Response to Reviewer 1

We place the reviewer comments in bold font and our response in normal font.

(1). I object to arbitrarily introducing a new production function (equation 3) without serious discussion. The discussion in the text, based on curves in Figure 1 is not nearly sufficient to justify equation 3. The standard Cobb-Douglas production function was introduced in 1928 for a good reason and the other production functions economists have introduced and tested since 1928, have histories also. I am not defending any of them, but the reasoning behind equation 3 cannot simply be based on the data represented by the curves in Figure 1.

We agree that Equation 3 is not a production function in the same sense as e.g. the Cobb-Douglas or Leontieff production functions, namely a function that calculates the output of economic production Y on the basis of inputs, or factors of production, typically capital K and labour L. As mentioned in the first paragraph, such “production functions" (deliberately put in quotation marks on line 16), grossly underestimate the role of energy in production by treating the energy sector as just one of many other sectors in the economy. By contrast, we are interested in the direct relationship between primary energy consumption and economic output, without going through the route of a production function in the economic sense. For this reason, we termed Equation 3 an “inflation-adjusted production relation" on line 67 (immediately before Equation 3), but regrettably used the term “production function" on line 72 (immediately after Equation 3), which we will change to “production relation".

Having said that, the result expressed in Equation 3 is a direct consequence of the 50 years of data in Figure 1 supporting a nearly fixed relationship between historically cumulative GDP and current energy consumption as given by $W(t) = \int_0^t Y(t')dt' = wE(t)$. Namely, we first *define* W as the integral of production Y in Equation 1 and then *observe* in Equation 2 that this quantity is related to energy E through a nearly constant ratio w over a period during which the GDP increased by a factor of 4.5. Equation 3 then follows purely mathematically from Equations 1 and 2 (namely by taking derivatives and assuming that w is constant), that is, $Y(t) = wdE/dt$. Equation 4 then further validates this relation by estimating w as the ratio between Y and dE/dt, with a value similar to the one found in Equation 2.

The physical arguments supporting why might the relationship hold are as described by Lotka (1922) in the introduction and elaborated upon in the discussion. It is not clear from the comment why empirical evidence and physical reasoning is considered “arbitrary" as the basis for establishing a link between economic production and growth in energy consumption.

(2). In another place, the authors note that the usual relationship for capital growth is [dK/dt = Y-C minus delta (capital depreciation)], where delta (the depreciation rate) is constant and C = cY where c also is assumed to be a constant. The last assumption is wrong. The ratio c = C/Y (the fraction consumed) may leave a significant surplus for capital investment now (and for the past 50 years) but 200+ years ago c was practically unity while the depreciation rate was smaller than it is now – the surplus for investment or saving back then was virtually zero, and what surplus there was came from coal mines. In other words, until very recently almost everybody needed every bit of their income to buy consumables, mainly food and fuel (for light and heat). So, in the long run c is not a constant; it can (and will) decrease. Neither is the depreciation rate constant, by the way. Most people will spend their time playing computer games.

Nowhere in the article is it stated as the reviewer claims that depreciation or the ratio of consumption to production c is constant. See lines 100 to 105 which in fact point to the opposite. Average rates are provided, but this is stated by acknowledging that the instantaneous rates have changed during the period under consideration. The point being made is that whatever new that is produced is either consumed or depreciates at a rate that appears too rapid to allow for societal contributions from the more distant past.

Also (3) the curve shown for capital stock K in Figure 1 is presumably based on prior work by Garrett but – being central for the rest of the argument – the underlying data also needs explanation and justification, especially since

Garret's earlier work in this field has not been widely accepted. (That is not a criticism). The underlying capital stock K data for Figure 1 should be published.

The underlying capital stock data are from the Penn World Tables and are referenced, as are all other quantities. See the Appendix for methods.