

## ***Interactive comment on “ $\Pi$ -theorem generalization of the ice-age theory” by Mikhail Y. Verbitsky and Michel Crucifix***

**Anonymous Referee #2**

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As ice-age dynamics is not my field of research, I have been kindly asked by the Editor to verify whether the math is correct and whether the application of the Buckingham-Pi theorem is adequate to this system. So from that perspective I haven't found any mistake and I find the application of the Pi theorem to be very nice.

Furthermore, the way the authors defined the non-dimensional  $V$  number as the ratio between the positive to negative feedback mechanisms is illuminating (although they have done it in a previous paper - Verbitsky et al., 2018, so this by itself is not a new result of the current paper). It is also quite impressive that, at least for a simple type of forcing, the overall response of the dynamical system depends to a high degree on the value of  $V$  rather than on the details of the different combinations of positive and negative feedback mechanisms. The authors themselves are honest to show the

C1

limitations of this approach for a more complex type of forcing like a multi sinusoidal forcing. So in terms of what is new in this paper - I think the authors managed now to incorporate their previous works into a more coherent mathematical framework based on, first - a drastic reduction of the number of parameters by defining the  $V$  number, and second by a further reduction using the Pi similarity theorem.

Therefore overall I think the paper deserves publication. I personally myself feel a bit uncomfortable with the starting point of a model of the type of eqs' (1-3). In one hand it is a low order model but on the other hand it is still quite complex. When I see such models I always get a feeling that maybe there are other equally important feedback mechanisms that are not included and maybe they will change dramatically the dynamical behavior.

Nonetheless, the robustness of the  $V$  number, at least when subject to a simple forcing, is impressive. Therefore, in order to strengthen the paper, and add more new material, my suggestion is that the authors will take this model and add several potential feedback mechanisms to obtain different variation of dynamical systems. Then they will have different  $V$  numbers for the different models. If for the same value of the different  $V$  numbers, for the different models, the dynamic response of the different models will be similar this will be highly cool and much more robust. This will mean that what truly matters is the ratio between positive to negative feedback mechanisms, not only within the same model but also with similar models of the same family. I will be happy to review the revised version.

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C2