Earth System
Dynamics

Discussions

Interactive comment on "Π-theorem generalization of the ice-age theory" *by* Mikhail Y. Verbitsky and Michel Crucifix

Anonymous Referee #1

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This paper uses an idealized model of ice age and the Buckingham-pi dimensional analysis theorem to gain insight into glacial cycles. The approach is interesting and can potentially benefit the community. I have a few comments and suggestions that I ask the authors to address.

Specific comments - I am a bit confused about why the authors have not gone beyond deriving equations such as (9) or (13) to actually find the full scaling relationship, as is often done (e.g., see the papers I mentioned in my very last comment). What I mean is to find the functional form of ϕ or X in these equations by computing the powers of Π_1 and Π_2 in Eq. (9) and $\Pi_1 - \Pi_4$ in Eq (13) using simulations. Even if the whole goal is to find scale invariances, then this is important: in the analysis of Eq. (13), the authors state that because Π_3 and Π_4 include P, then θ' is not expected to be scale

invariant. But it is possible that if you find the functional form of X, you find something like $\theta' \sim P \times \ldots \Pi_3^A \times \Pi_4^B$ with $A = -B$. In that case, P drops out from X and θ' would be scale invariant with P. The authors should do this analysis, or fully explain why it is not needed, and also address the issue I raised about their interpretation of Eq. (13).

- While there is great value in idealized models, and as the authors clearly stated, the dimensional analysis could be only effectively applied to an idealized model, I believe that the authors should at the end, test, or discuss the implications of, their findings in the context of data from more comprehensive models or actual observations (proxies). That would really demonstrate the power of this approach and increase the impact of this work.

Minor comments/suggestions

- Line 47: explicitly mention that in this case, one gets 18-4=14 pi groups

- It is up to the authors, but I suggest using the word "dimensionless" instead of "adimensional"

- Line 45: what is the unit of concentration in terms of fundamental dimensions?

- It is up to the authors, but I suggest using Kelvin (K) instead of degree Celsius (C) as the unit of temperature

- Fig 1: improve the clarity of the figure and expand the caption. Also, What is the line with $\beta_a = 1$?

- Lines 39-41: There are a few papers in which the Buckingham-pi theorem is applied to a problem in global climate dynamics or its low-dimensional model,

MJO: Yang, D. and Ingersoll, A.P., 2014. A theory of the MJO horizontal scale. Geophysical Research Letters, 41(3), pp.1059-1064.

Planetary circulation: Koll, D.D. and Abbot, D.S., 2015. Deciphering thermal phase curves of dry, tidally locked terrestrial planets. The Astrophysical Journal, 802(1), p.21.

Blocking events: Nabizadeh, E., Hassanzadeh, P., Yang, D. and Barnes, E.A., 2019. Size of the atmospheric blocking events: Scaling law and response to climate change. Geophysical Research Letters. 46

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