would be worth far less.

So simultaneously flutes can consume little energy while teaching flute playing can be highly valued, not because flute teaching is energy intensive but because flutes are embedded as an intrinsic part of a larger network – civilization – that consumes an immense amount of energy. Any given component of society, flutes or fancy cars, cannot be disconnected from any other, at least in any meaningful fashion, nor can be assigned a value uniquely related to its energy consumption without considering the relationship to other things in energy dissipative networks. This is why we focus here on civilization as a whole.

References

Garrett, T. J.: Long-run evolution of the global economy: 1. Physical basis, Earth's Future, 2, 127–151, https://doi.org/10.1002/2013EF000171, 2014.

Garrett, T. J.: Long-run evolution of the global economy - Part 2: Hindcasts of innovation and growth, Earth Syst. Dyn., https://doi.org/10.5194/esd-6-673-2015, 2015.