

## Interactive comment on " $\Pi$ -theorem generalization of the ice-age theory" by Mikhail Y. Verbitsky and Michel Crucifix

## Anonymous Referee #3

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This is an interesting contribution to our understanding of the ice ages and the structure of glacial cycles. However, a broader review of ice age dynamics is needed in the introduction and in the wider paper. This will make it more accessible to a wider audience of Quaternary scientists. For the introduction, some reference to studies of ice age dynamics would be useful. There are obviously lots of paper you can refer to here, such as Imbrie et al. 1993, Paillard 2001 and Lang and Wolff 2011, for example.

In particular, it would be useful if you can explain more clearly and explicitly the wider significance of your findings for understanding the nature of glacial cycles. Your paper is clearly important because it provides a mathematical solution for understanding ice-age dynamics whereas other approaches are more qualitative or semi-quantitative (e.g. Hughes and Gibbard, 2018). However, Hughes and Gibbard (2018) found that our

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understanding of glacial cycles, especially ice dynamics, is not always easily explained by external forcing such as solar radiation, although this does account for  $\sim$ 50-60% of glacier change and associated sea level change through glacial cycles. Internal glacierclimate dynamics account for the rest of the glacier variations. A complex interplay of various geographical factors was found to be responsible for the asynchronous spatial variation in global glacier dynamics, in both the largest high- and mid-latitude ice sheets as well as in smaller mountain ice caps and glaciers at a range of latitudes around the world. Your modelling appears to incorporate ice sheet dynamics only, and the feedbacks associated with this, and does not account for the complexity of the known spatial and temporal glacial patterns. Of course, I don't expect you to solve this in your modelling, but you should make the reader know that you aware of the limitations of your approach.

You conclude that only two factors define most of the ice age dynamics: a) a balance between intensities of climate positive and ice sheet negative dynamics and b) the period T and the amplitude of the external forcing. I can see how for b) this can be constrained from orbital parameters but the variables for a) are potentially very complex and only partially accounted for in your modelling. From this, if we can be confident about b) it would be useful to see a statement on the comparable effects of a) versus b). You may already do this, but I would like to see a much clearer statement on this matter. For example, be much clearer about the implications of what you mean by "the amplitude and duration of glacial cycles is governed by a property of scale-invariance that does not depend on the underlying positive and negative feedbacks incorporated by the system". Unless you make the wider significance your findings more explicit, then it will have a limited audience. I think the findings are potentially very important, and you need to communicate these more effectively with those researching ice age dynamics, who will then be able to refer to your work, thereby increasing the academic impact of this paper.

Refs:

Hughes, P.D., Gibbard, P.L., 2018. Global glacier dynamics during 100 ka Pleistocene glacial cycles. Quaternary Research 90, 222-243.

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Lang, N., Wolff, E.W., 2011. Interglacial and glacial variability from the last 800 ka in marine, ice and terrestrial archives. Climate of the Past 7, 361-380. doi:10.5194/cp-7-361-2011

Paillard, D., 2001. Glacial cycles: Towards a new paradigm. Reviews of Geophysics 39, 325-346.

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