Response to Comments of Anonymous Referee #2

Authors thank Reviewer #2 for the valuable comments and suggestions. We believe that the manuscript is improved considerably after the suggested revisions. Our responses are in blue color below.

Referee #2: It will be nice to have a global approach that has some background and reason for the research in the first paragraph of the introduction. Right now, it is jumping to the problem. … The reviewer suggests revising the introduction section with implicit assumptions behind them.

Authors' response: Thank you for the valuable suggestion. The Introduction section was revised to include the below section in the revised manuscript:

"Nearly 70 years ago in his hydrologic studies of the High Aswan Dam, Hurst (1951) has discovered that the flow time series of the Nile river demonstrated fluctuations whose rescaled range may not be proportional to the square root of the observation duration, but may be proportional to the duration raised to a power H (the so-called Hurst coefficient) that is larger than 0.5 but less than 1. This finding, now called as the "Hurst phenomenon" implies that in such river flows the integral scale (the integral of the flow autocorrelation function with respect to the time lag, over the range from zero to infinity) may not exist, putting the process outside the Brownian domain of finite-memory processes where the integral scale is finite. Since the Hurst phenomenon amounts to the clustering of wet years with wet years and the dry years with the dry years, the so-called "Joseph effect" in the Bible (Mandelbrot, 1977), it has important consequences on the planning and operation of water storage systems over long periods (Koutsoyiannis, 2005). Hurst phenomenon in hydrologic flow processes was later demonstrated convincingly by various researchers, including Eltahir (1996), Radziejewski and Kundzewicz (1997), Montanari et al. (1997) and Vogel et. al. (1998) among others. In order to model the Hurst phenomenon in river flows the fractional Gaussian noise (FGN), where the rescaled range for the time series of a flow process in a time interval [0,t] is proportional to t^H for $0.5<H<1$, was introduced by Mandelbrot and Wallis (1969). FGN model was later extended by Koutsoyiannis (2002) in order to model satisfactorily a range of time scales, including the conventional Brownian finite memory flow processes. Aside from the FGN models, physically-based models of the Hurst phenomenon were also developed by various authors, including Klemes (1974), Beran (1994) and Koutsoyiannis (2003). However, a physically-based model that explains the Hurst phenomenon explicitly in terms of the hydrologic process mechanisms is still missing. Yevjevich (1963, 1971) provided a plausible physical explanation for the Markovian structure of the annual river flows within a river basin by linking the annual evolution of the water storage in the basin to the exponential recession in baseflow of the basin runoff. Meanwhile, baseflow in basin runoff is mainly due to unconfined aquifer flow to the neighboring stream network of the basin. As shall be shown in a numerical example later in this paper, the conventional unconfined groundwater flow equation with integer powers does result in the hydraulic head of and the discharge from the aquifer to decay exponentially, that would result in the Markovian finite memory behaviour of the river outflow from the basin. Such exponentially decaying baseflow, while it can be explained by the mechanics of the conventional unconfined groundwater flow governing equation with integer powers, cannot produce the heavy tailed recession behaviour necessary for the long range dependence in river flows, the basic characteristic of the Hurst phenomenon, reported in annual river flow series in the above-mentioned studies. The conventional integer-power governing equations of the unconfined groundwater flow, having finite memory, are fundamentally in the Brownian domain, and cannot model the heavy-tailed baseflow recession behaviour that would be necessary to model the Hurst phenomenon in annual river flows. What is needed is a new structure for the governing equation of unconfined groundwater flow that can reproduce heavy tailed behaviour with time in the hydraulic head and aquifer discharge recession, that would then lead to heavy-tailed recession behaviour in the baseflow of the river basin. Furthermore, various researchers also reported long-range dependence in groundwater level fluctuations (e.g., Li and Zhang, 2007; Yu et al., 2016; Tu et al., 2017; and the references therein). One possible way to reproduce heavy-tailed recession behavior in the hydraulic head and discharge of an unconfined aquifer is by means of a new governing equation of unconfined groundwater flow with fractional powers. Such behavior in an anisotropic confined groundwater aquifer with time and space fractional operators in its governing equation was recently demonstrated (Kavvas et al. 2017a, Tu et al. 2017). Accordingly, the reported study will follow a similar approach to develop a new governing equation for unconfined groundwater aquifers."

Additional References

Beran, J., Statistics for Long-Memory Processes, Vol. 61 of Monographs on Statistics and Applied Probability, Chapman & Hall, New York, USA.

Eltahir, E.A.B., El Nino and the natural variability in the flow of the Nile River, Water Resour. Res. 32(1), 131-137, 1996.

Hurst, H. E., Long term storage capacities of reservoirs, Trans. Am. Soc. Civil Engrs. 116, 776-808, 1951.

Klemes, V., The Hurst phenomenon: a puzzle?, Wat. Resour. Res., 10(4), 675-688, 1974.

Koutsoyiannis, D., The Hurst phenomenon and fractional Gaussian noise made easy, Hydrol. Sci. J., 47(4), 573-595, 2002.

Koutsoyiannis, D., Climate change, the Hurst phenomenon, and hydrological statistics, Hydrol Sci. J., 48(1), 3-24, 2003.

Koutsoyiannis, D., Hydrologic Persistence and The Hurst Phenomenon, Chp. 5 in The Encyclopedia of Water, DOI: 10.1002/047147844X.sw434, 2005.

Mandelbrot, B.B., *The Fractal Geometry of Nature*, Freeman, New York, 1977.

Mandelbrot, B.B. and Wallis, J.R., Computer experiments with fractional Gaussian noises, Part 2: Rescaled ranges and spectra, Wat. Resour. Res., 5(1), 242-259, 1969.

Montanari, A., Rosso, R. and Taqqu, M.S., Fractionally differenced ARIMA models applied to hydrologic time series, Wat. Resour. Res. 33(5), 1035-1044, 1997.

Radziejewski, M. and Kundzewicz, Z. W., Fractal analysis of flow of the river Warta, J. Hydrol., 200, 280-294, 1997.

Yevjevich, V., Fluctuations of wet and dry years: Part 1: Research data assembly and mathematical models, Hydrology Papers, Colorado State University, Ft. Collins, Colorado, 1963.

Yevjevich, V., *Stochastic Processes in Hydrology*, Wat. Resour. Pub., Ft. Collins, Colorado, 1971.

Referee#2: The introduction section has adequate literature reviewed to come to the present research. Such is good but there are numerous jargon to be defined or clearly mentioned. For example, Riemann-Liouville fractional derivative, local Caputo derivatives. The text explained the intensive use of such derivatives. But for the general audience, the questions could arise how such derivatives were used. What could be the assumptions?

Authors' response: Thank you for the valuable suggestion. The Introduction section was revised to address the issues raised by the reviewer by providing detailed references to the Riemann-Liouville and Caputo fractional derivatives, and by providing the physical reasons for preferring the Caputo fractional derivative over the Riemann-Liouville derivative in this study.

Referee #2: It seems that the authors tried to stick to a book chapter. It is not clear why this problem is strictly considered. Since this is the research paper, one should try with a real problem, not the virtual ones. The conclusion made by the authors is too technical. The reviewer does not see any possible application as well as future research behind this.

Authors' response: The particular numerical example was chosen because the analytical solution of the corresponding unconfined ground water flow is available in the referenced book chapter. This problem was chosen to demonstrate that our numerical solution can reproduce the standard unconfined groundwater flow problem and how the solution varies by changing the fractional derivative powers of the proposed fractional governing equations.

In order to further satisfy the reviewer's concerns about the applicability of the research, we added a second numerical example. The second problem deals with a transient unconfined groundwater flow from a hillslope toward a stream (Figure 4 in the revised manuscript). The upstream boundary vertical plane separates the region of flow from the adjacent hillslope that feeds the adjacent tributary system, therefore $\frac{\partial h}{\partial x} = 0$ (Freeze, 1978).

As shown in Figure 5c in the revised manuscript, the newly-developed governing equations can produce heavy-tailed recession behavior in unconfined aquifer discharges.

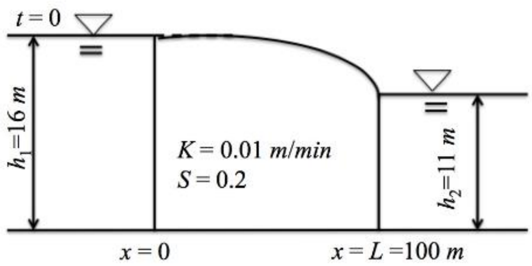

Figure 4. The sketch of numerical application 2: The downstream groundwater head is fixed at 11 m and the initial upstream groundwater head is 16 m.

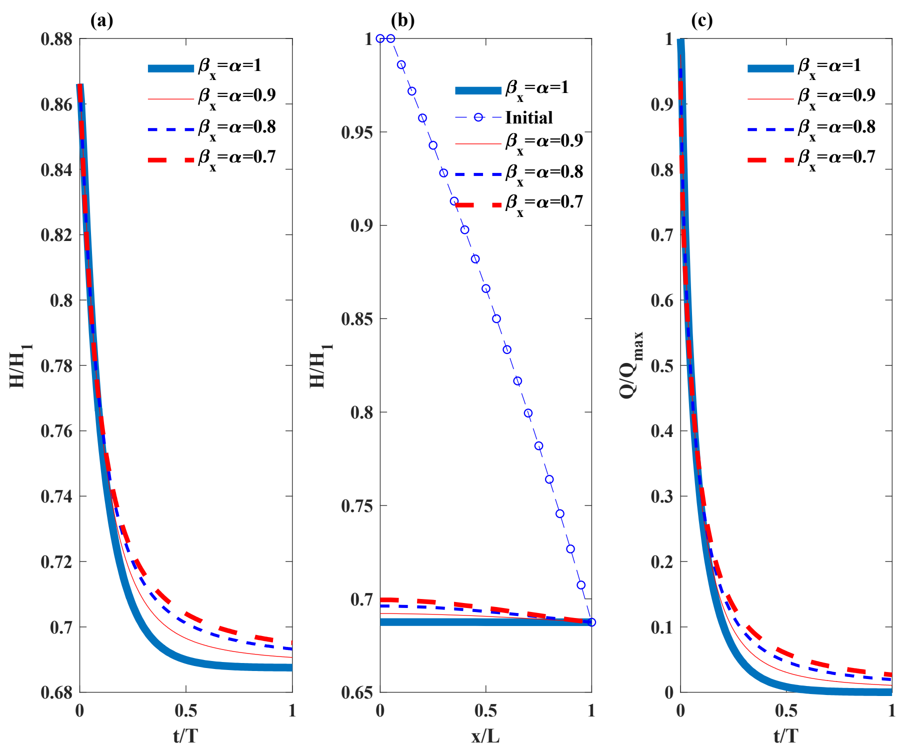

Figure 5. Results for numerical application 2: (a) The normalized groundwater head H/H_1 at $x=L/2$ through time under different fractional derivative powers; (b) The normalized initial groundwater head in the unconfined aquifer, and the normalized groundwater head H/H_1 at time t=60,000 min through length of the aquifer under different fractional derivative powers; (c) The normalized groundwater discharge per unit width at $x=L/2$ through time under different fractional derivative powers; t is time and the simulation time T is 60,000 min.

Reference:

Freeze, R. A.,Mathematical models of hillslope hydrology, Chap. 6 in *Hillslope Hydrology*, ed. by Kirkby, M.J., John Wiley & Sons, Ltd, New York, 1978.

Referee #2: In equation (1) the definition of shi is strictly missing. In line 168, a comma is extra.
Authors' response: Revised as suggested.

Referee #2: In lines 286-287, the phrase "the network streamflow" should be better if it is like "the streamflow network."
Authors' response: Revised as suggested.