1. Regarding your answer to my Comment #1, I know that I used notations different from yours. All I am trying to say is that you have to mention that the specific form insolation is for the annual average.

2. Regarding your answer to my Comment #2, we can show by using $T(\phi)=T_0(\cos\phi)^{1/4}$ (your equation (7)) that

$$\bar{T}=\frac{1}{2}\int_{-\pi/2}^{\pi/2}T(\phi)\cos\theta d\phi=\frac{T_0}{2}\int_{-\pi/2}^{\pi/2}(\cos\theta)^{5/4}d\phi=0.93088T_0, \qquad (1)$$

and

$$\overline{T^4}=\frac{T_0^4}{2}\int_{-\pi/2}^{\pi/2}\cos^2\phi d\phi=0.7854T_0^4. \qquad (2)$$

Therefore,

$$\bar{T}=0.9309T_0 \quad \text{and} \quad \sqrt[4]{\overline{T^4}}=0.9414T_0. \qquad (3)$$

Thus, the difference between the two is only ~1%. Further, you also used a similar approximation in your derivation (see equation above (10)).

The reason why you have such a strange result is that you assumed temperature to be a step function in Θ, i.e.,

$$T(\varphi,\Theta)=\sqrt[4]{\frac{(1-\alpha)S\cos\varphi\cos\Theta}{\varepsilon\sigma}}\times 1_{[-\pi/2<\Theta<\pi/2]}(\Theta). \qquad (4)$$

Obviously, this is not realistic at all. You have to assume that energy absorbed during the day is not emitted immediately. Perhaps for that reason, you introduced heat capacity, which makes it possible for energy to be stored during the day and released during the night.

Now, what the author is trying to convey is a little clearer. Nonetheless, presentation of the material is still very confusing. Further, the role of heat capacity should be compared and contrasted with that of heat redistribution (not only in the meridional direction but also in the zonal direction), since the latter is already proven to produce reasonable temperature distribution on the surface of the earth.