

Dear Editor, Dear Professor Kravitz,

We thank you for the opportunity to submit our revised paper to be published in ESD. We hope that with the modifications detailed below it can now be accepted for publication. Overall, based on the reviewers' comments and on your suggestions, we believe that in this new version there is an improved presentation of our results.

We reply below your comments and suggestions and indicate the new parts of the paper based on your suggestions. Replies to the reviewers' suggestions have already been uploaded, but we include them for reference.

Editor Decision: Reconsider after major revisions (27 May 2020) by Ben Kravitz

Comments to the Author:

The reviewers have recognized that this manuscript may have some value as an idealized framework that provides context for more detailed applications. There is certainly a place for this sort of mathematical formalism. Reviewer 2 rightly points out that such formalisms need to demonstrate clear application to the real world. One example could include verifying that the model can, to some degree, reproduce real world data. Another example could include ensuring that, although the model may be missing some processes, it still contains the right ones - phrased in the converse, one needs to be certain that increasing the complexity of the model will not totally change the answers one gets.

The authors need to instill confidence in readers that their model is useful. I believe this could be the case, but those points are not as clearly communicated as they need to be. I encourage the authors to focus on this when revising the manuscript.

In our paper we present a conceptual model. Its relative simplicity allows us to treat it mathematically and to carry out a detailed numerical investigations. Our conclusions are robust in a mathematical sense, meaning that small changes in our model (inclusion of noise, for example) will not alter our main findings. As long as the addition of other interactions only provide a small perturbation, our conclusions remain valid.

There is no guarantee, however, that adding more complex and extended interactions, degrees of freedom to our model will not lead to different trends. This is precisely the point of our paper: we separate the most important variables, feedbacks and interactions to reveal their effects on global trends. We do not wish to conclude that the addition of other interactions cannot lead to different trends. However, the results obtained from our model are confirmed by comparing them with the outcome of more complex models, we see in our model the same trends. In fact, the new citations included after the suggestions of reviewer 2 are probably the best confirmation that our very conceptual model can give a simple account for the trends seen in more realistic models. The difference is that the reason of these trends is transparent in our model, whereas its origin may remain hidden among the multitude of variables and interactions found in more complex models.

To further strengthen this line of thought, the following changes have been made to the text of the paper.

We added the following new text to the Introduction:

In spite of the current trend to include biogeochemistry in climate models (see e.g. Schluneger et al, 2019), a basic understanding of such processes is still limited. It is still under debate whether net primary production is increasing or decreasing in coupled carbon-climate models as a consequence of warming induces production increase and stronger nutrient limitations induced by increased stratification (Laufkötter et al, 2015). The situation appears to be similar to the understanding of thermal or fluid dynamical concepts decades ago. The study of e.g. the energy balance Ghil (1976) or of the thermohalin circulation Stommel (1961) started with elementary conceptual models which later evolved into more complex ones, and are by now decisive components of cutting-edge climate models. We therefore propose here to study a conceptual atmosphere-plankton model where emphasis is on a proper choice of couplings (feedbacks).

We added the following new text into the Conclusions:

As far as we know, our work is the first step in the direction of studying the feedbacks between the atmosphere and the biosphere by a simple conceptual model. Our conclusions are robust in a mathematical sense, meaning that small changes in our model (inclusion of noise, for example) will not alter our main findings since snapshot attractors are robust. As long as the

addition of other interactions only provide a small perturbation, our conclusions remain valid. In general, it is an open question in complex nonlinear systems whether neglected couplings to other subsystems and other simplifications could cause qualitative change in the dynamical behavior of a model. However, we see two important reasons why we believe our model goes in the right direction. First, the trends we find in our model are in accordance with the trends observed in the majority of complex models as mentioned above. Second, we believe that in our model the origin of trends are more transparent than in more complex models where this origin can be hidden among the multitude of variables, feedbacks and interactions. Our model is a conceptual model, and as such, both the biological and climate models are highly simplified. However, one can consider it as a starting module of an extendable model system. On the one hand, more trophic levels and inorganic resources can be easily added to the biological side of our model, on the other hand, simple ocean circulation models can extend the climate side of our model in order to make a first step to build more complex coupled models (Daron and Stainforth, 2013). We think that mutual interactions and iterations between conceptual models and detailed Earth System Models (ESM) help to reveal the distinction between relevant and less relevant mechanisms and feedbacks behind climate change. We expect deeper insight into these feedbacks by studying conceptual and ESMs parallelly in the future.

We also added to the Conclusions the following new text:

One of our main results is that an increase in the global temperature reduces mixing intensity, which is the leading factor in decreasing the total biomass of primary producers. Interestingly, this result is in concordance with numerous studies applying Earth System Models with vastly more detailed plankton models (Bopp et al, 2013; Fu et al, 2016; Kwiatkowski et al, 2019), although other works report different observations (Laufkötter et al, 2015; Flombaum et al, 2020).

We thank you again for the comments and suggestions. The new and changed parts of the manuscript are typeset in **boldface**. We hope the presentation of our work is now acceptable to be published in ESD.

Reply to reviewer 1

We thank the reviewer for a thorough reading of our paper and for the useful suggestions. Below we reply them and indicate the changes made to the manuscript.

This work presents an analysis of the feedbacks between atmosphere and ocean life with simplified mathematical models and detailed mathematical analysis. The questions and science considered in the paper are of broad relevance to researchers across many slices of the life sciences. Overall, the study is very good and offers broadly applicable insights relevant to the Earth Sciences. However, the paper does not sufficiently put the Earth Science relevant findings and broader implications front and center for ESD and its audience. The findings are there, but the paper (and especially the introduction and conclusion) would benefit from expansion in this direction. Overall, I suggest re-arranging the material to highlight broader relevance.

We thank the reviewer for considering the study “very good” which “offers broadly applicable insights relevant to the Earth Sciences”. We are grateful for his/her suggestions (below) that gives us the opportunity to highlight the broader relevance of our findings, as detailed below.

Major Comments:

1. *As mentioned, a greater focus on the broader Earth science issues and relevance is needed for an ESD paper. This can likely be accomplished through changes to the introduction and discussion/conclusions. Questions I wish the paper had addressed are along the lines of: what do these findings mean for more complex, process-based Earth System Models? I wanted more than Lines 355-356, and I think more could be said.*

In the revised form of the Introduction, we express our view that the understanding of the interplay between biogeochemistry and climate is still limited, and the situation of this problem is similar to the state climate science faced decades ago. This requires the use of a hierarchy of conceptual models increasing in details to shed light on the importance of various processes. Our simple conceptual model is an attempt to make the first step in this direction by coupling biogeochemistry and climate to identify the relative importance of some basic feedback mechanisms. In the Conclusions we also added that this model can be developed into a sequence of gradually more complex ones.

We added the following new text into the Introduction:

In spite of the current trend to include biogeochemistry in climate models (see e.g. Schluneger et al, 2019), a basic understanding of such processes is still limited. It is still under debate whether net primary production is increasing or decreasing in coupled carbon–climate models as a consequence of warming induces production increase and stronger nutrient limitations induced by increased stratification (Laufkötter et al, 2015). The situation appears to be similar to the understanding of thermal or fluid dynamical concepts decades ago. The study of e.g. the energy balance Ghil (1976) or of the thermohalin circulation Stommel (1961) started with elementary conceptual models which later evolved into more complex ones, and are by now decisive components of cutting-edge climate models. We therefore propose here to study a conceptual atmosphere–plankton model where emphasis is on a proper choice of couplings (feedbacks).

We added the following new text into Conclusions:

As far as we know, our work is the first step in the direction of studying the feedbacks between the atmosphere and the biosphere by a simple conceptual model. As such, both the biological and climate models are highly simplified. However, one can consider it as a starting modul of an extendable model system. On the one hand, trophical levels and inorganic resources can be easily added to the biological side of our model, on the other hand, simple ocean circulation models can extend the climate side of our model in order to make a first step to build more complex coupled models (Daron and Stainforth, 2013). We think that mutual interactions and iterations between conceptual models and detailed Earth System Models (ESM) help to reveal the distinction between relevant and less relevant mechanisms and feedbacks behind climate change. We expect deeper insight into these feedbacks by studying conceptual and ESMs parallelly in the future.

1a. *Section 3 is a good example of how the paper is heavily focused on the details of the math. There’s good scientific insight there: Lines 179-181 “The relation indicates that in the case of a positive*

enrichment parameter the phytoplankton dynamics weakens the climate change, weakens the trend from D_0 to D in the temperature contrast, as expected. Quite surprisingly, however, the effect is rather weak since $\alpha\beta$ is quadratically small.” Is there a way to make that point up front in this section, with fewer references to equations, and to move even more of the equations to the SI? Adjustments along these lines throughout would be beneficial to appeal to a broader audience of researchers.

We relegated the derivation of our formulae into the SI, and only kept those mathematical results in the main text that are explicitly used to reach the conclusions. We added a short paragraph about what a naive expectation suggests without any mathematical treatment, then reach the conclusion by analysing the results of the detailed calculation (obtained in the SI).

We added the following new text to Sec. 3:

Naively, one expects that an increased CO₂ level (smaller F in (1)) leads to a higher carrying capacity and concentration of the plankton, and a slower decrease of the temperature contrast, i.e., $S(D)$ should increase (decrease) with the enrichment parameter. However, only by calculating the precise dependence can reveal whether these trends are important or hardly discernible.

2. One easy change would be to include a table of variable and parameter notations, the quantities each notation represents, and any assumed values or boundaries imposed on the variables/parameters (such as alpha). This could be included in the SI, but is important to include, given the number of variables, parameters, and values being considered.

We thank the reviewer for the suggestion. The table has been added as Supplementary Material III.

3. Similarly, any kind of figure/model schematic illustrating the setup and feedbacks (and their notations where possible) would be beneficial in the main text Section 2.

A schematic drawing, illustrating the main feedbacks used in the paper, has been added to Section 2 as a new Fig. 1. We think that this drawing indeed helps the reader by making the set of feedbacks used in the paper easier to overview. We also attach this new figure to the reply.

Specific comments:

1. Lines 42-57: some of this text would be better suited in a methods section than the introduction.

The first part of the mentioned lines provides a general qualitative introduction to the ensemble method, heavily used in our approach. The next part, describing the concept of snapshot attractors, is indeed too strongly mathematics oriented, and we hence moved it to the beginning of Section 5.

The moved sentences:

The mathematical concepts underlying the ensemble view are snapshot (Romeiras et al, 1990) or pullback (Ghil et al, 2008) attractors. One might consider the ensemble of all permitted climate realizations over all times as the pullback attractor of the problem, and the set of the permitted states of the climate at a given time instant as the snapshot attractor belonging to that time instant (their union over all time instants is the pullback attractor). Both views express that the climate system possesses a plethora of possibilities. In the terminology of climate science, climate has a strong internal variability (e.g. Stocker et al (2013)). The concept of snapshot or pullback attractors is nothing but a reformulation of this fact in dynamical terms.

In numerical simulations, we consider the members of an ensemble simulation to describe parallel climate realizations only after the initial conditions are “forgotten”, transient dynamics disappears. Due to dissipation, this time is typically short compared to the time span of interest. Such an ensemble approach was shown to be the only method providing reliable statistical predictions in systems with underlying nonpredictable dynamics (since in this class the traditional approach based on single time series is known to provide seriously biased results). A number of papers illustrate these statements within the physics literature (see. e.g. (Romeiras et al, 1990; Lai, 1999; Serquina et al, 2008)), as well as in low order climate models (Chekroun et al, 2011; Bódai et al, 2011; Bódai and Tél, 2012; Bódai et al, 2013; Drótos et al, 2015), in general circulation models (Haszpra and Herein, 2019; Kaszás et al, 2019; Pierini et al, 2018, 2016; Drótos et al, 2017; Herein et al, 2017; Bódai et al, 2020; Haszpra et al, 2020; Haszpra and Herein and Bódai, 2020) and also in experimental situations (Vincze, 2016; Vincze et al, 2017).

2. *Line 60: What is the relevance to this work that the ensemble approach has been used in other adjacent but distinct studies that presumably consider different models of different variables?*

The ensemble method turns out to be the only reliable method in processes taking part in the presence of climate change. The traditional approach based on a single time evolution is not representative, and might lead to biased conclusions. This we emphasize now in the Introduction.

The adjusted text in the Introduction:

An appropriate treatment of even elementary models describing climate change is not obvious since basic parameters change with time and, therefore, traditional long-time averages cannot be used to define (in the sense of any statistical quantifiers) a state of the climate. An emerging new view, already embraced by Drótos et al (2015), follows a different route to obtain information on instantaneous statistical quantifiers (e.g. expected, average properties) of the climate. Since our information on the actual state of the climate is incomplete, one imagines an ensemble of parallel Earth systems carrying parallel climate realizations subjected to the same set of physical laws, boundary conditions and external forcing, but with different initial conditions. Then the chaotic or turbulence-like properties of the climate dynamics allows for distinct climate realizations (for a review see Tél et al, 2019). These realizations, however, cannot be arbitrary since only those are permitted that are compatible with physical laws and the given forcing. The ensemble of realizations defines a probability distribution of all the relevant variables at any instant of time from which one can obtain expected, ensemble average properties of the climate (for more details, and mathematical aspects, see Sec. 5). It is therefore natural to use the ensemble view in our conceptual biogeochemistry model, too. The ensemble approach in it corresponds to generating parallel atmosphere–phytoplankton realizations from different initial conditions.

3. *Figure 8 : Could you add a colorbar rather than (or in addition to) writing it out in the caption?*

We added the colorbar to Figure 8 (now Fig. 9, due to the addition of the new schematic drawing, Fig. 1, in Section 2).

4. *Section 4: overall I like this section very much but please make explicit mention that angled brackets always correspond to ensemble average, for every variable, early in the section.*

We have made this explicit at the beginning of Section 4.

The new text added to Section 4:

Here and in what follows angled brackets $\langle \rangle$ will always denote averages taken with respect to our ensemble at a given time instant, t .

5. *Please consider making code and possibly some archive of the ensembles you run available to support open-access, reproducible science.*

We upload the code as supplementary information upon acceptance of the paper.

We thank again the reviewer for the insightful comments, and we hope that with the indicated changes the paper can be accepted for publication in ESD.

Reply to reviewer 2

We thank the reviewer for a thorough reading of our paper and for the constructive comments and suggestions that, we believe, improved the presentation of our results. We reply all comments and indicate the changes made in the manuscript.

This manuscript describes a conceptual modelling study of how phytoplankton respond to changes in temperature, atmospheric CO₂, and ocean mixing. In this study the authors attempt to use a relatively simple model that contains many assumptions to simulate phytoplankton growth as a function of changes in anthropogenic forcing. While I am not opposed to simple conceptual models that focus on key processes or dynamics, I feel that the conceptual model presented here makes many assumptions that are not well justified.

We thank the reviewer that he is also in favor of conceptual models like the one we present here. We agree that our model only grasps the most important processes only in the simplest possible way, and neglects many effects that need to be addressed in an extended model. We believe, however, that the processes we include in the model are well justified. To better support this statement, we added a new schematic drawing (new Fig. 1, also attached to the reply) to illustrate the processes and feedbacks we include in our model. We believe that it is possible to extend our model by sequentially adding more and more details into it, as we now describe this in the Conclusions. We believe that applying this approach allows us to identify how individual components contribute to the overall behavior of complex models. We also explain this more explicitly in the Introduction.

To give more explanation why we think our model is well justified as a conceptual model, we added the following new text into the Introduction:

In spite of the current trend to include biogeochemistry in climate models (see e.g. Schluneger et al, 2019), a basic understanding of such processes is still limited. . . . The situation appears to be similar to the understanding of thermal or fluid dynamical concepts decades ago. The study of e.g. the energy balance Ghil (1976) or of the thermohalin circulation Stommel (1961) started with elementary conceptual models which later evolved into more complex ones, and are by now decisive components of cutting-edge climate models. We therefore propose here to study a conceptual atmosphere–plankton model where emphasis is on a proper choice of couplings (feedbacks).

We also added the following new text into the Conclusions:

As far as we know, our work is the first step in the direction of studying the feedbacks between the atmosphere and the biosphere by a simple conceptual model. As such, both the biological and climate models are highly simplified. However, one can consider it as a starting modul of an extendable model system.

And a bit later we emphasize:

We think that mutual interactions and iterations between conceptual models and detailed Earth System Models (ESM) help to reveal the distinction between relevant and less relevant mechanisms and feedbacks behind climate change. We expect deeper insight into these feedbacks by studying conceptual and ESMs parallelly in the future.

In addition, many relevant dynamics seem to be left out, e.g., other bottom-up controls on phytoplankton growth such as light and nutrients like N, P, and Fe, as well as top-down controls like grazing and mortality. While, it may be possible that global phytoplankton-climate dynamics and responses to forcing can be explained without such factors (although I doubt it), in order for me to have confidence in their model I would need to see better evidence that justifies the model simplifications and model parameterizations. Many other conceptual studies of phytoplankton use laboratory studies or observations to justify their model structure and parameterizations, surely this can be done here as well.

We agree with the reviewer that our treatment of phytoplankton neglects many important factors, this is why we call it a “conceptual” model already in the title of the paper. We point out that our choice for the ecological component in the form of a very simple logistic equation is motivated by the similarly simple representation of the full atmosphere by merely three ordinary differential equations of the Lorenz’84 model. We feel that the coupling of this simple atmospheric model to a detailed ecological

model would be inconsistent. By taking such a simple set-up, we intend to strengthen the conclusions of more complex models by obtaining exact results in a simple transparent conceptual model. We prefer to work with a well understood, but at the same time paradigmatic description which enables one to explore the mechanism of basic feedbacks. Thus, for example, our approach makes possible to study the feedback of the primary producer on the temperature contrast which, in turn, drives the atmospheric dynamics. This might remain hidden in the complexity of current GCMs.

In the new closing section of the Conclusions we formulate that the present approach is considered to be a starting project on which a full hierarchy of models can be built. The next step in this hierarchy can be a still conceptual, combined atmosphere-ocean model to which a simple ecology model with more trophic levels and inorganic resources can be coupled.

These points are now added to the text in the Conclusions:

On the one hand, more tropical levels and inorganic resources can be easily added to the biological side of our model, on the other hand, simple ocean circulation models can extend the climate side of our model in order to make a first step to build more complex coupled models (Daron and Stainforth, 2013).

In addition, and perhaps more importantly, for me to have confidence in the model there needs to be some validation, i.e., comparisons to actual data. I know that the goal is to simulate climate change and obviously one cannot validate future projections. However, it should still be possible to come up with a clever way to validate the model (or key underlying equations) using observations. There is also no real attempt to contrast the results of this study with other phytoplankton focused climate studies that have used ocean-only or Earth system models (e.g., see citations listed below). Without satisfactory justification and validation of the model and the results this is just a mathematical exercise that while interesting, leaves the reader wondering if it has meaning. Therefore, I must recommend that the manuscript be rejected.

We thank the reviewer for the citations, we found them immensely useful to put our work into context, and we added all of them to the list of references. Papers like these give the most important motivation for conceptual models like the one we present in our paper. These papers shed light on the fact that it is still under debate whether net primary production is increasing or decreasing in coupled carbon-climate models as a consequence of warming induces production increase and stronger nutrient limitations induced by increased stratification. We also compare now our findings with the conclusions of these papers.

To sum this up, we added the following new text to the Introduction:

In spite of the current trend to include biogeochemistry in climate models (see e.g. Schluneger et al, 2019), a basic understanding of such processes is still limited. It is still under debate whether net primary production is increasing or decreasing in coupled carbon-climate models as a consequence of warming induces production increase and stronger nutrient limitations induced by increased stratification (Laufkötter et al, 2015).

And:

The direct effect of increased CO₂ concentration on phytoplankton dynamics can be stimulating or inhibiting, we study both scenarios.

We added to the Conclusions the following new text:

One of our main results is that an increase in the global temperature reduces mixing intensity, which is the leading factor in decreasing the total biomass of primary producers. Interestingly, this result is in concordance with numerous studies applying Earth System Models with vastly more detailed plankton models (Bopp et al, 2013; Fu et al, 2016; Kwiatkowski et al, 2019), although other works report different observations (Laufkötter et al, 2015; Flombaum et al, 2020).

We thank the reviewer again for his insightful comments, and especially for the very useful citations that support our conclusions that global warming is expected to reduce the total biomass of primary producers. We hope that with the changes we made to the text, our paper can be accepted for publication in ESD.

Climate change in a conceptual atmosphere–phytoplankton model

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Abstract. We develop a conceptual coupled atmosphere–phytoplankton model by combining the Lorenz’84 general circulation model and the logistic population growth model under the condition of a climate change due to a linear time dependence of the strength of anthropogenic atmospheric forcing. The following types of couplings are taken into account: a) the temperature modifies the total biomass of phytoplankton via the carrying capacity, b) the extraction of carbon dioxide by phytoplankton slows down the speed of climate change, c) the strength of mixing/turbulence in the oceanic mixing layer is in correlation with phytoplankton productivity. We carry out an ensemble approach (in the spirit of the theory of snapshot attractors) and concentrate on the trends of the average phytoplankton concentration and average temperature contrast between the pole and equator, forcing the atmospheric dynamics. The effect of turbulence is found to have the strongest influence on these trends. Our results show that when mixing has sufficiently strong coupling to production, mixing is able to force the typical phytoplankton concentration to always decay globally in time and the temperature contrast to decrease faster than what follows from direct anthropogenic influences. Simple relations found for the trends without this coupling do, however, remain valid, just the coefficients become dependent on the strength of coupling with oceanic mixing. In particular, the phytoplankton concentration and its coupling to climate is found to modify the trend of global warming, and is able to make it stronger than what it would be without biomass.

15 *Copyright statement.*

1 Introduction

Large scale general circulation models typically take into account the interaction of the atmosphere with land vegetation and marine biomass production in the form of a huge number of parametrized processes (see e.g. Marinov et al, 2010; Zhong et al, 2011; Mongwe et al, 2018; Wilson et al, 2018). A basic understanding of such coupling is, however, easier to obtain in low

20 order conceptual models, where even analytic results may be available. Probably the most important component that needs to be included in such conceptual models is the phytoplankton content of the ocean.

Oceans are the major sink for the atmospheric CO₂ (Hader et al, 2014; Li et al, 2012). CO₂ is either stored as dissolved inorganic carbon or transferred to the underlying sediment by biological carbon pump. The motor of the biological pump is phytoplankton which is one of the major components of the global carbon cycle hence influencing decisively atmospheric CO₂ (Hutchins and Fu, 2017; Sanders et al, 2014; Turner, 2015; Falkowski et al, 2000). Besides, phytoplankton is responsible for nearly half of the total primary production on Earth (Basu and Mackey, 2018). Consequently, it is extremely important to understand the interaction of primary production in oceans with effects contributing to global climate change. However, the task is very challenging: change in atmospheric CO₂ level can have opposite impact on processes influencing the phytoplankton and the intensity of the biological carbon pump. Increased atmospheric CO₂ level increases ocean temperature, decreases pH, increases water stratification, influences general oceanic circulation. These can all modify the net productivity and the composition of phytoplankton, and can have either positive or negative net effect on the biological carbon pump (Basu and Mackey, 2018, and references therein).

In spite of the current trend to include biogeochemistry in climate models (see e.g. Schlunegger et al, 2019), a basic understanding of such processes is still limited. It is still under debate whether net primary production is increasing or decreasing in coupled carbon–climate models as a consequence of warming induces production increase and stronger nutrient limitations induced by increased stratification (Lauferkötter et al, 2015). The situation appears to be similar to the understanding of thermal or fluid dynamical concepts decades ago. The study of e.g. the energy balance Ghil (1976) or of the thermohaline circulation Stommel (1961) started with elementary conceptual models which later evolved into more complex ones, and are by now decisive components of cutting-edge climate models. We therefore propose here to study a conceptual atmosphere–phytoplankton model where emphasis is on a proper choice of couplings (feedbacks). Thus, in our model, an increase of the global temperature affects the global primary production of ocean. As we emphasize above, phytoplankton plays significant role in the global CO₂ balance (De La Rocha and Passow, 2014; Falkowski, 2014; Guidi et al, 2016), hence our aim is to take an elementary description of phytoplankton dynamics coupled to an elementary model of the atmosphere. **The direct effect of increased CO₂ concentration on phytoplankton dynamics can be stimulating or inhibiting, we study both scenarios.** As atmospheric model, we use Lorenz' elementary global circulation model (Lorenz, 1984), which was extended to mimic climate change (Drótos et al, 2015). The global phytoplankton concentration is represented by a simple logistic model in which the carrying capacity is coupled with the CO₂ content (direct effect) depending also on the concentration itself and on the wind energy influencing the oceanic mixing layer (indirect effect of climate change).

An appropriate treatment of even elementary models describing climate change is not obvious since basic parameters change with time and, therefore, traditional long-time averages cannot be used to define (in the sense of any statistical quantifiers) a *state of the climate*. An emerging new view, already embraced by Drótos et al (2015), follows a different route to obtain information on instantaneous statistical quantifiers (e.g. expected, average properties) of the climate. Since our information on the actual state of the climate is incomplete, one imagines an ensemble of parallel Earth systems carrying parallel climate realizations subjected to the *same set of physical laws*, boundary conditions

55 and external forcing, but with *different initial conditions*. Then the chaotic or turbulence-like properties of the climate dynamics allows for distinct climate realizations (for a review see Tél et al (2019)). These realizations, however, cannot be arbitrary since only those are permitted that are compatible with physical laws and the given forcing. The ensemble of realizations defines a probability distribution of all the relevant variables at *any instant of time* from which one can obtain expected, ensemble average properties of the climate (for more details, and mathematical aspects, see Sec. 5).

60 It is therefore natural to use the ensemble view in our conceptual biogeochemistry model, too. The ensemble approach in it corresponds to generating parallel atmosphere–phytoplankton realizations from different initial conditions. In our model, the number of variables is 4, hence the snapshot attractor in the full state space is difficult to visualize. We therefore concentrate on ensemble averages, and the internal variability will be expressed in terms of variances. We include, in a simple, heuristic form, *important feedbacks* in the model: a) the change in the atmospheric temperature modifies phytoplankton concentration, b) the extraction of CO₂ by phytoplankton, and c) wind energy enhances the strength of turbulence in the oceanic mixing layer which increases the phytoplankton production (Estrada and Berdalet, 1997; Peters and Marrasé, 2000; Jäger et al, 2010).

The paper is organized as follows. In section 2 we describe the model and define the relevant coupling parameters. Without mixing, exact relations can be derived, the most important of these are summarized in Section 3, while details of the calculations are relegated to Supplementary Material I. In the presence of mixing, numerical simulations are carried out in the spirit of snapshot attractors. The results are summarized in Section 4 where one learns that the extraction effect of CO₂ has the least influence on the general behavior in the presence of mixing. The feedback of the temperature contrast on the phytoplankton concentration has important consequences, but these are suppressed by a sufficiently strong mixing, which converts the typical phytoplankton concentration to always decay in time, and surprisingly, the typical temperature contrast is found to decrease faster than that solely by direct anthropogenic effects. Planar sections of the 4-dimensional snapshot attractor underlying the dynamics are presented in Section 5, and our conclusions are drawn in Section 6. Additional figures are presented in Supplementary Material II. A list of variables and parameters is given in Supplementary Material III, while Supplementary Material IV contains a sample of the C code applied during numerical simulations.

2 The model

80 The physical content of Lorenz’s atmospheric circulation model for the midlatitudes (Lorenz, 1984, 1990) on one hemisphere is the following. The main forcing is the temperature difference $T_e - T_p$ between the Equator and the Pole. This is proportional to model variable F influencing most directly the wind speed of the Westerlies represented by x . As an effect of baroclinic instability, cyclonic activity facilitates poleward heat transport, two modes of which are represented by y and z . The model

reads as follows:

$$85 \quad \dot{x} = -y^2 - z^2 - ax + aF(t), \tag{1a}$$

$$\dot{y} = xy - bxz - y + G, \tag{1b}$$

$$\dot{z} = xz + bxy - z. \tag{1c}$$

For the parameter setting we take the common choice: $a = 1/4$, $b = 4$, $G = 1$. The equations appear in a dimensionless form with the time unit corresponding to 5 days.

90 By using time-dependent forcing, $F(t)$, as Drótos et al (2015), we also model the contribution of the varying CO₂ content in association with the greenhouse effect. Besides the variation of CO₂ due to effects appearing in $F(t)$, the extraction of CO₂ by phytoplankton is also included into our model. The CO₂ content stored in marine ecosystems, or buried in the sea bed, is correlated with primary production (Falkowski et al, 1998, 2003). Thus, as discussed in the Introduction, modelling the interaction of phytoplankton and atmospheric dynamics is a good proxy for studying marine ecosystem interaction with
 95 atmospheric dynamics. Hence we couple the Lorenzian atmospheric dynamics to that of the photosynthesizing oceanic biomass, assumed to be dominated by phytoplankton of concentration $c(t)$. The temperature contrast parameter thus also depends on the global phytoplankton concentration c : $F(t) \rightarrow F(c(t), t)$, with a form to be given below.

Spatial inhomogenities in nutrient and consequently phytoplankton content due to e.g. oceanic eddies and upwellings are known to play an important local role in Nature. However, a global atmospheric model like (1) can adequately be coupled
 100 only to a global phytoplankton dynamics model. Therefore, the concentration itself is assumed in this simple set-up to follow a logistic population growth

$$\dot{c} = rc \left(1 - \frac{c}{K(t)} \right). \tag{2}$$

Carrying capacity K is taken to depend on the average temperature of the hemisphere, or, equivalently, on the temperature contrast F . As a consequence, K depends on time also via the concentration c : $K(t) = K(c(t), t)$. We shall see that an
 105 important oceanic effect, that of the turbulence in the mixing layer, can be incorporated into carrying capacity K , although only on a global scale. Parameter r sets the growth rate of the phytoplankton. If, e.g., $r = 1$, the phytoplankton characteristic time is $5/r = 5$ days, as that of the atmosphere. This latter choice will be kept throughout the paper. The assumption of Eq. (2) for the global phytoplankton dynamics tacitly implies that phytoplankton biomass determines the total biomass of the oceans, and also that no catastrophic events (no mass extinction or invasion of species) can take place in this model.

110 A basic feature of the observed climate change on Earth is that the polar temperature $T_p(t)$ increases, while the equatorial one T_e remains practically constant (Serezze and Francis, 2006; Blunden and Arndt, 2013). We can thus write in suitable units the temperature contrast parameter as $F(t) = T_e - T_p(t)$. The mean temperature in these units is then $T(t) = (T_e + T_p(t))/2 = T_e - F(t)/2$. We are interested in dominant, leading order effects and assume therefore the carrying capacity to be coupled linearly by a small coupling constant to the mean temperature relative to some reference state of mean temperature T_r (in
 115 which the temperature contrast parameter is $F_r = 2(T_e - T_r)$). The temperature difference $T - T_r$ is then $-(F - F_r)/2$. We

therefore write

$$K(t) = K_r - \alpha(F(c(t), t) - F_r), \quad (3)$$

where K_r is the carrying capacity in the reference state characterized by F_r . Coefficient α represents coupling a) between the carrying capacity K and climate change, represented by F , and shall be called the *enrichment parameter*, see **Fig. 1** where
 120 **the full set of feedbacks considered in the model is schematically presented.** This coupling may be either positive or negative. For example, increased CO_2 level enhances the efficiency of photosynthesis ($\alpha > 0$), however, acidification because of increased CO_2 levels depresses respiration ($\alpha < 0$) (Reid et al, 2009; Mackey et al, 2015). Similarly, increased water temperature can have both positive and negative effect on phytoplankton biomass in different regions of Earth (Chust et al, 2014; Roberts et al, 2017). We shall, therefore, allow for both positive and negative values of α with $|\alpha|$ small.

125 Plankton dynamics influences the temperature contrast. If concentration c increases, the temperature contrast F increases, too, because the biomass extracts more CO_2 . In leading order, we therefore express the concentration-dependent temperature contrast parameter as a linear function of the concentration:

$$F(t) = F(c(t), t) = \beta(c(t) - c_r) + F_0(t) \quad (4)$$

with a small $\beta > 0$, where c_r is the phytoplankton concentration in the reference state. Coefficient β represents coupling b) due
 130 to the extraction of CO_2 by phytoplankton, and we therefore call β the *extraction parameter* (see **Fig. 1**). The second term, $F_0(t)$, represents the primary external forcing due to the CO_2 content of anthropogenic origin. The increase of both F_0 and $(c(t) - c_r)$ leads to an increase in the temperature contrast $F(t)$. With this form of F the carrying capacity (3) is

$$K(t) = K_r - \alpha[\beta(c(t) - c_r) + F_0(t) - F_r]. \quad (5)$$

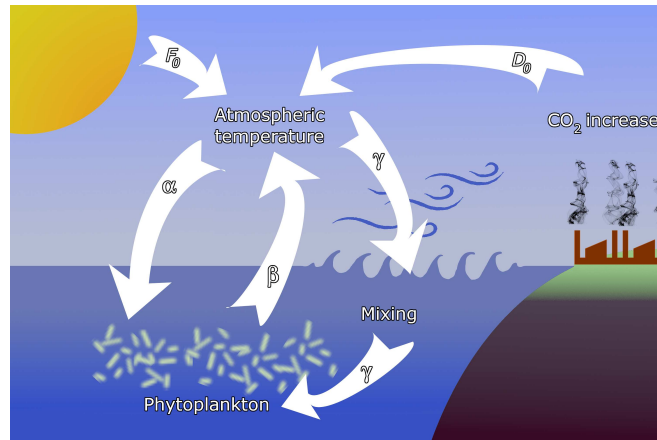


Figure 1. Sketch of the feedbacks considered in the model. Temperature contrast F_0 between the Pole and the Equator, containing also seasonal variability, is augmented by anthropogenic effects D_0 . The main interactions are α) the effect of atmospheric temperature on biomass, β) the extraction of CO_2 by phytoplankton, and γ) oceanic mixing, driven by atmospheric dynamics, affecting phytoplankton productivity.

Without a restriction of generality, we can choose the reference carrying capacity to be $K_r = 1$, implying a reference concentration $c_r = 1$. This choice only rescales parameters α and β in (3) and (4), respectively.

Starting from negative times, we assume the Earth system to be in climatic and population dynamical equilibrium up to time $t = 0$. This state, chosen as the reference state, is characterized by a time independent mean temperature T_r , concentration $c_r = K_r$, and $F_0(t) = F_r$. At time zero, climate change sets in expressed by a linear decrease in the primary temperature contrast:

$$140 \quad F_0(t) = F_r - D_0 t, \tag{6}$$

expressing direct anthropogenic effects, with a decrease parameter $D_0 = 2/7300$ for $t > 0$ (Drótos et al, 2015). Since one year corresponds to 73 time units (365 days), $1y = 73$, this form expresses that the temperature contrast decreases by 2 units over 100 years. We shall take $F_r = 9.5$, with which the temperature contrast would go down, after a climate change period of 150 years, and without any change in the biomass concentration, to 6.5. We stop the climate change scenario in year 150 because model (1) loses its global chaotic property, which is a prerequisite even for a minimal climate model, for small F .

With this scenario (6) of the anthropogenic influence, the carrying capacity $K(t)$ is, in rescaled units,

$$K(t) = 1 - \alpha[\beta(c(t) - 1) - D_0 t], \tag{7}$$

where $D_0 = 0$ for $t \leq 0$ and $D_0 = 2/7300 = 2.7 \cdot 10^{-4}$ for $t > 0$.

We can model seasonality, too, as Lorenz also did (Lorenz, 1990), by augmenting (6) with a periodic term:

$$150 \quad F_0(t) = F_r - D_0 t + A \sin(\omega t). \tag{8}$$

His choice was $A = 2$ with $\omega = 2\pi/73$, which we shall adopt. Our climate change starts with year 0, and this year begins at the time instant $t = 0$. Note that this time instant belongs to an autumnal equinox according to (8), and, furthermore, $F_r - D_0 t$ can be considered as the annual mean temperature contrast. Any time $t \bmod 73 = 0$ coincides with other autumnal equinoxes, and results will be presented on this day of the year throughout the paper.

Up to this point, the atmospheric variables have not entered the concentration dynamics. Without the linear and constant terms (representing dissipation and forcing, respectively), Eqs. (1) would conserve the total kinetic energy

$$E = \dot{x}^2 + \dot{y}^2 + \dot{z}^2$$

155 of the atmosphere. From the point of view of the biomass, it is natural to assume that the activity of the atmosphere influences the ocean dynamics within its uppermost mixing layer (Sverdrup, 1953; Whitt et al, 2017), in particular, the strength of turbulence, and hence the depth of mixing layer. Note that component \dot{x}^2 represents the contribution of zonal winds to the total atmospheric energy, while $\dot{y}^2 + \dot{z}^2$ represents wind energy staming from cyclonic activity. The depth of the mixing layer, and consequently the carrying capacity, are assumed to increase linearly with E in our model, with a small coupling constant. The most general form of the carrying capacity K is thus

$$160 \quad K(t) = 1 - \alpha[\beta(c(t) - 1) - D_0 t + A \sin(\omega t)] + \gamma(\dot{x}^2 + \dot{y}^2 + \dot{z}^2). \tag{9}$$

Here $0 \leq \gamma \leq 0.2$ is the strength of a weak coupling c due to oceanic mixing what we call the (oceanic) *mixing parameter*. This provides a feedback between the phytoplankton dynamics and the climatic variables (see Fig. 1).

3 Analytic results without mixing

165 Without mixing ($\gamma = 0$), Eq. (2) can be solved by a simple ansatz of $c(t)$, irrespective of the atmospheric dynamics. This leads to analytic results concerning some properties of the model, which are summarized in Supplementary Material I. As an example, we give here two simple relations which help to understand the general tendencies of the system. Eq. (2) with (9) is shown for $\gamma = 0$ to possess linear behavior for long times, inherited from the temperature contrast of anthropogenic origin:

$$c(t) \sim St, \quad F(t) \sim -Dt. \quad (10)$$

170 **Naively, one expects that an increased CO_2 level (smaller F in (1)) leads to a higher carrying capacity and concentration of the phytoplankton, and a slower decrease of the temperature contrast, i.e., S (D) should increase (decrease) with the enrichment parameter. However, only by calculating the precise dependence can reveal whether these trends are important or hardly discernible.** The linear coefficient, slope S in the phytoplankton concentration's time dependence is found to be

$$175 \quad S = \frac{D_0 \alpha}{1 + \beta \alpha} \approx D_0 \alpha. \quad (11)$$

The approximate equality reflects that the product $\alpha \cdot \beta$ is quadratically small since both the enrichment parameter α and the extraction parameter β are small quantities. Hence the leading order behavior in α is linear. This relation shows that for a positive (negative) coupling α the phytoplankton concentration increases (decreases) proportionally with the enrichment parameter α , and with the slope D_0 of the anthropogenic temperature contrast.

180 The linear coefficient in the temperature contrast is

$$D = \frac{D_0}{1 + \beta \alpha} \approx D_0(1 - \beta \alpha). \quad (12)$$

The approximate equality provides, again, the leading order behavior in α . The relation indicates that in the case of a positive enrichment parameter α the phytoplankton dynamics *weakens* the climate change, weakens the trend from D_0 to D in the temperature contrast, as expected. Quite surprisingly, however, the effect is rather weak since $\alpha \cdot \beta$ is quadratically small.
185 Relations (11,12) also suggest that the role of (a weak) extraction coupling is not essential: the leading behavior in S is independent of β . Its effect is weak also in D , this quantity coincides with the anthropogenic slope D_0 for $\beta = 0$ (as also follows from (4)), it deviates from D_0 very little otherwise.

It is worth noting that relations (11,12) remain valid for the time-averaged trends in the presence of a seasonal periodicity, as also shown in Supplementary Material I. Relations (11,12) are independent of initial conditions, they represent the snapshot
190 attractor of the problem projected on variable c . This attractor is fixed point-like, but changes in time (moves uniformly, or with an oscillation superimposed when seasonality is taken into account). There is, however, no internal variability in the

concentration variable c , although an extended, fractal snapshot attractor underlies the atmospheric variables exactly as in the model of Drótos et al (2015) where no phytoplankton dynamics was taken into account.

4 Numerical results with mixing: trends in the fully coupled model

195 In the interesting case of nonnegligible mixing, no analytic result can be obtained. This implies a nontrivial biomass dynamics for $\gamma > 0$, a dynamics exhibiting internal variability in variable c , too. To explore this regime, we carried out a sequence of numerical simulations of the full 4-variable dynamics. The following parameters are kept fixed (as indicated in the previous section): $r = 1$, $D_0 = 2/7300$, $F_r = 9.5$, $A = 2$, $\omega = 2\pi/73$, and we vary α , β , and γ . Equations (1) and (2) with (4), (8) and (9) are solved with the classical 4th-order Runge-Kutta method with a fixed time-step $dt = 0.01 \approx 1.37 \times 10^{-4}$ y.

200 To start with, Fig. 2a shows a few individual concentration realizations (colored lines) $c(t)$ for a mixing parameter $\gamma = 0.1$ ($\alpha = 0.05$, $\beta = 0.1$), along with the ensemble average $\langle c \rangle (t)$ of 50 000 realizations initiated at $t = -20$ y (purple line). **Here and in what follows angled brackets $\langle \rangle$ will always denote averages taken with respect to our ensemble at a given time instant, t .** The individual cases are all rather different. For $t < 0$ there is no climate change, nevertheless, the individual time series $c(t)$ exhibits strong variance, very similar to those observed for $t > 0$, i.e., they are unable to properly reflect the ensemble, and, in particular, the lack of climate change for $t < 0$. The ensemble average, $\langle c \rangle (t)$, however, provides a plateau

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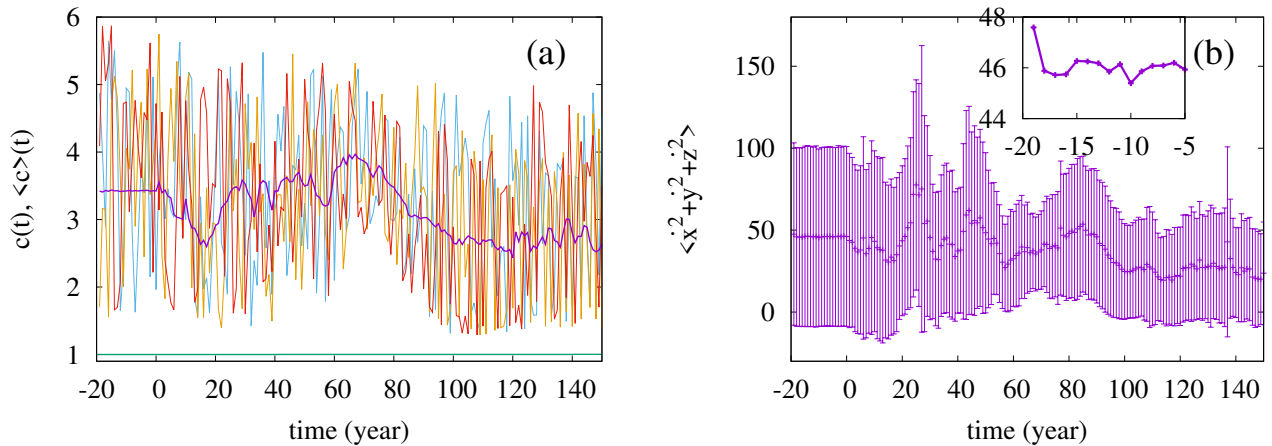


Figure 2. Ensemble properties before ($t < 0$) and after ($t > 0$) the onset of climate change. a) Phytoplankton concentration c as a function of time for three random initial conditions in different colors for $\alpha = 0.05$, $\beta = 0.1$, $\gamma = 0.1$. The purple line is the ensemble average $\langle c \rangle (t)$ for 50000 trajectories started with random initial positions in the range $x \in [-0.5, 3]$, $y, z \in [-2.5, 2.5]$, $c \in [0.9, 1.1]$ at year -20. The green line (close to $c = 1$) shows the expected phytoplankton concentration without any mixing ($\gamma = 0$) as predicted by Eq. (10). (The increase for $t > 0$ is so weak that one hardly recognizes it on this graph.) b) Time-dependence of the ensemble average (dark violet “+” marks) of the total atmospheric kinetic energy $\langle \dot{x}^2 + \dot{y}^2 + \dot{z}^2 \rangle$ for the same ensemble as the one used for c in (a). Violet bars indicate the standard deviation. The inset shows the blow up of the initial part of the average in b).

here up to $t = 0$ indicating clearly the stationarity of the climate and, therefore, of the biomass dynamics in this range. In Fig. 2b we display the source of the time variability of the phytoplankton concentration, the total kinetic energy $\dot{x}^2 + \dot{y}^2 + \dot{z}^2$ of the atmosphere at each time instant. The deviation of the individual ensemble member time series from the average is represented here by means of the standard deviation evaluated over the ensemble (violet bars). The average kinetic energy, along with its ensemble variance, is also constant before the climate change and starts an irregular time dependence right after $t = 0$. One can observe that the kinetic energy strongly influences the phytoplankton concentration (via the carrying capacity K in (9)), but the concentration itself contributes to the CO_2 content, and to the temperature contrast F , see (4), forcing the atmosphere (as will be demonstrated in Fig. 3). The feedback of the atmosphere on phytoplankton is rather strong in this set-up with $\gamma = 0.1$, also expressed by the strong difference between the green line (obtained for $\gamma = 0$) and the purple line in Fig. 2a illustrating that this coupling leads to an enormously enhanced biomass concentration.

It is visible in the inset to Fig. 2b that the ensemble average curve shows some change during the first 5 years (between $t = -20$ and -15 y). This indicates (along with several other simulations, not shown) that the convergence to the snapshot attractor takes about $t_c = 5$ y. The numerical data after $t = -15$ y thus represent parallel atmosphere-phytoplankton realizations on the snapshot attractor of the system.

The considerable deviation of the individual time series from the ensemble average indicates that the formers are not representing properly the mean climate state, as also pointed out by Drótos et al (2015). Therefore, from here on, we shall concentrate on ensemble averages, and consider the variance about these as a measure of the internal variability (the size of the snapshot attractor in the chosen variable).

We carried out similar simulations with other extraction parameter values from the range $\beta \in [0.0, 0.5]$ and found that β does not have much effect on the average phytoplankton concentration, the curves for various values of β are close to each other (see Fig. S1 in Supplementary Material II). In what follows, therefore, we stick to a single value, $\beta = 0.1$.

The time-dependence of the typical (ensemble averaged) temperature contrast $\langle F \rangle(t)$ forcing the atmospheric variables in (1) is shown in Fig. 3. The value of $\langle F \rangle(t)$ at each time instant is computed from Eqs. (4), (8), with the average values $\langle c \rangle$ (ensemble average over 50000 trajectories at that time instant) in place of c . The fluctuations in the $\langle F \rangle(t) = F(\langle c \rangle(t), t)$ curve of Fig. 3 follow the fluctuations in the average phytoplankton concentration, but, for small values of γ , the linear decrease of $F(t)$ is recovered. In other words, for weak mixing (small values of γ) the trend in the forcing $F(t)$ follows quite closely the direct anthropogenic trends. For strong mixing ($\gamma \leq 0.1$), however, the fluctuations have longer time-scale, hence the trends imposed by anthropogenic effects are less obvious, in particular, on shorter time-scales. A comparison of Figs. 3 a and b belonging to $\alpha = 0.05$ and $\alpha = -0.05$, respectively indicates that a change in the sign of the enrichment parameter leads to only minor differences in the general trends.

Next, we study the dependence of the ensemble average of the phytoplankton concentration on the strength of mixing. We have seen in Fig. 2 that for $\gamma = 0.1$ strong deviations appear from the trend, αD_0 , occurring without mixing. The time dependence of $\langle c \rangle$ for mixing parameters on this order of magnitude, shown in Fig. S2 of Supplementary Material II, confirms the existence of large fluctuations. The time dependence of $\langle c \rangle$ for much smaller values of γ are shown in Fig. 4.

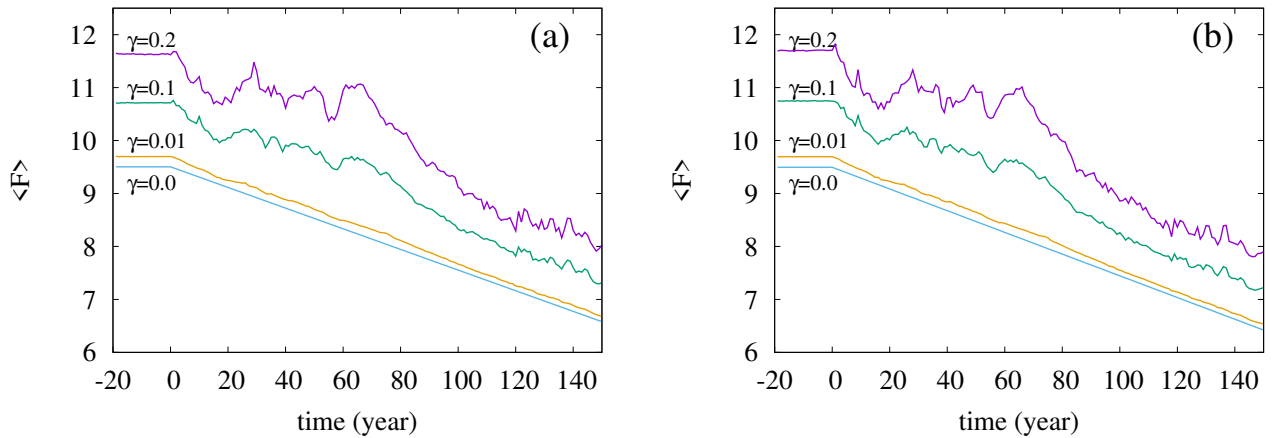


Figure 3. Time-dependence of the ensemble averaged atmospheric forcing $\langle F \rangle(t) = F(\langle c \rangle(t), t)$ in case of $\gamma = 0, 0.01, 0.1$ and 0.2 for a) $\alpha = 0.05$, b) $\alpha = -0.05$. The slope of the blue line for $\gamma = 0$ corresponds to D in (12).

240 The linearly increasing trend in harmony with (10) and (11) gradually disappears, and large scale fluctuations are visible even for $\gamma = 0.005$.

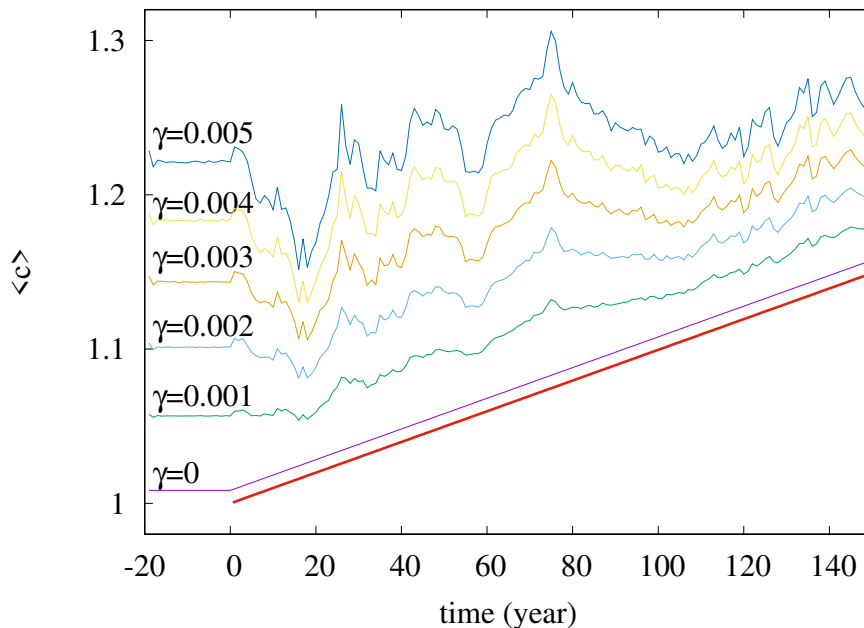


Figure 4. Ensemble averaged phytoplankton concentration $\langle c \rangle$ as a function of time for $\alpha = 0.05$ for various values of γ ($\gamma = 0, 0.001, 0.002, 0.003, 0.004$ and $\gamma = 0.005$). The thick red line shows the expected phytoplankton concentration in lack of mixing ($\gamma = 0$) as predicted by Eq. (11).

It seems that even a small coupling of the atmospheric variables x , y and z to the phytoplankton dynamics will result in large variations of $\langle c \rangle$ and in the suppression of the anthropogenic trends on short terms. One can also conclude from these figures that a coupling with $\gamma \geq 0.002$ should already be considered strong in the atmosphere-ocean interaction, at least from
 245 the point of view of the phytoplankton dynamics.

Now we investigate the effect of the enrichment parameter α on the phytoplankton concentration. We have seen in Fig. 4 that for small $\alpha = 0.05$, the short term trends are destroyed for $\gamma > 0.002$. We see in Fig. 5 that with an increase of $|\alpha|$, a trend might reappear at even higher values of the mixing parameter $\gamma = 0.01$. Indeed, for α between roughly -0.05 and 0.05 , no trend is visible, large scale fluctuations stemming from the internal variability of the dynamics rule the behavior of the average
 250 phytoplankton population. For $|\alpha| \geq 0.1$, however, we see that trends emerge, there is an increasing trend for positive, and a decreasing trend for negative α with a slope similar to the one given by the analytic calculation valid for $\gamma = 0$.

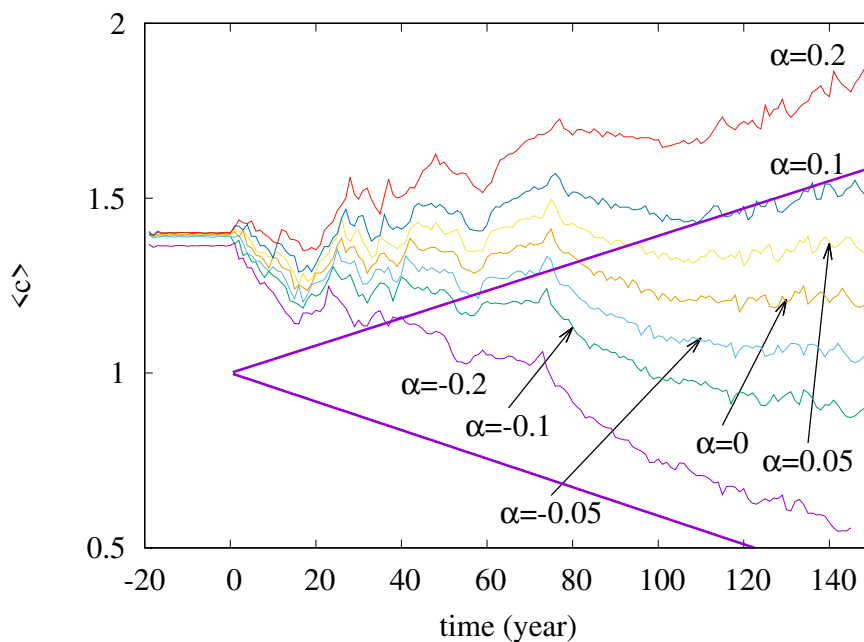


Figure 5. Average phytoplankton concentration $\langle c \rangle$ as a function of time for $\gamma = 0.01$ for various values of α (-0.2 , -0.1 , -0.05 , 0.0 , 0.05 , 0.1 and 0.2). The thick purple lines show the expected phytoplankton concentration without mixing ($\gamma = 0$) as predicted by Eq. (11) for $\alpha = -0.2$ (lower line) and $\alpha = 0.2$ (upper line).

From the same set of α values used to construct Fig. 5, we show the time-dependence of the average forcing $\langle F \rangle(t)$ for an intermediate ($\gamma = 0.01$) and a large ($\gamma = 0.1$) mixing parameter in Fig. 6a and b, respectively. Interestingly, for each value of α and γ , the $\langle F \rangle(t)$ graphs show a nearly linear decay, the slope depending somewhat on α . It seems that the direct anthropogenic component is dominant in the average forcing term, in particular for $\gamma = 0.01$, but this also holds qualitatively
 255 for $\gamma = 0.1$ (see Fig. 6b). We thus conclude that a mixing parameter on the order of 0.1 is not yet strong from the point

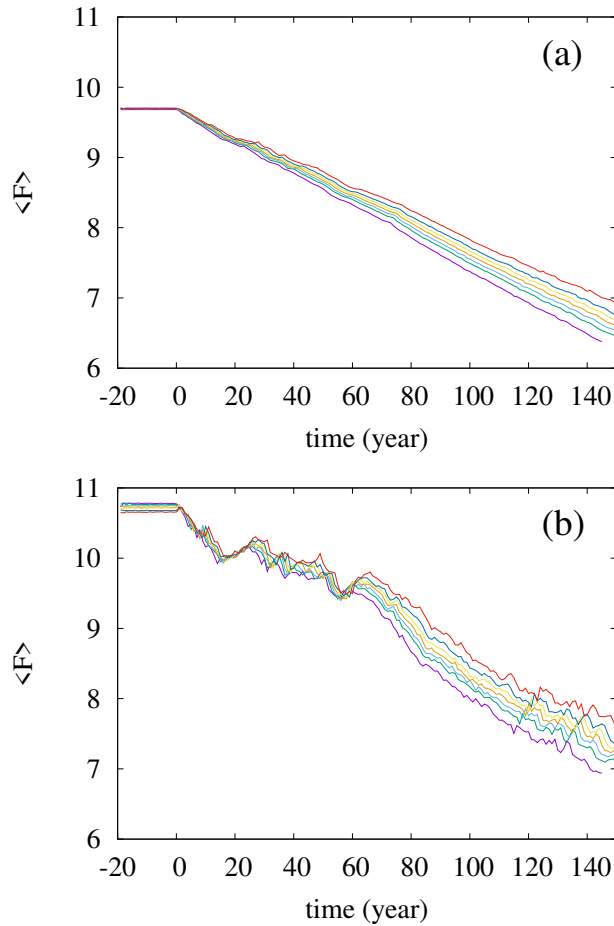


Figure 6. Time dependence of the average forcing $\langle F \rangle (t)$ in case of (a) $\gamma = 0.01$ and (b) $\gamma = 0.1$. The values of the enrichment parameter from top to bottom are $\alpha = 0.2, 0.1, 0.05, 0, -0.05, -0.1$ and -0.2 .

of view of the forcing. This is in harmony with the observation that the atmospheric kinetic energy hardly depends on the mixing strength (see Fig. S3 of Supplementary Material II): the atmosphere is rather resistant against the feedback from the biomass. Although an increased (decreased) amount of phytoplankton present in the system results in an increased (decreased) temperature contrast and hence in a decreased (enhanced) climate change, this effect is quite small. The order of magnitude of the effect of the phytoplankton concentration on $\langle F \rangle (t)$ can be assessed by observing in Fig. 5 that $\langle (c) - 1 \rangle$ falls between -0.5 and 1 at $t = 150$ y. Multiplied by our fixed $\beta = 0.1$, as (4) requires, one finds a range of 0.15 , which is much smaller than the final value of F , about 7 , at 150 y. This is comparable with the spread of the temperature contrast at the end of year 150 in Fig. 6a and b. Note that these conclusions are drawn from the average temperature contrast. No trend can be extracted if instantaneous values of a single simulation are used instead of the ensemble average, in the same spirit as in Fig. 2a.

Next, we study quantitatively how the trend observed in the ensemble average of the phytoplankton concentration changes with the parameters. To this end, we fit a straight line to the time dependence of the ensemble average $\langle c \rangle(t)$ of the phytoplankton concentration for $t > 0$ for various values of parameters α and γ . The slope $S(\alpha, \gamma)$ of the best fit line in the presence of mixing gives information on the trend of the phytoplankton concentration, that is, on how quickly the concentration changes with time on (ensemble) average. We have also computed the standard deviations of this fit from the measured values to gain information on the fluctuations appearing in individual members of the ensemble. We found (not shown) that in case of a strong trend (slope of time-dependence far from zero) we find small fluctuations and vice versa.

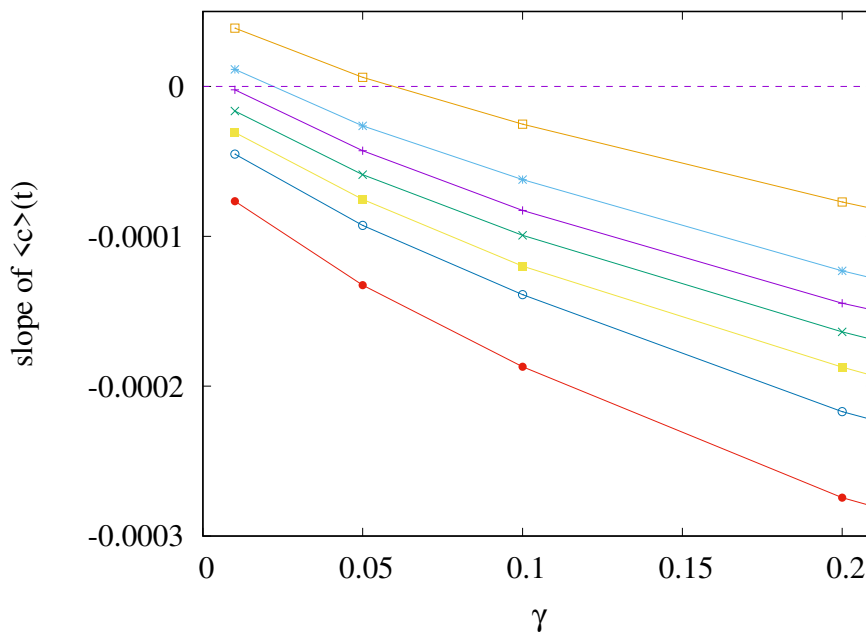


Figure 7. Slope S of the ensemble average of the phytoplankton concentration, $\langle c \rangle$, for $t > 0$ as γ is varied, data shown for various values of α , from top to bottom, $\alpha = 0.2, 0.1, 0.05, 0, -0.05, -0.1$ and -0.2 .

In Fig. 7 we show the approximate slope $S(\alpha, \gamma)$ of the $\langle c \rangle(t)$ curves as a function of the mixing parameter γ . We see that the measured slopes, that is, the trends in the time-dependence, decrease with increasing values of γ . We also found that the fluctuations (not shown) are enhanced when γ increases. This implies that when mixing becomes stronger, not only the phytoplankton concentration is decreasing for any α (the slope is negative), but even drops faster (the slope is decreasing). Note that the initial concentration from which the decrease starts at $t = 0$ is higher for larger γ (stronger mixing), see Fig. 4 and Fig. S2 of Supplementary Material II. Concerning the fluctuations, we call the attention to the fact that in nearly all figures exhibiting time dependence one can observe a decrease in the amplitude of variations for longer times, for $t > 100y$ approximately. This appears to be a consequence of the decrease of the total atmospheric kinetic energy with time, due to the overall decrease of the temperature contrast in time, as Fig. S3 of Supplementary Material II. also illustrates. At a fixed mixing

parameter γ , the strength of mixing is proportional to the kinetic energy, which is thus decreasing in time. Since the carrying capacity is assumed to linearly depend on the kinetic energy (see (9)), K also decreases in time. Thus, the phytoplankton concentration and its fluctuations are also decreasing with time.

285 It is worth also noting that even if for $\gamma = 0$ the trend in $\langle c \rangle$ would be increasing for positive enrichment parameters, see (11), it is the increase of γ that converts all trends to be negative. It remains true, however, that the trend for a positive α is less negative than for a negative α . In other words, for sufficiently strong mixing, the phytoplankton concentration always decreases with time due to climate change, the sign of the enrichment parameter only influences the strength of decrease.

If we plot the same data shown in Fig. 7 as a function of α instead of γ , see Fig. 8a, we see that the increase in the enrichment parameter increases the trend in the phytoplankton concentration. It is a surprising observation that even if the change in the mixing parameter changes the slopes essentially, their α -dependence remains similarly linear as for $\gamma = 0$ given in (11). Plotting the slope $-D(\alpha, \gamma)$ of the time-dependent ensemble averaged forcing $\langle F \rangle$ as a function of α , see Fig. 8b, a very weak dependence is found (note the vertical scale). On a closer look, the α -dependence is linear, and is increasing. This is in harmony with the expectation that the CO_2 extraction is weaker when the phytoplankton concentration is lower. With the exception of small γ values, the slopes are more negative than the direct anthropogenic one, $-D_0$. It is a remarkable finding supported by our results that a large mixing parameter enhances the speed of the climate change, irrespective of the sign of the enrichment parameter.

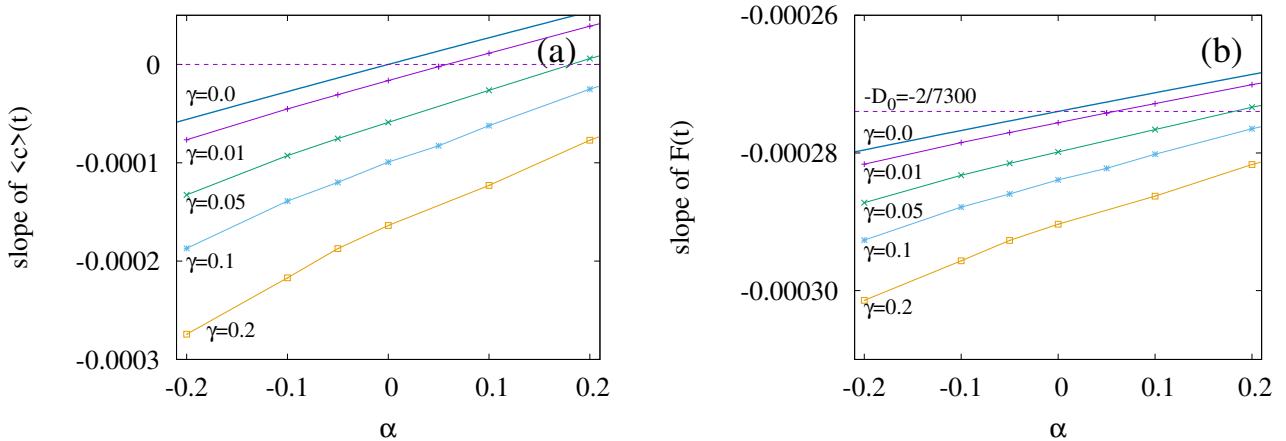


Figure 8. Slope (a) S of the ensemble averaged phytoplankton concentration $\langle c \rangle(t)$ and (b) $-D$ of the average forcing $\langle F \rangle$ for $t > 0$ as α is varied, data shown for various values of γ . The $\gamma = 0$ curve shows the α -dependence of (a) S and (b) $-D$ from Eqs. (11,12). Dashed lines mark the slopes for $\alpha = 0, \gamma = 0$.

We see that the trends predicted by Eqs. (11,12) are approached when γ is decreased. What is even more interesting, the dependence of the trends on α remains the same for any γ . In particular, we find a numerical fit of the slope S of $\langle c \rangle(t)$ for

300 $\beta = 0.1$ as

$$S(\alpha, \gamma) = \alpha D_0(1 + 3.8\gamma) - 2D_0\gamma^{0.75}. \quad (13)$$

A similar expression is obtained from the slopes of the averaged forcing $\langle F \rangle(t)$ that replaces D_0 found in (12) for $\gamma = 0$ by

$$D(\alpha, \gamma) = D_0[1 - \alpha\beta(1 + 3.8\gamma)] + 2\beta D_0\gamma^{0.75}. \quad (14)$$

It is surprising that the leading order linear behavior in the enrichment parameter α found for S and D without any mixing
305 remains valid for practically the entire γ range investigated, just the coefficients become γ -dependent.

5 Snapshot attractors

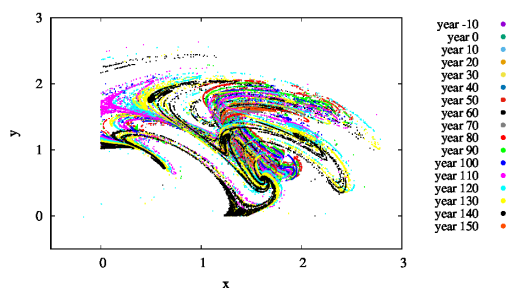


Figure 9. The projection of the $z = 0, z > 0$ section of the snapshot attractors on the x, y plane for $\beta = 0.1, \alpha = 0.05, \gamma = 0.1$. The snapshot attractors at intervals of 10 years are shown with purple ($t = -10$ y), green ($t = 0$ y), cyan ($t = 10$ y), light orange ($t = 20$ y), yellow ($t = 30$ y), dark cyan ($t = 40$ y), dark red ($t = 50$ y), dark grey ($t = 60$ y), grey ($t = 70$ y), red ($t = 80$ y), light green ($t = 90$ y), blue ($t = 100$ y), pink ($t = 110$ y), light blue ($t = 120$ y), bright yellow ($t = 130$ y), black ($t = 140$ y), dark orange ($t = 150$ y). They are generated by initiating 7×10^7 random initial conditions at year -20.

The mathematical concepts underlying the ensemble view are snapshot (Romeiras et al, 1990) or pullback (Ghil et al, 2008) attractors. One might consider the ensemble of all permitted climate realizations over all times as the pullback attractor of the problem, and the set of the permitted states of the climate at a given time instant as the snapshot attractor
310 belonging to that time instant (their union over all time instants is the pullback attractor). Both views express that the climate system possesses a plethora of possibilities. In the terminology of climate science, climate has a strong internal variability (e.g. Stocker et al (2013)). The concept of snapshot or pullback attractors is nothing but a reformulation of this fact in dynamical terms.

In numerical simulations, we consider the members of an ensemble simulation to describe parallel climate realiza-
315 tions only after the initial conditions are “forgotten”, transient dynamics disappears. Due to dissipation, this time is typically short compared to the time span of interest. Such an ensemble approach was shown to be the only method providing reliable statistical predictions in systems with underlying unpredictable dynamics (since in this class the

traditional approach based on single time series is known to provide seriously biased results). A number of papers illustrate these statements within the physics literature (see. e.g. (Romeiras et al, 1990; Lai , 1999; Serquina et al, 2008)), as well as in low order climate models (Chekroun et al, 2011; Bódai et al, 2011; Bódai and Tél, 2012; Bódai et al, 2013; Drótos et al, 2015), in general circulation models (Haszpra and Herein, 2019; Kaszás et al, 2019; Pierini et al, 2018, 2016; Drótos et al, 2017; Herein et al, 2017; Bódai et al, 2020; Haszpra et al, 2020; Haszpra and Herein and Bódai, 2020) and also in experimental situations (Vincze, 2016; Vincze et al, 2017).

For several parameter values, we also determined the snapshot attractors of the coupled model. An example is given in Fig. 9 where we see the attractor on the $z = 0$ slice of the atmospheric dynamics with the corresponding c -values not shown directly. Different colors indicate different time instances separated by 10 y, clearly indicating that the attractor is changing in time. As the colors indicate, the projection to the (x, y) plane of the $z = 0$ cross-section of the snapshot attractor has a minimum size in years 60–80, after which it increases again, and the maximum extension is reached by about year 150. Note that one cannot decide how much of the time dependence is a consequence of $F_0(t)$ or of the phytoplankton concentration. Due to the couplings between the biomass and the atmosphere, the direct anthropogenic effect cannot be separated from the effect of the biomass.

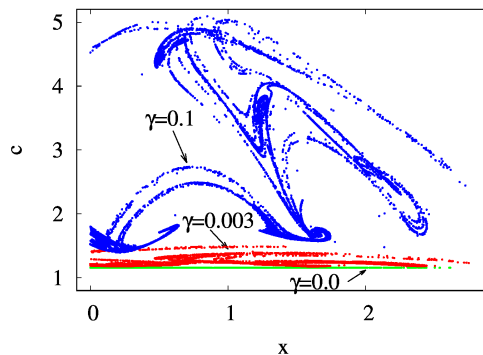


Figure 10. The projection of the $z = 0, \dot{z} > 0$ section of the snapshot attractors at year 150 on the x, c plane for $\beta = 0.1, \alpha = 0.05$, and for $\gamma = 0.0$ (green), 0.003 (red) and 0.1 (blue).

By investigating a projection of the snapshot attractor on a plane containing concentration c as one of the axes, the influence of mixing on the internal variability within c can be visualized. In Fig. 10, the $z = 0$ slice of the snapshot attractor of a given time instant is shown for three values of γ , projected to the x, c plane, that is, the y values are not shown. We see that the extension of the snapshot attractor in the c direction is greatly affected by the strength of mixing: the c extension is zero for $\gamma = 0$, but increases rapidly for increasing γ . Parallel to this, the pattern becomes interwoven in the space of variables, suggesting that the c -dynamics becomes more and more complex in time, too. It is the increasing size and complexity of the snapshot attractor in the c direction which is reflected in the increase of the strength of fluctuations in Fig. 4 and Fig. S2 of Supplementary Material II.

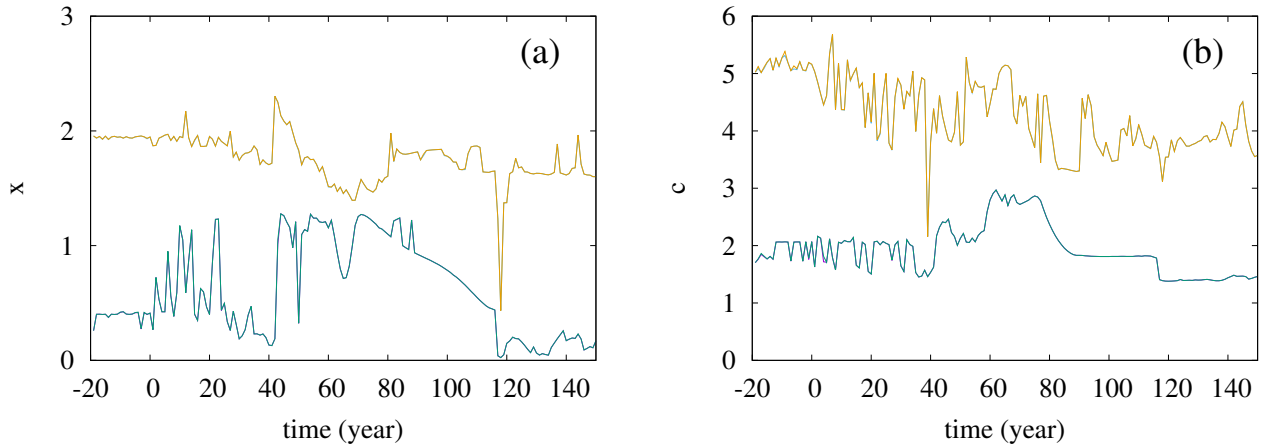


Figure 11. The extremes of the snapshot attractor for $\alpha = 0.05$, $\beta = 0.1$, $\gamma = 0.1$, Only 10 % of the points are found above (below) the higher (lower) values for each time instant for (a) x and (b) c .

340 We also investigated the extremes of the snapshot attractors. That is, at a fixed time instant, we looked for those values of e.g. x , for which only 10 % of values can be found on the snapshot attractor below (lower extreme) or above (higher extreme) x . These values of x are shown in Fig. 11a as a function of time. The interval between these thresholds is a measure of the size of the extension of the snapshot attractor at a given time instant. Clearly, this size undergoes strong variations as a function of time. The same is shown in Fig. 11b for the time dependence of the c extension of the snapshot attractor: the upper (lower) 345 curve shows the value of c above (below) which only 10 % of the values appear on the snapshot attractor. Again, we see considerable variations in time. It is interesting to note that, as these figures indicate, there is no unique trend in the size of the snapshot attractors, although trends can be seen in averages taken with respect to the ensemble designating the attractor itself, like e.g. in $\langle c \rangle$ or $\langle F \rangle$.

6 Conclusions

350 We have set up a conceptual coupled atmosphere–phytoplankton model by combining the Lorenz’84 general circulation model and the logistic equation under the condition of a climate change due to a linear decrease in the strength of direct anthropogenic forcing. The novel features of the model are in the choice of the possible forms of couplings. We allow for an influence of the biomass on the atmospheric forcing, modeling this way the extractions of CO_2 by phytoplankton, but the same forcing is able to modify the carrying capacity via its coupling to the temperature contrast characterized by the enrichment parameter. 355 An additional atmosphere-ocean coupling is also taken into account mimicking the enhancement of phytoplankton primary production via increased atmospheric activity, i.e., via turbulent mixing. Our intention has been to include leading order effects, and hence the couplig constants are chosen intentionally to be small. Nevertheless, interesting consequences are found.

By investigating the parameter dependence of the ensemble average of the atmospheric forcing and the phytoplankton content, we have shown that

- 360 – even without mixing, the phytoplankton biomass interacts with the atmospheric forcing, and the coupling between the phytoplankton concentration and the temperature might weaken or strengthen the anthropogenic warming trend, the increase or decrease of the phytoplankton biomass depends on the sign of the enrichment parameter. In this regime, analytic results are available, see Eqs. (10, 11 and 12).
- increased mixing parameter enhances the total phytoplankton population biomass. Stronger coupling may enhance fluctuation to a degree that the anthropogenic component practically disappears (Fig. 4 and Fig. S2 of Supplementary Material II.).
- 365 – in contrast, mixing appears to depress the trend of the extraction of CO₂ by phytoplankton, and may force the phytoplankton population to globally decrease in time (see Fig. 7), although starting from a higher initial level.
- the coupling of mixing with phytoplankton biomass has a much weaker effect on the atmospheric forcing (see Fig. 6),
370 as it is minimally expected from a coupled atmosphere-phytoplankton model.
- despite the strong modifications due to mixing, the dependence of trends on the strength of the coupling between the phytoplankton concentration and the temperature (the enrichment parameter) remains practically the same as without mixing (see Fig. 8).

We have obtained these results in a conceptual coupled atmosphere–phytoplankton model which contains a tractable number
375 of variables and parameters. To our knowledge, this is the first attempt to understand the general and robust features of the interplay between the atmosphere and the biosphere in a climate change framework. **One of our main results is that an increase in the global temperature reduces mixing intensity, which is the leading factor in decreasing the total biomass of primary producers. Interestingly, this result is in concordance with numerous studies applying Earth System Models with vastly more detailed plankton models (Bopp et al, 2013; Fu et al, 2016; Kwiatkowski et al, 2019), although other**
380 **works report different observations (Laufkötter et al, 2015; Flombaum et al, 2020).**

As far as we know, our work is the first step in the direction of studying the feedbacks between the atmosphere and the biosphere by a simple conceptual model. Our conclusions are robust in a mathematical sense, meaning that small changes in our model (inclusion of noise, for example) will not alter our main findings since snapshot attractors are robust. As long as the addition of other interactions only provide a small perturbation, our conclusions remain valid. In
385 general, it is an open question in complex nonlinear systems whether neglected couplings to other subsystems and other simplifications could cause qualitative change in the dynamical behavior of a model. However, we see two important reasons why we believe our model goes in the right direction. First, the trends we find in our model are in accordance with the trends observed in the majority of complex models as mentioned above. Second, we believe that in our model the origin of trends are more transparent than in more complex models where this origin can be hidden among the

390 **multitude of variables, feedbacks and interactions. Our model is a conceptual model, and as such, both the biological
and climate models are highly simplified. However, one can consider it as a starting modul of an extendable model
system. On the one hand, more trophical levels and inorganic resources can be easily added to the biological side of our
model, on the other hand, simple ocean circulation models can extend the climate side of our model in order to make
a first step to build more complex coupled models (Daron and Stainforth, 2013). We think that mutual interactions
395 and iterations between conceptual models and detailed Earth System Models (ESM) help to reveal the disctinction
between relevant and less relevant mechanisms and feedbacks behind climate change. We expect deeper insight into
these feedbacks by studying conceptual and ESMs parallelly in the future.**

Code availability. The C language code applied during the simulations is included in the Supplementary Material.

Author contributions. Gy. Károlyi, I. Scheuring and T. Tél worked out the outline of the applied model, with a special contribution of I.
400 Scheuring to the biological background, Gy. Károlyi and T. Tél carried out the analytical calculations, Gy. Károlyi developed the simulation
code, Gy. Károlyi and R.D. Prokaj carried out the simulations, all authors contributed to the preparation of the manuscript.

Competing interests. The authors declare that they have no conflict of interest.

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