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Earth System Dynamics
Discussions

Interactive comment on "Π-theorem generalization of the ice-age theory" *by* Mikhail Y. Verbitsky and Michel Crucifix

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Dear Anonymous Referee #1,

Thank you for your detailed review and insightful suggestions. We are pleased to learn that you find our approach to be interesting and helpful. The following is our response to your comments and suggestions.

Comment: I am a bit confused about why the authors have not gone beyond deriving equations such as (9) or (13) to actually find the full scaling relationship, as is often done (e.g., see the papers I mentioned in my very last comment). What I mean is to find the functional form of φ or X in these equations by computing the powers of $\Pi1$ and $\Pi2$ in Eq. (9) and $\Pi1 - \Pi4$ in Eq (13) using simulations. Even if the whole goal is

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to find scale invariances, then this is important: in the analysis of Eq. (13), the authors state that because $\Pi3$ and $\Pi4$ include P, then θ is not expected to be scale invariant. But it is possible that if you find the functional form of X, you find something like θ' âĹij P$\times$. . . ..($\Pi3$)ˆA $\times$ ($\Pi4$)ˆB with A = $-$B. In that case, P drops out from X and θ' would be scale invariant with P. The authors should do this analysis, or fully explain why it is not needed, and also address the issue I raised about their interpretation of Eq. (13).

Answer: As we discuss in the paragraph 4.1, the property of the scale invariance does not depend on the physical nature of the underlying positive and negative feedbacks that define the V-number ($\Pi1$). At the same time, the function Φ of the equation (9) and the function X of the equation (13) do depend on the underlying physics. To calculate functions Φ or X as powers of $\Pi1$ and $\Pi2$ in Eq. (9) and $\Pi1 - \Pi4$ in Eq (13), we would need to span the space of eight (8) parameters forming the V-number. Obviously, this would defeat the purpose of this research, and therefore, we have limited ourselves with the discovery of the scale invariance only. Nevertheless, your observation regarding the equation (13) is correct. We have also observed experimentally that when the amplitude of the external forcing, ε, is reduced, the equation (13) becomes scale invariant with a frequency slope equal 1. In this case θ' $\sim$ X($\Pi1$, $\Pi2$, $\Pi3/\Pi4$). We did not include this analysis into the paper because the effect of a reduction of the amplitude of the astronomical forcing is not something we expect to see in the real world. However, in retrospect and given your comment, we see the benefit of bringing this analysis back, because it will hopefully make our thinking more explicit.

Action: We will add this discussion into the text

Comment: While there is great value in idealized models, and as the authors clearly stated, the dimensional analysis could be only effectively applied to an idealized model, I believe that the authors should at the end, test, or discuss the implications of, their findings in the context of data from more comprehensive models or actual observations (proxies). That would really demonstrate the power of this approach and increase the impact of this work.

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Answer: This is a good point, but difficult to address in practice. We would like to take this opportunity to share our views about how our study, we believe, contributes to filling a gap in the literature. Palaeoclimate simulations with “more sophisticated models”, including the seminal paper by Abe-Ouchi et al., 2013, and the simulations with CLIMBER provided by Ganopolski et al. 2010, tend to focus on the response of the ice-sheet climate system to orbital forcing, and discuss the respective amplitudes of the 100-ka, 41-ka, and 21-23-ka periods, but none discuss the slope of the power spectrum down to the millennium scale. Yet, empirical analysis of paleoclimate series shows that there is a rich spectral content and point to the existence of “spectral slopes” (to cite by a few, Huybers and Curry 2006 and Lovejoy and Schertzer, 2013). Lovejoy and Schertzer evoke some generic process, such as the principle of “cascades” and which is tightly linked to the concept of scale invariance of the equations. For example, the scale invariance of fluid-dynamics equations is exploited to provide inferences about spectral slopes of turbulent flows. However, to our knowledge, there is no available theory supporting scale invariance in regimes associated with glacial-interglacial dynamics. So, we believe that we have here been providing at least some important elements that should help us to bridge both approaches. If the sensitivity of the stationary state is effectively determined by a dimensionless number (the V-number) in the way our model does, then we satisfy a necessary condition to produce relationship between the amplitude and duration of glacial cycles over a reasonably wide range of periods, including the millennial scales. It would indeed be useful to see whether a similar response-scaling structure appears with more sophisticated ice-sheet-atmosphere model. This might not be too difficult to verify with an adequate set of experiments, but we must obviously leave this task to the scientists who know and develop these models. Perhaps, though, it is worth restating the physical roots of our enterprise. Our model was developed with attention to scaling invariance of ice flow conservation laws (Verbitsky et al. 2018), and was also tested against the ice-sheet-ice-shelf model of Pollard and De Conto (2012). Of course, we fully appreciate that there is some mileage left before delivering of a full theory of the fluctuation spectrum, from millennial to glacial-interglacial cycles. This objective, among others, requires understanding better the structure of the millennial variability, which was here merely postulated as a forcing. Hopefully the reviewer will understand that need to proceed step by step.

Action: We will add this discussion into the text

Minor comments/suggestions

Line 47: explicitly mention that in this case, one gets 18-4=14 pi groups; Action: Will be done

It is up to the authors, but I suggest using the word “dimensionless” instead of “adimensional” Action: Will be done

Line 45: what is the unit of concentration in terms of fundamental dimensions? It is up to the authors, but I suggest using Kelvin (K) instead of degree Celsius (C) as the unit of temperature Answer: CO2 concentration is usually measured in ppm, parts per million, or mg/L. Since we mention these units in a reference to a specific model and its variables (Saltzman and Verbitsky, 1993), we think we need to keep the units of measurements that the authors used in their model.

Fig 1: improve the clarity of the figure and expand the caption. Also, what is the line with βa = 1? Action: Higher quality pictures (including better captions) will be provided

Lines 39-41: There are a few papers in which the Buckingham-pi theorem is applied to a problem in global climate dynamics or its low-dimensional model, MJO: Yang, D. and Ingersoll, A.P., 2014. A theory of the MJO horizontal scale. Geophysical Research Letters, 41(3), pp.1059-1064. Planetary circulation: Koll, D.D. and Abbot, D.S., 2015. Deciphering thermal phase curves of dry, tidally locked terrestrial planets. The Astrophysical Journal, 802(1), p.21. C2 ESDD Interactive comment Printer-friendly version Discussion paper Blocking events: Nabizadeh, E., Hassanzadeh, P., Yang, D. and Barnes, E.A., 2019. Size of the atmospheric blocking events: Scaling law and response to climate change. Geophysical Research Letters. 46

Answer: We agree. Indeed, if we say “low-order models of global climate dynamics” we should mention the references you provided. Otherwise, we need to narrow our statement, like, for example “low-order models of the Pleistocene climate” Action: The sentence will be edited.

References:

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Ganopolski A., R. Calov and M. Claussen (2010), Simulation of the last glacial cycle with a coupled climate ice-sheet model of intermediate complexity, Climate of the Past, (6) 229–244 doi:10.5194/cp-6-229-2010

Huybers P. and W. Curry (2006), Links between annual, Milankovitch and continuum temperature variability, Nature, (441) 329-332 doi:10.1038/nature04745

Lovejoy, Shaun, and Daniel Schertzer. The weather and climate: emergent laws and multifractal cascades. Cambridge University Press, 2013.

Pollard D. and R. M. DeConto (2012), Description of a hybrid ice sheet-shelf model, and application to Antarctica, Geoscientific Model Development, (5) 1273–1295 doi:10.5194/gmd-5-1273-2012

Verbitsky M. Y., M. Crucifix and D. M. Volobuev (2018), A theory of Pleistocene glacial rhythmicity, Earth System Dynamics, (9) 1025–1043 doi:10.5194/esd-9-1025-2018

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