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Interactive comment on "Π-theorem generalization of the ice-age theory" *by* Mikhail Y. Verbitsky and Michel Crucifix

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Dear Anonymous Referee #2,

Thank you very much for your thorough review and a very interesting suggestion. We are delighted to hear that you find our research to be illuminating and impressive. The following is our response to your suggestion.

Suggestion: Therefore overall I think the paper deserves publication. I personally myself feel a bit uncomfortable with the starting point of a model of the type of eqs' (1-3). In one hand it is a low order model but on the other hand it is still quite complex. When I see such models I always get a feeling that maybe there are other equally important feedback mechanisms that are not included and maybe they will change dramatically

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Discussion paper

the dynamical behavior. Nonetheless, the robustness of the V number, at least when subject to a simple forcing, is impressive. Therefore, in order to strengthen the paper, and add more new material, my suggestion is that the authors will take this model and add several potential feedback mechanisms to obtain different variation of dynamical systems. Then they will have different V numbers for the different models. If for the same value of the different V numbers, for the different models, the dynamic response of the different models will be similar this will be highly cool and much more robust. This will mean that what truly matters is the ratio between positive to negative feedback mechanisms, not only within the same model but also with similar models of the same family. I will be happy to review the revised version.

Answer: The Verbitsky et al (2018) model is, to our knowledge, unique because it is the only low-order ice-age model that, instead of being postulated, has been parsimoniously reduced from the conservation equations of viscous ice flow. Equations (1) and (2) have been derived from the ice mass balance and the ice-flow energy equations, correspondingly. For this reason, we would prefer to keep them untouched. The equation (3) of the "rest-of-the-climate temperature" is, indeed, ambiguous, but since it is linear, it can be split into several equations:

ω=ω_1+ω_2+ ... +ω_n

(dω_1)/dt=γ_11 - γ_21 (S-S_0) - γ_3 ω_1

(dω_2)/dt=γ_12 - γ_22 (S-S_0) - γ_3 ω_2

...

(dω_n)/dt=γ_1n - γ_2n (S-S_0) - γ_3 ω_n

Each of the above equations may represent different feedback mechanisms. Therefore our experiments with increased (or reduced) $\gamma 2$ may be also understood as experiments with additional feedbacks of different nature ($\gamma 2 = \gamma 21 + \gamma 22 + \ldots + \gamma 2n$), though of the same time-scale $1/\gamma 3$.

Certainly, if we introduce in our model more dramatic - although not necessarily more realistic - changes, the dynamics of the system may be different. As an illustration, let us consider the van der Pol oscillator. It was previously suggested as a minimal model capturing ice-age dynamics (Crucifix, 2012):

dx/dt=(-y+β+γF)/τ

dy/dt=(-α(y^3/3-y-x))/τ

Here all variables and parameters, except τ, are dimensionless; τ is measured in units of time. Variable x is thought to represent the global ice volume, and variable y makes the "rest-of-the climate" response. Using the same π-theorem technique, let's determine the period P and the amplitude x' of the system response to the external forcing F of the period T.

P=ψ(α,β,γ,τ,T)

P=TΨ(α,β,γ,τ/ T)

Since α, β, and γ are constants

P= TΨ(τ/ T)

Similarly,

x'=φ(α,β,γ,τ,P)

x'=Φ(α,β,γ,τ/P)= Φ(τ/P)

It means that the amplitudes of forced fluctuations in the van der Pol model are not expected to be scale invariant. We have tested this conclusion experimentally for τ = 36.2 kyr (this reference value of τ produces auto-oscillations with a 100-kyr period) and a forcing period T ranging from 5 kyr to 100 kyr.

Therefore, instead of comparing our model with other existing low-order models or creating a new low-order model for the sole purpose of a comparison, we think it would be more advantageous, in the future work, to compare our results with calibrated simulations of intermediate-complexity models and 3-D spatially-resolving models. Having said that, we are confident that the discussion you initiated would benefit our paper, and therefore. . .

Action: . . . we will add the above discussion into the text

Reference:

M. Crucifix "Oscillators and relaxation phenomena in Pleistocene climate theory." Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences 370, 1962, 1140-1165, 2012

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