Dear Prof. Levermann,

Thank you for your decision. We have adopted minor revisions that you and two anonymous reviewers suggested.

Mikhail Verbitsky and Michel Crucifix

November 18, 2020

Anonymous Referee #1.

Comment: In your model Eqs. (1) - (5) you represent two ice sheets (Laurentide and Greenland) which are independent from each other. To which extent can the Laurentide and Greenland ice sheets be considered to be independent from each other considering their close proximity? It seems to me that the whole argument rests on this assumption.
Answer: The equations (1) – (5) and corresponding diagram of Fig. 1(a) describe the dynamical system when the Greenland ice sheet is coupled one-way via the climate temperature ω acting as an external forcing for it. We used this configuration for our scaling reasoning. For the numerical experiment, we added a term $-\gamma_2 S_G$ in the equation (3) thus making full coupling of the Laurentide and Greenland ice sheets (lines 122-124). Full coupling is, indeed, important. As we demonstrated earlier (Verbitsky et al, 2019, https://doi.org/10.5194/esd-10-257-2019), millennial forcing may penetrate upscale and under some conditions may significantly change the orbital-timescale dynamics.
Action: To articulate this more clearly, we suggest adding one more panel to Fig. 1 that would show both dynamical configurations:

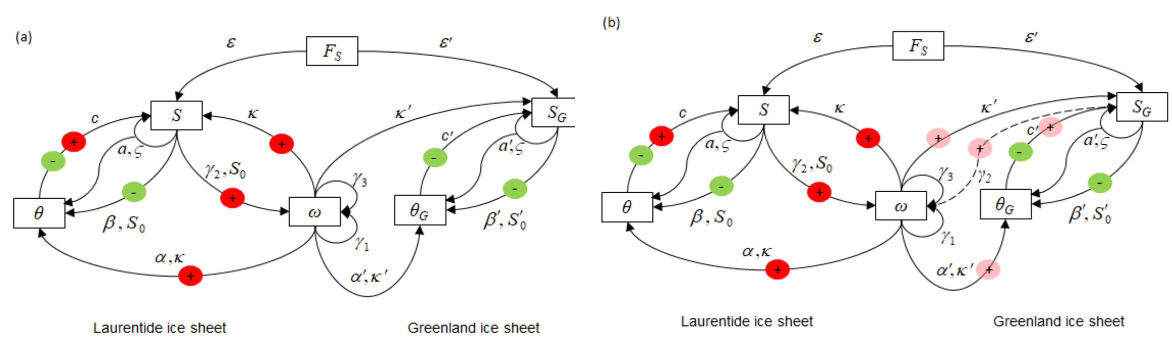

Fig. 1: (a) the dynamical system (1) – (5) as it has been used for scaling reasoning. The Greenland ice sheet is coupled one-way via the climate temperature ω acting as an external forcing for it. Red circles mark positive feedback loops and green circles mark negative feedback loops; (b) the dynamical system (1) – (5) as it has been used in the numerical experiment. The Laurentide and Greenland ice sheets are fully coupled. Pink circles mark weaker positive feedback loops. **Done: Lines 125-127, Fig. 1(b), lines 235-239**

Comment: Or could Greenland be substituted with Antarctica (or would the size of Antarctica change the dynamics)?
Answer: Your observation regarding the proximity is correct: We do not consider here a number of mechanisms that might be important in the close proximity of Laurentide ice sheet - our coupling is made through the global albedo and global temperature only. Nevertheless, we cannot simply replace the Greenland ice sheet with the Antarctic ice sheet. Even though the area of the Antarctica is also limited, we may anticipate that the larger area of the Antarctic glaciation implies thicker ice sheet, bigger Peclet number, weaker internal negative feedback, larger amplitude and period of the relaxation oscillations, and,

therefore, possibly stronger influence on the global climate including Laurentide ice sheet. Besides, Antarctic ice sheet volume changes cause stronger sea-level variations, which are immediately distributed worldwide and would immediately affect Laurentide ice sheet, not via the global temperature but via sea-level. For that reason, to answer your question confidently, an additional study may be needed.
Action: We will add this discussion into the text. **Done: Lines 139-143**

Anonymous Referee #2.

Main comments

Comment: The key element of this paper is the Peclet number defined as “aH/k”, with “a” the mass balance, “H” the ice thickness and “k” the temperature diffusivity. The authors assume that this number is an increasing function of ice-sheet size. But standard knowledge of ice-sheets suggests that this is very likely to be just the opposite. Indeed, today we have a rather good knowledge of two ice-sheets (Greenland and Antarctica).The size of Greenland is smaller than the size of Antarctica and its height H is roughly 70% or 80% of the one of Antarctica. But the advection parameter “a” (ice accumulation minus ablation) is certainly much higher in Greenland (about 5 to 10 times higher). This is partly due to atmospheric circulation and continental set-up, but mostly due to ice sheet height: the larger the ice sheet, the higher its surface, the drier the climate. I therefore do not think reasonable to assume that the Peclet number defined above would increase with ice sheet size. On the contrary, I expect it to decrease strongly. Overall, diffusion should dominate the dynamics of large ice sheets, since they have very little precipitations. This is likely the case for the Laurentide ice sheet in the past, as it is for Antarctica today. I therefore strongly disagree with the main message of this manuscript.

Answer: Let us first examine the claim that “Overall, diffusion should dominate the dynamics of large ice sheets, since they have very little precipitations”. The established theory for ice sheets (Grigoryan et al, 1976) concludes that above the basal diffusive boundary layer “the temperature propagates along the particle trajectories… and conductive heat transfer causes small variations in the temperature field”. Indeed, an ice sheet is considered here in a “thin-layer” approximation, i.e., it is a glacial object with a vertical dimension that is much smaller than the horizontal dimension. Therefore, horizontal diffusion of heat can be neglected, and the relevant length scale to be included in the Peclet number is the vertical length scale. Now, let us examine the Peclet number. For ice thickness H ~ 3,000 m, ice temperature diffusivity $k = 10^{-6}$ m^2/s, and $\hat{a}$ ~ $3x10^{-9}$ m/s (the latter, 10 sm/yr, would correspond to the dry conditions of, for example, East Antarctic ice sheet), the Peclet number $Pe = \hat{a}H/k$ ~ 10. It means that temperature advection dominates vertical temperature diffusion even for very dry conditions. Extremely low temperature diffusivity of ice is responsible for this phenomenon. We therefore conclude that this first claim of the reviewer is incorrect.

Let us now consider the second claim that the Peclet number would “decrease strongly” for growing ice sheets because ice accumulation is smaller. What should be observed here is that over a full glaciation phase, starting from no ice sheet to a fully mature ice sheet before the deglaciation, the ice sheet undergoes more or less rapid phases of expansion. For example, the final growth phase between marine isotopic stages 3 and 2 was pretty rapid, indicating substantial accumulation. The reason for this is that the mass balance at a given time results from the interplay of multiple feedbacks, and the quoted by the reviewer “continental feedback”, causing less precipitation as the ice sheet grow, is only one of them. If, for the sake of the argument, we consider that the accumulation has been roughly constant throughout the growth phase, then we can reasonably conclude that the Peclet number increased along with the growth of the ice sheet. In other words, the Peclet number of *a same ice sheet growing over time* under constant positive mass balance increases (as we say in the paper “*the Peclet number, changing in concert with the glaciation size*”). The main premise of the paper **-** The Laurentide ice sheet has the space to grow its Peclet number over time, but the Greenland ice sheet does not - is based on this property. We thus conclude that the second claim of the reviewer is incorrect.

Comparing the Antarctic and Greenland ice sheets is partly misleading for two reasons: first this is a specific instant in the glacial-interglacial cycle (precisely when these ice sheets no longer grow), which is not representative of the dynamics of ice sheet formation and growth. Second, it compares *different ice sheets of different geographical locations*. Nevertheless, the reviewer argument is helpful to us because it lead us to identify an inaccuracy in lines 81-83: When we compare Greenland and hypothetical "big ice sheet", we stated that Greenland's Peclet number may be smaller because the ice sheet is thinner. In fact we tacitly assumed that the mass balance of these two ice sheets is similar. In the new version we will clarify this point.

Action: We will include some elements of the above discussion in the text to make our thinking more explicit. **Done: Line 82**

Comment: I also find it difficult to appreciate the relevance of such a model without any result in the time domain. If I understand well the wavelet diagram, the model exhibits a very strong 400-ky oscillation that is certainly not observed (the famous 400-kyr problem…). I therefore doubt that this (rather complex) model can bring any insight in the problem of Quaternary climate variability.

Answer: First, we would like to reassure the reviewer about the absence of spurious 400-ka periodicity in the VCV18 (Verbitsky et al. 2018) model. The original publication (Verbitsky et al. 2018) shows side by side continuous wavelet spectra of the LR04 benthic stack and of model output (Figure 13), and the slight yellow band at 400 ka is present in both the record and the model output, suggesting at least that the model passes the spectral test. In fact, that article discusses how 400-ka periodicity could be generated by further altering the balance between positive and negative feedbacks in this model (the so-called *V*-number). Hence, we would consider that, in the context of this note, this otherwise interesting subject does not deserve more discussion.

Our choice of a continuous wavelet representation has been guided by its convenience in allowing visualization of both orbital and millennial variations on the same diagram. Given the space restriction of the "idea" ESD format we propose to maintain this approach. However, for the reference to the reviewer (and interested reader) and for the sake of transparency, the time series of the Greenland area of glaciation that has been used as a source of the wavelet diagram is provided here (Fig. AC2-1).

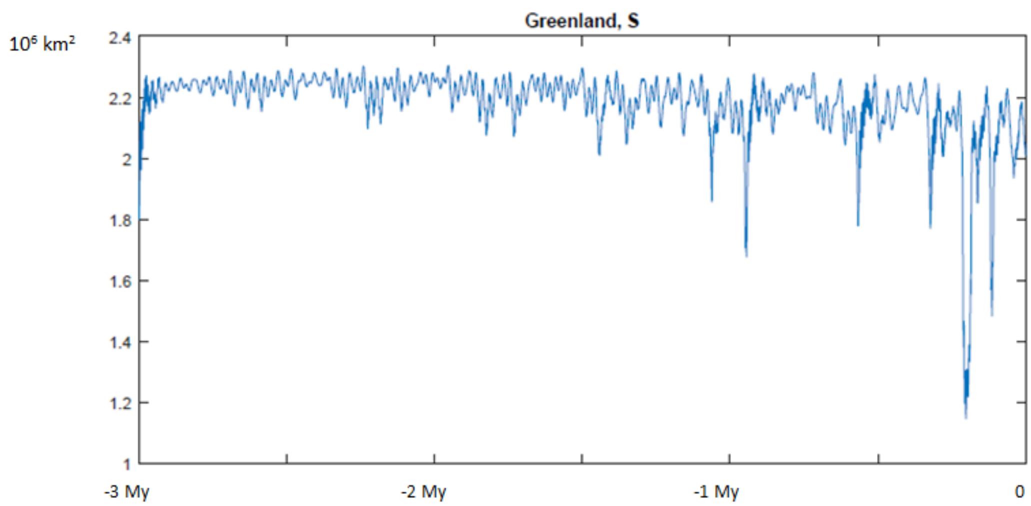

Fig. AC2-1: Simulated 3-My time-series of the Greenland ice sheet area S (10^6 km^2)

Action: We do not suggest any actions here. **No action.**

Other comments

Comment: The output of the model is not actually compared to observation, but only shows "more variability" in the both the 100-kyr band and the millennial band. In other words, I find that the complexity of the model does not scale reasonably well with its results.

Answer: The purpose of our paper is to provide novel explanation to the observed coherence between millennial and orbital-scale variabilities: specifically, after the MPT, millennial events are more frequent and more intense than before. Therefore, our focus on the model output that demonstrates such coherence should not be surprising.

The success of a theory is indeed in good part relates to how much it explains given a small number of adjustable parameters (e.g. Hitchcock and Sober, 2004). This is in fact the main selling point of our VCV18 model which, in our view, is a step forward on previous low order models of glacial-interglacial cycles. We have demonstrated (Verbitsky and Crucifix, 2020) that our model has a property of similarity and a number of parameters can be reduced to only two adimensional parameters. Therefore we have the following arguments to bring here:

- Our reasoning remains heavily dependent on physical constraints and established principles of glaciology. This does not leave so much room for playing around with parameters in a way that would force the conclusions that we want to see

- We are here in the format of an “idea” paper: our objective must be to frame a hypothesis on the basis of established theoretical elements and observations. We certainly agree that this “idea” requires further investigation and deserves being challenged, but the question here is whether we have provided enough arguments to make it credible; that is, whether this augmented VCV18 model provides a formal framework which is credible to make the hypothesis plausible. We definitely believe that this is the case.

Action: We welcome suggestions of the editor if further clarifications about the epistemological status of our hypothesis are needed. **No action.**

ESD Ideas: The Peclet number is a centerstone of the orbital and millennial Pleistocene variability

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Abstract. We demonstrate here that a single physical phenomenon, specifically, a naturally changing balance between intensities of temperature advection and diffusion in the viscous ice media, may influence the entire spectrum of the Pleistocene variability from orbital to millennial time-scales.

Introduction. About 1 million years ago, the dynamics of glacial-interglacial cycles experienced a major change, transitioning from the predominant 40-kyr periodicity to approximately 100-kyr variability (e.g., Ruddiman et al., 1986; Lisiecki and Raymo, 2005). This change of the glaciation rhythmicity is called the middle-Pleistocene transition (MPT). On the other hand, Hodell and Channell (2016) presented the analysis of 3.2 Myr records of stable isotopes and sediment physical properties, and observed coherence between millennial and orbital-scale variabilities: after the MPT, millennial events are more frequent and more intense than before. In our previous work (Verbitsky et al., 2019), we have shown that millennial variability may penetrate upscale and influence the slower pace dynamics of glacial-interglacial cycles. Here, we will demonstrate that the same ice sheet non-linearity that is responsible for the penetration of the millennial oscillations into the orbital domain may also contribute — without invoking any additional physics — to the millennial variability and explain its coherence with orbital-scale dynamics. To underline the significance of this proposal, we briefly summarize the current understanding of millennial variability.

Classically, millennial variability during the glacial periods is explained by reference to two families of mechanisms acting in concert. The first one relates to iceberg calving, which occurs when ice sheets have reached the ocean margins and ice-shelves developed. Episodes of intense ice-rafting generate a halocline and reduce ocean ventilation (the *Heinrich* events); on the other hand, changes in ocean temperature and sea-level influence the stability of ice shelves and may pre-condition calving events. The second family of mechanisms involves the dynamics of ocean circulation, deep-water mixing and sea-ice formation at high latitudes. The ocean circulation effectively causes horizontal and vertical transports of buoyancy, which have a role in maintaining the circulation itself. Because of the non-linear character of this feedback, the geographical structure of convection in the North Atlantic realm can shift very rapidly (within years) between different configurations. There is converging evidence that the *Dansgaard-Oeschger* events identified in Greenland correspond to such circulation changes. Millennial variability associated with all these mechanisms is most likely to occur during cold periods (ice sheets must be large enough) but not too cold (sea-ice must not be locked into a deep freeze): a so-called "sweet-spot" (Mitsui et al., 2018; Pedro et al., 2018; Li and Born, 2019) during which iceberg calving and ocean-circulation changes can entertain a rich pattern of oscillations (Schulz et al., 2002). The mechanism that we suggest here is not subject to the same sweet spot, and can actually explain a pervasive variability during both glacial *and* interglacial periods.

Methods. We consider the non-linear dynamical model of the global climate system presented in (Verbitsky et al, 2018). It is derived from the scaled conservation equations of the non-Newtonian ice flow, and combined with a linear feedback equation of the climate temperature:

$$\frac{dS}{dt} = \frac{4}{5}\zeta^{-1}S^{3/4}(a - \varepsilon F_S - \kappa\omega - c\theta) \tag{1}$$

$$\frac{d\theta}{dt} = \zeta^{-1}S^{-1/4}(a - \varepsilon F_S - \kappa\omega)\{\alpha\omega + \beta[S - S_0] - \theta\} \tag{2}$$

$$\frac{d\omega}{dt} = \gamma_1 - \gamma_2[S - S_0] - \gamma_3\omega \tag{3}$$

Equations (1) - (3) describe dynamical properties of a large continental (e.g., Laurentide) ice sheet. Here S (m^2) is the glaciation area, θ (°C) is the basal ice sheet temperature, and ω (°C) is the global climate temperature. The profile factor ζ ($m^{1/2}$) is determined by ice viscosity, density, and acceleration of gravity; a (m/s) is snow precipitation rate; F_S is normalized mid-July insolation at 65°N (Berger and Loutre, 1991); ε (m/s) is the amplitude of F_S; κ (m s^{-1} °C^{-1}) and c (m s^{-1} °C^{-1}) define ice mass balance sensitivity to ω and θ correspondingly; the adimensional coefficient α is the basal temperature sensitivity to ω changes, β (°C /m^2) defines basal temperature sensitivity to the changes of the ice sheet area S, S_0 (m^2) is a reference glaciation area; γ_1 (°C/s), γ_2 (°C m^{-2} s^{-1}) and γ_3 (s^{-1}) define the relaxation dynamics of ω.

We now supplement this system with equations (4) and (5) that describe the thermodynamical properties of a smaller-size (e.g., Greenland) ice sheet.

$$\frac{dS_G}{dt} = \frac{4}{5}\zeta^{-1}S_G^{3/4}(a' - \varepsilon' F_S - \kappa'\omega - c'\theta_G) \tag{4}$$

$$\frac{d\theta_G}{dt} = \zeta^{-1}S_G^{-1/4}(a' - \varepsilon' F_S - \kappa'\omega)\{\alpha'\omega + \beta'(S_G - S'_0) - \theta_G\} \tag{5}$$

Equations (4) and (5) are identical to equations (1) and (2). Here S_G (m^2) and θ_G (°C) are the area and the basal temperature of the Greenland ice sheet; all other parameters have the same dimensions and meaning as the corresponding parameters in the equations (1) and (2). Some (but not all) of them may have different numerical values, and for this reason we mark them with an apostrophe. The area of the Greenland ice sheet is limited by the size of the island and its variations have limited impacts on the global climate. Therefore, for reasoning on scaling relationships, we can neglect its contribution to the ω-evolution and leave equation (3) unchanged. Hence, the Greenland ice sheet model may be reduced to a system formed by equations (4) and (5), forced by the astronomical forcing and the global climate temperature ω. The dynamical system formed by the equations (1) – (5) is represented graphically on Fig. 1(a). Without external forcing, the evolution of S_G and θ_G is fully determined by 5 parameters (namely a', ς, S'_0, β', and c'). Some combinations of these parameters generate a relaxation-oscillation behavior, and as we show now, it is reasonably straightforward to estimate the period P of this oscillation. Indeed, if we take parameters a', S'_0, and β', as parameters with independent dimensions, and consider parameters ς and S'_0 as constant, then, using the generalized π-theorem (Sonin, 2004), the time scale P must obey:

$$P = (a')^{-1}S'^{1/2}_0\Psi(\Pi_1) \tag{6}$$

where Ψ is dimensionless function of $\Pi_1 = a'/(\beta' c' S'_0)$. It can be determined experimentally that the period P is smaller when Π_1 is smaller. Following Verbitsky and Chalikov (1986), we now introduce the Peclet number as $Pe = \hat{a}H/k$, where $\hat{a}$ is a characteristic mass influx, i.e., accumulation minus ablation, H is ice thickness, and k is temperature diffusivity. It measures the balance between temperature advection and diffusion. Since the Greenland ice sheet is relatively thin, then for comparable $\hat{a}$, its Peclet number is smaller than that of a big ice sheet, and therefore the effect of the geothermal heat flux on the basal temperature is, in relative terms, stronger. Verbitsky et al (2018) showed that the parameter β, which determines the basal temperature response to the changes of the ice sheet size, emerges as a delicate balance between vertical advection, internal friction, and geothermal heat flux, and that it is proportional to $Pe^{-1/2}$. Thus, the relatively small size of the Greenland ice sheet implies a higher β' and, according to the equation (6), its relaxation oscillations have a higher frequency than, say, the Laurentide ice sheet. In fact, numerical experiments with equations (4) - (5) and with a reasonable set of parameters produce millennial-range relaxation oscillations of the Greenland ice sheet.

With these considerations in hand, let us foresee the consequences of the MPT. As the Laurentide ice sheet reached gradually increasing footprints on the large American continent, its Peclet number increased, implying that temperature advection became the dominant process in the moving ice media. Consequently, β became smaller. We previously showed (Verbitsky et al, 2018, Verbitsky and Crucifix, 2020) that the dynamics of the system described by equations (1) - (3) is largely determined by a V-number measuring the balance between positive and negative model feedbacks:

$$V = \frac{1}{\beta}\left(\alpha + \frac{\kappa}{c}\right)\left(\frac{\gamma_2}{\gamma_3} - \frac{\gamma_1}{S_0\gamma_3}\right) \tag{7}$$

Thus, high values of the parameter β (V ~ 0.1) imply a weak positive feedback in the system, and, conversely, low values of the parameter β (V ~ 0.75) imply a strong positive feedback. As it has been previously shown (Verbitsky et al, 2018), when the orbital forcing is large enough compared to the average ice accumulation (high enough ε/a ratio), an increase in the V-number generates a period-doubling bifurcation, that is, an escape from the 40-kyr oscillation regime towards longer glacial-interglacial cycles. The increased period of the system response to the astronomical forcing also allows the glaciation area S to become larger, meaning that the amplitude of the oscillation is larger after the MPT than before. This amplitude increases may be understood as a consequence of a scale-invariance property (Verbitsky and

Crucifix, 2020). In this case, we see that the natural evolution of the Peclet number generates higher-amplitude oscillations of the larger ice sheets (i.e., Laurentide ice sheet), along with longer glacial-interglacial cycles. On the other hand, over Greenland, the Peclet number cannot grow. It remains bounded by a small value which allows millennial relaxation oscillations. The large post-MPT oscillations of the Laurentide ice sheet may then excite more vigorously the millennial oscillations of Greenland. The idea of the present contribution is therefore the following: *the Peclet number is a key quantity for explaining the joint emergence of large-amplitude oscillations of the bigger ice sheets, along with the increase in the occurrence of the millennial oscillations of the smaller-size ice sheets.*

To test our theoretical perspective, we reproduce the Pleistocene glaciation history, solving system (1) – (5) jointly, thus calculating a coevolution of the Laurentide and Greenland ice sheets. For the Laurentide ice sheet, we change linearly the ε/a ratio over the last 3 million years from ε/a = 0.3 to ε/a = 1.7 and change the reference glaciation area S_0 from S_0 = 2 10^6 km^2 to S_0 =12 10^6 km^2 thus invoking a hypothetical trend in processes that control long-term CO_2 levels. The ε/a ratio is reduced by changing the *a*-component only, following the assumption that the Pleistocene cooling trend goes along with a decrease in the snow precipitation rate. The parameter β is modified as being proportional to $Pe^{-1/2} \sim H^{-1/2} \sim S_0^{-1/8}$ (Since $H \sim S^{1/4}$, Verbitsky et al, 2018), changing from β = 2.4 ^oC 10^{-6} km^{-2} to β = 1.9 ^oC 10^{-6} km^{-2}. To account for the relatively small effect of Greenland changes on climate, which we have neglected for our scaling reasoning, we add a term $-\gamma_2(S_G - S'_0)$ into the right side of the equation (3). The dynamical system (1) – (5) as it has been used in the numerical experiment is shown in Fig. 1 (b). The Laurentide and Greenland ice sheets are fully coupled here. For the Greenland ice sheet, we assume that the reference glaciation area depends on climate temperature, i.e., $S'_0 \sim -\omega$, $\beta' \sim {S'}_0^{-1/8}$, and the ratio Π_1 is an order of magnitude less than the corresponding ratio of the Laurentide ice sheet. In Fig. 1(c) we present the results as a wavelet spectrum over the past 3 million years for the Greenland glaciation area S_G. We can see that though the Greenland ice sheet itself has a limited influence on the global climate temperature ($\gamma_2 S_G << \gamma_2 S$), the changes of Greenland's geometry reflect all major events of the Pleistocene history – a transition from the double-precession 40-kyr variability to increased amplitudes of double-obliquity oscillations. The global climate temperature ω, driven by the Laurentide ice sheet (taken here as a proxy for all the large ice sheets of the Northern Hemisphere), shifts the equilibrium state of the Greenland ice sheet, and the latter responds with millennial-period oscillatory adjustments. Such shifts are more prominent in the late Pleistocene and therefore, consistently with Hodell and Channell (2016), the millennial events of the late Pleistocene are more frequent and more intense. The amplitude of S_G millennial variability is ~ 0.1 10^6 km^2, corresponding to ~ 0.15 10^6 km^3 of ice volume or ~ 0.4 m of the sea level change. Talking about the sea-level, we must advise our readers that the coupling of the Greenland and Laurentide ice sheets has been done here via global albedo and global temperature only. In other situations (e.g., consider the Antarctic ice sheet) ice sheet volume changes may cause stronger sea-level variations, which are instantly distributed worldwide and would immediately affect the Laurentide ice sheet.

Discussion. Until recently, the time-scale separation approach (Saltzman, 1990) has been implicitly or explicitly used to consider separately glacial-interglacial cycles and millennial variability. Accordingly, it is generally accepted that the spectrum of Pleistocene variability in the orbital domain is defined by continental ice sheets, while the millennial part of the spectrum is to be attributed to ocean-atmosphere interactions. We first challenged this approach by demonstrating that ice-sheet non-linearity allows millennial variability to propagate upscale and influence ice-age dynamics (Verbitsky et al., 2019). Here, we show that the same non-linear ice-flow dynamics can also affect the millennial part of the spectrum.

From equation (7), it appears that different mechanisms would explain an increase in the *V*-number throughout the Pleistocene. Therefore, several scenarios can be invoked as the origin of the MPT. For example, a steady decrease in the background climate temperature (S_0, γ_1), an increase in climate sensitivity to the ice volume (γ_2), a change in the sensitivity of ice mass balance and its temperature to global climate temperature (κ and α), or even a decrease in the intensity of ice sliding (c). At the same time, all these mechanisms produce the changes in the Peclet number which we have described. For this reason, we claim that *the Peclet number, changing in concert with the glaciation size, is the key similarity parameter connecting the millennial and orbital Pleistocene variations.* We find it remarkable that the same physical phenomenon can explain all major events of the Pleistocene in the orbital domain if the evolution of an ice sheet is not restricted, and in addition, generate variability in the millennial domain when the size of the ice sheet is bounded.

Author contributions: MYV conceived the research and developed the formalism. MYV and MC contributed equally in writing the paper.
Competing interests: The authors declare that they have no conflict of interest.
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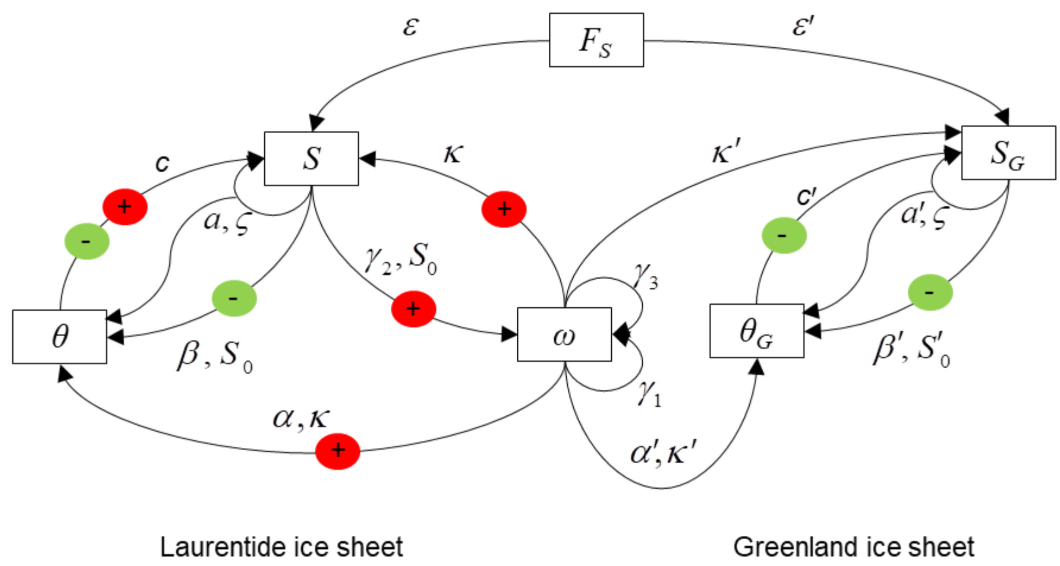
(a)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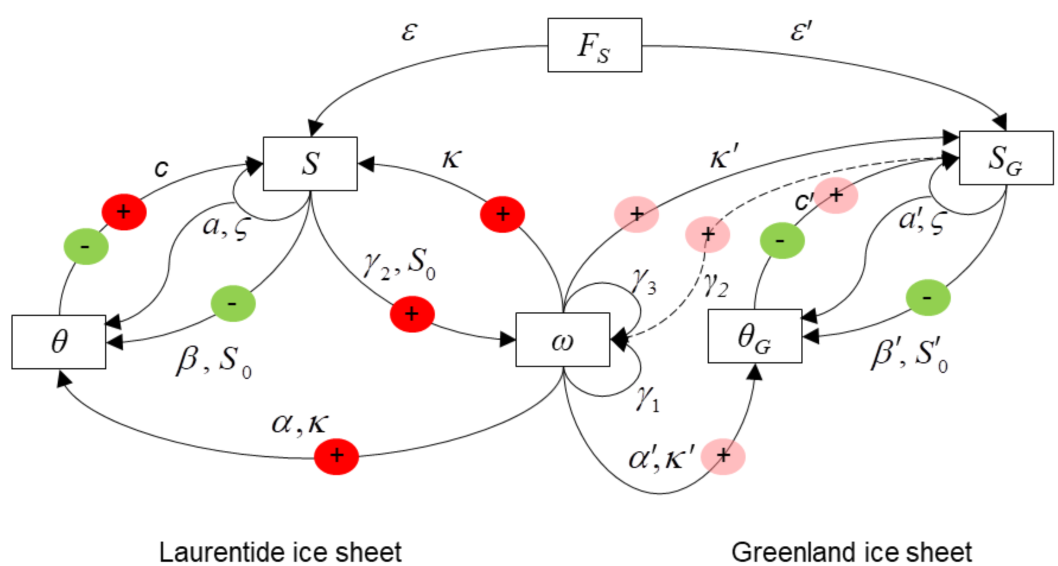
Laurentide ice sheet
Greenland ice sheet

(b)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Laurentide ice sheet
Greenland ice sheet

(c)

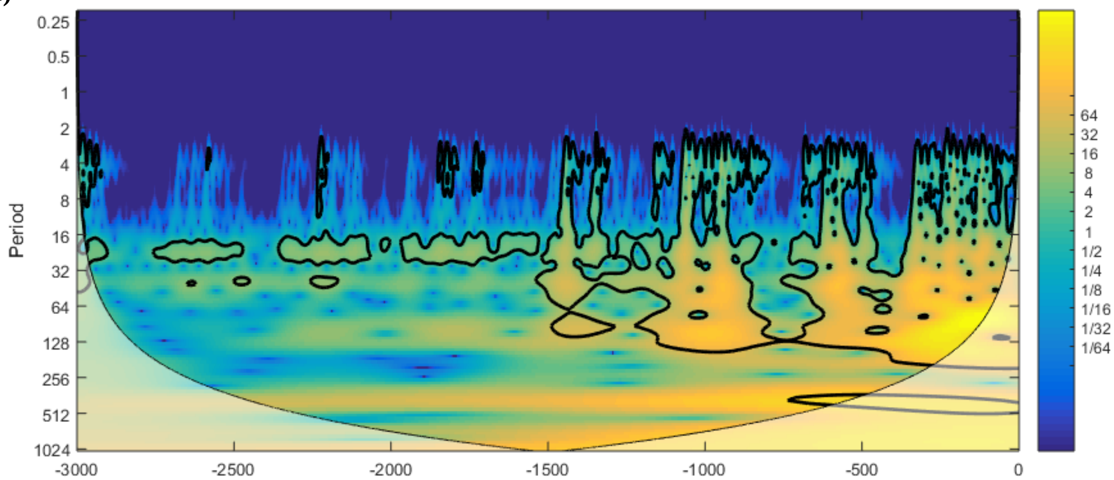

Fig. 1: (**a**) the dynamical system (1) – (5) as it has been used for scaling reasoning. The Greenland ice sheet is coupled one-way via the climate temperature ω acting as an external forcing for it. Red circles mark positive feedback loops and green circles mark negative feedback loops; (**b**) the dynamical system (1) – (5) as it has been used in the numerical simulation. The Laurentide and Greenland ice sheets are fully coupled. Pink circles mark weaker positive feedback loops. (**c**) Evolution of wavelet spectra over the past 3 million years for the Greenland glaciation area S_G (10^6 km^2). The color scale shows the continuous Morlet wavelet amplitude, the thick line indicates the peaks with 95 % confidence, and the shaded area indicates the cone of influence for wavelet transform.