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Interactive comment

Interactive comment on "Π-theorem generalization of the ice-age theory" *by* Mikhail Y. Verbitsky and Michel Crucifix

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Dear Anonymous Referee #3,

Thank you very much for your detailed review and helpful suggestions. We appreciate that you consider our findings to be important. The following is our response to your suggestions.

Suggestion: This is an interesting contribution to our understanding of the ice ages and the structure of glacial cycles. However, a broader review of ice age dynamics is needed in the introduction and in the wider paper. This will make it more accessible to a wider audience of Quaternary scientists. For the introduction, some reference to studies of ice age dynamics would be useful. There are obviously lots of papers you

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Discussion paper

can refer to here, such as Imbrie et al. 1993, Paillard 2001 and Lang and Wolff 2011, for example.

Answer: We agree that the introduction may be expanded. Currently, it is focused on the unique properties of our model that allows us to use the π-theorem insightfully. At the same time, it does not provide enough background that would allow our readers to better appreciate the importance of the obtained results.

Action: We will expand the introduction with a brief review of the state-of-the-art ice age research.

Suggestion: In particular, it would be useful if you can explain more clearly and explicitly the wider significance of your findings for understanding the nature of glacial cycles. Your paper is clearly important because it provides a mathematical solution for understanding ice age dynamics whereas other approaches are more qualitative or semi-quantitative (e.g. Hughes and Gibbard, 2018). However, Hughes and Gibbard (2018) found that our understanding of glacial cycles, especially ice dynamics, is not always easily explained by external forcing such as solar radiation, although this does account for 50-60% of glacier change and associated sea level change through glacial cycles. Internal glacier climate dynamics account for the rest of the glacier variations. A complex interplay of various geographical factors was found to be responsible for the asynchronous spatial variation in global glacier dynamics, in both the largest high- and mid-latitude ice sheets as well as in smaller mountain ice caps and glaciers at a range of latitudes around the world. Your modelling appears to incorporate ice sheet dynamics only, and the feedbacks associated with this, and does not account for the complexity of the known spatial and temporal glacial patterns. Of course, I don't expect you to solve this in your modelling, but you should make the reader know that you aware of the limitations of your approach.

Answer: Your observation, that the ice-sheet dynamics (equations (1) and (2)) is the most comprehensive and most physically substantiated part of the model, is correct. The equation (3) of the "rest-of-the-climate" is ambiguous but its ambiguity allows us to interpret our experiments with increased (or reduced) $\gamma 2$ as experiments with additional feedbacks of different nature (see also our response to Anonymous Referee #2 https://www.earth-syst-dynam-discuss.net/esd-2019-65/esd-2019-65-AC2-supplement.pdf). In other words, we are uncertain about some key mechanisms that we have chosen to describe using the "rest-of-the-climate" linear equation. Among others, non-linear effects related to the carbon cycle, non-linear effects related to sea-level destabilization of ice sheets and related synchronization, non-linear effects related to atmospheric circulation, or non-linear effects related to biogenic calcifiers and their action on alkalinity, etc. A challenger might thus claim that these effects are so important that they should be taken off $\gamma 2$ and be considered more explicitly. However, we have the hope that even after accounting for these processes, we might end up with a model that still has grossly the same mathematical structure as the Verbitsky et al (2018) model, even though the meaning of some of the variables will have changed.

Action: We will add the above discussion into the text.

Suggestion: You conclude that only two factors define most of the ice age dynamics: a) a balance between intensities of climate positive and ice sheet negative dynamics and b) the period T and the amplitude of the external forcing. I can see how for b) this can be constrained from orbital parameters but the variables for a) are potentially very complex and only partially accounted for in your modelling. From this, if we can be confident about b) it would be useful to see a statement on the comparable effects of a) versus b). You may already do this, but I would like to see a much clearer statement on this matter. For example, be much clearer about the implications of what you mean by "the amplitude and duration of glacial cycles is governed by a property of scale-invariance that does not depend on the underlying positive and negative feedbacks incorporated by the system". Unless you make the wider significance your findings more explicit, then it will have a limited audience. I think the findings are potentially very important, and you need to communicate these more effectively with those researching ice age dynamics, who will then be able to refer to your work, thereby increasing the academic impact of this paper.

Answer: Unlike many models of the ice-age climate that postulate internal 100-kyr oscillator, in our model, 100-kyr cycle is produced as a non-linear system response to the astronomical forcing. Nothing happens without astronomical forcing and nothing happens without system internal dynamics. Therefore, it is not possible to quantify precisely the impact of the astronomical forcing versus internal climate dynamics. Furthermore, a similar system response may be observed with different forcing and different internal climate dynamics. For illustration, let us consider equation (7)

P=TΨ(V,ε/a)

and equation (9)

S $'$=εˆ2 Pˆ2 Φ(V,ε/a)

Here the V-number is defined by the climate dynamics, ε/a is the relative intensity of the astronomical forcing, T is the forcing period, P is the period of system response, and S' is the amplitude of the system response. Function Ψ(V,ε/a) defines a forcing-period doubling domain and it may be the same (let say, Ψ=2) for different combinations of V and ε/a (see also Fig. 1). We can only say that the obliquity-period doubling requires both well-developed positive feedbacks in the system ($0.6 < V < 0.8$) and relatively high climate sensitivity to the astronomical forcing ($\varepsilon/a \approx 1$). Moreover, different sets of parameters may lead to the same V-number. Function Φ(V,ε/a) and function Φ'(V,ε/a) corresponding to the same value of the V-number but formed by the different parameters may not be the same. Similarly, function Ψ(V,ε/a) and function Ψ'(V,ε/a) may differ. Most remarkably though, the power degree "2" of the response-period in the equation (9) is defined by the fundamental dimensionality requirements and does not depend on the underlying physics. This is what gives us the property of scale invariance but at the same time makes our efforts to disambiguate historical records even more challenging.

As a result, we can't claim to have a full picture of the mechanisms of ice ages, but if ice age physics are well captured by the mathematical structure that we have obtained, then this scale invariance linking amplitude and response periods applies. We further suggest that a model that would indeed be a bit different than the Verbitsky et al (2018) model because it includes some other important (may be non-liner) mechanisms, might still retain an important property that we have discovered: there is a connection between the sensitivity of the fixed point (since the V-number is indeed constructed by consideration to the sensitivity of the fixed point) and a scale invariance linking period and amplitude of response. This seems to be the fundamental proposal, for which we welcome challengers equipped with bigger models.

Action: We will add the above discussion into the text.

References:

Verbitsky M. Y., M. Crucifix and D. M. Volobuev (2018), A theory of Pleistocene glacial rhythmicity, Earth System Dynamics, (9) 1025–1043 doi:10.5194/esd-9-1025-2018

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