Earth Syst. Dynam. Discuss.,
https://doi.org/10.5194/esd-2019-41-AC3, 2019

Earth System Dynamics Discussions
Open Access
EGU

Interactive comment on "Estimates of climatic influence on the carbon cycle" *by* Ian Enting and Nathan Clisby

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Received and published: 13 November 2019

Note This author comment on Laplace transform is posted to provide context for detailed reply to review comments.

We have found the Laplace transform has been a useful way of exploring relations between components of the carbon cycle. However, our main reason for introducing the Laplace transform in our paper is as a way of emphasising the relation

$$p\,r(p) = \frac{1}{1 + \beta_O(p) + \beta_L(p)} \qquad (AC3.1)$$

which links our approach in terms of response functions to the description in terms of β and γ which has been used in the majority of studies of climate to carbon influence:

$$q(p) = \frac{s(p)/p}{1+\beta_O(p)+\beta_L(p)} + \frac{\gamma_O(p)+\gamma_L(p)}{1+\beta_O(p)+\beta_L(p)} \qquad \text{(Enting and Clisby (5))}$$

This maps one-to-one as a generalisation of the relation given by Friedlingstein et al (2003) (see equation (1) of Enting and Clisby) which in turn has defined the concepts and notation used in the majority of studies of climate-to-carbon feedback.

As summarised below, the Laplace transform captures both the asymptotic form of the airborne fraction and also the relations between the β factors and reservoir-specific response functions used in much earlier work, in particular the characterisation of ocean carbon models. Similarly, the Laplace transform provides a compact way of showing how our estimates relate to the weighted averaging used by Bauska et al. (2015). Our final response to review comments will indicate how we propose to revise our paper in order to emphasise these issues.

It is well known that in a linear system, subject to exponential forcing, all the system components will respond with the same exponential growth rate. The results from Oeschger et al (1980) cited in AC2 are a special case of this. An exponential response at time t requires exponential forcing over all times $t' \in (-\infty, t]$.

If the system behaviour is described using a response function $R(t)$, forcing $S(t) = A\exp(\lambda t)$ leads to response

$$Q(t) = A\int_{-\infty}^{t} \exp(\lambda t')R(t-t')dt' \qquad (AC3.2)$$

or putting $t'' = t - t'$

$$Q(t) = A\exp(\lambda t)\int_{0}^{\infty} \exp(-\lambda t'')R(t'')dt'' = A\exp(\lambda t)r(\lambda) \qquad (AC3.3)$$

Thus the instantaneous airborne fraction is given by

$$\dot{Q}(t)/S(t) = \lambda\, r(\lambda)$$

and similarly the cumulative airborne fraction is

$$Q(t) \Big/ \int_{-\infty}^{t} S(t')\, dt' \; = \lambda\, r(\lambda)$$

This is the asymptotic limit of Laplace transform relations which apply for functions defined on $[0, \infty)$, where for an exponential response function $\exp(-\alpha t)$, the Laplace transform gives

$$q(p) = \frac{A}{p-\lambda} \times \frac{1}{p+\alpha} = \frac{A}{\alpha - \lambda}\left[\frac{1}{p-\lambda} + \frac{1}{p+\alpha}\right] \qquad (AC3.4)$$

which has limit as per eqn (AC3.3) plus a transient term decaying as $\exp(-\alpha t)$.

This quantifies role of transients, as discussed by Gloor et al. (2010). The generalisation to when $R(t)$ is expressed as a sum of exponentials is obvious. (The special case of term with $\alpha = 0$ redefines the origin of Q).

As noted above (and our comment SC1) relation (AC3.1) above is not widely recognised. Thus reviewer 2 did not recognise the relation between the β factors and the relation given by Oeschger et al (1980). Similarly in the comprehensive review of feedbacks by Gregory (2009), the abstract states that “The concentration to carbon feedback is negative, it has generally received less attention in the literature . . . ” [i.e. less compared to the climate-carbon feedback].

In reality, as $R(t)$, the concentration-to-carbon feedback is widely discussed as representing carbon cycle response in carbon cycle studies. $R(t)$ is also very widely discussed because it defines the reference level for GWP. (Note that more recent usage has $R_{FB}(t)$ rather than $R(t)$ in the definition of GWP).

Of course, all the relations that we express as Laplace transforms can be expressed using integro-differential equations in the time domain. We would argue that actually doing so would deter readers more than the use of Laplace transforms.

Assuming that one wants to avoid the use of infinite sums of successively higher order integrals (representing binomial expansions of denominators of Laplace transform expressions), the fractions have to be multiplied out and equation (4) of our paper becomes:

$$Q(t)+\int_0^t Q(t')[B_O(t-t')+B_L(t-t')]\,dt' = \int_0^t S(t')\,dt' - \int_0^t W(t')[\Gamma_O(t-t')+\Gamma_L(t-t')]\,dt' \quad (AC3.5)$$

where $B_O(t)$, $B_L(t)$, $\Gamma_O(t)$, $\Gamma_L(t)$ are response functions, with Laplace transforms $\beta_O(p)$, $\beta_L(p)$, $\gamma_O(p)$, $\gamma_L(p)$ that characterise the feedbacks from concentration and temperature for the ocean and land pairs.

The $B_O(t)$, $B_L(t)$ that describe responses to concentration, can be related to responses $R_O(t)$, $R_L(t)$ that describe responses to fluxes. Here $R_O(t)$ is an ocean-only response (e.g. as used by Oeschger and Heimann (1983) and many subsequent studies) and $R_L(t)$ is a biota-only response (e.g. as calculated by Friedlingstein and reported in Enting et al (1994)).

In terms of Laplace transforms, the connection is (see eqn (13b) of Enting (2007)):

$$\beta_O = \frac{1}{p r_O(p)} - 1$$

and

$$\beta_L = \frac{1}{p r_L(p)} - 1$$

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Thus in the time domain

$$\int_0^t R_L(t-t')\Phi(t')dt' + \int_0^t R_L(t-t') \int_0^{t'} B_L(t-t'')\Phi(t'')dt'' = \int_0^t \Phi(t')dt' \quad (AC3.6)$$

serves to connect the response calculated by Friedlingstein in 1994 to the β factor introduced by Friedlingstein et al in 2003. (An obvious correspondence relates the $B_O(t)$ (and thus its Laplace transform β_O) to the ocean-only response $R_O(t)$ used by Oeschger and Heimann (1983) and in many later studies.)

We would argue that relations such as (AC3.5) and (AC3.6) are much more comprehensible as Laplace transforms.

Additional reference (for this comment, not proposed for paper)

Oeschger, H. and Heimann, M. (1983). Uncertainties of predictions of future atmospheric CO_2 concentrations. *J. Geophys Res.*, **883**, 1258–1262.

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