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Interactive comment on "The Half-order Energy Balance Equation, Part 1: The homogeneous HEBE and long memories" *by* Shaun Lovejoy

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Anonymous Referee #2

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Interactive comment on "The Half-order Energy Balance Equation, Part 1: The homogeneous HEBE and long memories" by Shaun Lovejoy Anonymous Referee #2 Received and published: 28 June 2020

Review of "The Half-order Energy Balance Equation, Part 1: The homogeneous HEBE and long memories" by Lovejoy

Recommendation: Major revisions

This study derived a new version of the energy balance model based on non-integer derivatives. These models seamlessly contain long memory characteristics. This manuscript might be acceptable for publication in ESM after a major revision.

Author: We thank the referee for his/her comments that suggest a few clarifications. These are indicated in the detailed responses below (in italics).

1) Certain parts of the paper are confusing. For instance, the model is called a "zero dimensional" model though it has a vertical dimension. I assume this is because traditionally the vertical axis has been neglected and only a horizontal average considered. I strongly suggest to find a different terminology for this.

Author: We apologize for the admittedly confusing jargon, but we did not invent it! "Zero-dimensional" is the standard term for climate models without HORIZONTAL degrees of freedom. We do indicate this but we will gladly underline it and use alternative expressions when possible.

2) You refer many times to Part II. I think this is distracting; in my opinion it would make the paper easier to read to remove those references or to just have a short outlook on Part II in the conclusions section.

Author: We apologize if references to the second part of the paper are distracting. Many of these references were added after the initial submission at the explicit request of the editor Anders Levermann who thought that the linkage between the two parts was not strong enough. Since the editor was mostly concerned about adding linkages near the beginning of the paper, I could try to remove a few later on, although most of the references to the second part are quite pertinent.

The specific correspondence is on the site, I reproduce it here:

Editor Initial Decision: Start review and discussion after technical corrections (02 Apr 2020) by Anders Levermann Comments to the Author: Dear Shaun See my comment to part no. 2. The two papers need to be clearly linked. Bests, Anders

The initial comment in part II alluded to above: Editor Initial Decision: Start review and discussion after technical corrections (19 Mar 2020) by Anders Levermann Comments to the Author: Dear Shaun, you have to reference the first part of the paper clearly in the very beginning of the paper, so that the reader can easily find it. I would actually prefer if you could reference it already in the abstract. I did not look very hard, but I was not able to find the reference to the part 1 in the paper. Please help us here. Bests, Anders

3) Is your approach valid for all time scales? A long memory climate response should lead to infinite climate sensitivity. So your climate response operator is probably only valid for certain time scales.

Authors: As discussed in the paper, while the model itself may well be valid over a very wide range of time scales, it has two regimes: one shorter than the relaxation time and one longer. Both regimes are scaling and therefore both could be considered to have long memories. However there is a common - but restrictive - definition of long memory processes that is often applied to Gaussian processes (a divergent integral time scale). If this definition is used for the HEBE, and the forcing is assumed to be a Gaussian white noise, this definition will only apply to the scales below the relaxation scale. According to this definition, the different long-time scaling regime has short memory. Therefore we will clarify this distinction in the revised manuscript.

4) Line 15: BC needs to be defined.

Author: OK.

5) Line 26: I do not think "macroweather" is a widely known term. So please define.

Author: OK.

6) Line 32: "latitudinally" probably should be "zonally"

Author: OK.

7) I am confused by the z-coordinate system. It is not clear to me what z=0 means? Surface or top of the atmosphere? Also all z values seem to be negative. Also Figure1 does not help at all in that respect.

Author: On line 114 it was stated:

"We consider that vertical (radiative and conductive), and horizontal (conductive and advective) heat transport occurs on the surface and in the half-volume (x,y,z<0) respectively. Although physically, this means that the atmosphere and ocean are modelled as regions with $z \leq 0$, as mentioned, only the vertical surface temperature derivative is ultimately needed and this is well defined if the surface layer is of the order of a few diffusion depths (hundreds of meters)."

As for figure 1, it clearly shows the positive z direction as "up" with radiation only in this region and with heat conduction into the z< region. Could the referee be more specific about how to clarify this further?

In any case, I will add a short discussion about the physical meaning of z=0: the surface.

8) Line 175: Your linearization is either accurate or not, but not both.

Author: I will rework the sentence.

9) Line 266: What do you exactly mean by "top"?

Author: I mean at z = 0. However this was already stated in the parentheses following the word "top":

"At the top (z = 0), the system is forced by the conductive - radiative surface boundary condition. . ."

The sentence could be reworked to make this clearer.

10) in (33) you develop an asymptotic expansion. Why do you stop at the 1/2 term? There are also higher order term which might lead to different orders on fractional derivatives.

Author: Eq. 33 does not stop at $\frac{1}{2}$ order terms but rather at orders 3/2, 5/2 (G0,1/2), 3, 3/2 (G1,1/2), 3, 3/2 (G2,1/2). In any case, I could easily have given the general nth order term since it is in the literature. The high order terms are simply high and low frequency corrections to the scaling - they do not define their own separate scaling regimes. I will state this in the revised ms. However, the high and low frequencies are dominated by the $\frac{1}{2}$ order part and this is supported by empirical analyses performed prior to the discovery of the HEBE. Indeed, the text immediately following eq. 33 states this:

"The asymptotic equation for the step response (G1,1/2) shows that thermodynamic equilibrium is approached slowly: as t-1/2. It is this power law step response (with empirical exponent 0.5$\pm$0.2) that was discovered semi-empirically by [Hebert, 2017], [Lovejoy et al., 2017] and was successfully used for climate projections through to 2100. Similarly, $\approx$ t0.4 behaviour was used for macroweather (monthly, seasonal) forecasts close to the short time t1/2 expansion [Lovejoy et al., 2015], [Del Rio Amador and Lovejoy, 2019]."

11) Line 350: I am not sure many ESM readers are very familiar with long memory. I suggest that explain why (37) implies long memory.

Author: Eq. 37 is simply the definition of a fractional derivative. Since such derivatives are based on power laws, it is common for fractional derivatives to be used in the context of long memory processes. I will add some material to clarify this.

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