Review

Title: Temperature from energy balance models: the effective heat capacity matters
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General Comments

The author attempted to investigate the energy balance in terms of temperature on the surface of the earth. He concluded that effective heat capacity is essential in explaining the observed temperature. Unfortunately, an essential process of heat distribution is not accounted for in the discussion, resulting in a faulty conclusion. It is unfortunate that I cannot recommend the publication of this paper in its present form.

Specific Comments

1. Eq. (4): $\varepsilon\sigma T^4(\varphi,\Theta)=(1-\alpha)S\cos\varphi\cos\Theta\times 1_{[-\pi/2<\Theta<\pi/2]}(\Theta)$ is not a good description of the energy balance on the surface of the earth. A description of the spatial distribution of energy requires an introduction of energy redistribution by ocean currents, eddies, etc. Further, sun's declination angle should be taken into account in order to describe reasonable spatial distribution of energy at any specific time of the year. What is described here is, at best, an energy balance in an annual-mean sense when there is no physical mechanism for redistribution of strong energy surplus in the equatorial region and strong energy deficit in the polar region.

2. P3 Eq. (8): This is a strange derivation. Let us consider outgoing longwave radiation and incoming solar radiation in the form

$$\varepsilon\sigma T^4(\phi,\theta)=\left(1-\alpha\right)S\cos\phi\cos\theta\times I_{[-\pi/2<\phi<\pi/2]}(\phi), \qquad (1)$$

where ϕ is longitude and θ is latitude. Equation (1) defines energy per unit time per unit area as the dimension of $\sigma=5.670373\times10^{-8}$ W m^{-2} K^{-4} indicates. Thus, total incoming radiation can be written as

$$\int_{-\pi/2}^{\pi/2}\int_{-\pi}^{\pi}\left(1-\alpha\right)S\cos\phi\cos\theta\times I_{[-\pi/2<\phi<\pi/2]}(\phi)R\cos\theta\, d\phi R d\theta$$

$$=\left(1-\alpha\right)SR^2\int_{-\pi/2}^{\pi/2}\int_{-\pi}^{\pi}\cos\phi\cos^2\theta\, d\phi d\theta$$

$$=\left(1-\alpha\right)SR^2\int_{-\pi/2}^{\pi/2}2\cos^2\theta\, d\theta=\left(1-\alpha\right)SR^2\int_{-\pi/2}^{\pi/2}\left(1+\cos 2\theta\right)d\theta$$

$$=\left(1-\alpha\right)SR^2\int_{-\pi/2}^{\pi/2}2\cos^2\theta\, d\theta=\left(1-\alpha\right)S\pi R^2. \qquad (2)$$

Similarly, total outgoing radiation can be written as

$$\int_{-\pi/2}^{\pi/2}\int_{-\pi}^{\pi}\varepsilon\sigma T^4(\phi,\theta)R\cos\theta\, d\phi R d\theta$$

$$=\varepsilon\sigma\bar{T}^4R^2\int_{-\pi/2}^{\pi/2}\int_{-\pi}^{\pi}\cos\theta\, d\phi d\theta=\varepsilon\sigma\bar{T}^4 4\pi R^2\,, \tag{3}$$

where

$$\bar{T}^4=\int_{-\pi/2}^{\pi/2}\int_{-\pi}^{\pi}T^4(\phi,\theta)\cos\theta\, d\phi d\theta \bigg/ \int_{-\pi/2}^{\pi/2}\int_{-\pi}^{\pi}\cos\theta\, d\phi d\theta\,. \tag{4}$$

Thus, we arrive at

$$\bar{T}=\sqrt[4]{\frac{(1-\alpha)S}{4\varepsilon\sigma}}\,. \tag{5}$$

As stated in my Comment #3, Eq. (1) is not quite correct since it lacks the heat redistribution mechanism. On the other hand, global averaging of heat redistribution should be zero, since there is no source or sink of heat. Thus, the addition of heat redistribution to Eq. (1) does not change the result addressed above. See also my Comment #3.

3. P3 L14: Specific heat is not needed to reproduce the reasonable spatial distribution of temperature. For example, 1D energy balance model with meridional heat flux can be written as (see North and Kim, 2017; p123-134)

$$-\frac{d}{d\mu}\left(D(1-\mu^2)\frac{dT(\mu)}{d\mu}\right)+A+BT(\mu,t)=Qa(\mu)s(\mu)\,, \tag{6}$$

where divergence of heat flux is approximated in the form of a diffusive heat transport as $\nabla\cdot\vec{q}_{\text{heat}}=-\nabla\cdot\left(D\nabla T\right)$, the outgoing longwave radiation is linearized as $\varepsilon\sigma T^4\approx A+B\left(T-273.15\right)$, $a(\mu)=1-\alpha(\mu)$ is colatitude, $s(\mu)$ is insolation distribution function (in more general form than the author used in his Eq. (4)), and $\mu=\sin\theta=\cos\vartheta$ (sine of latitude = cosine of colatitude). Solving (1) with a solution in the form

$$T(\mu)\approx T_0+T_2P(\mu) \tag{7}$$

with a realistic insolation distribution function and a realistic albedo (see North and Kim, 2017 for details), we obtain a solution as in Figure 1. The model solution is fairly similar to the observational data. Further, the diffusive heat transport in a zonal mean sense looks very reasonable compared to that derived from satellite observations (see Figure 2).

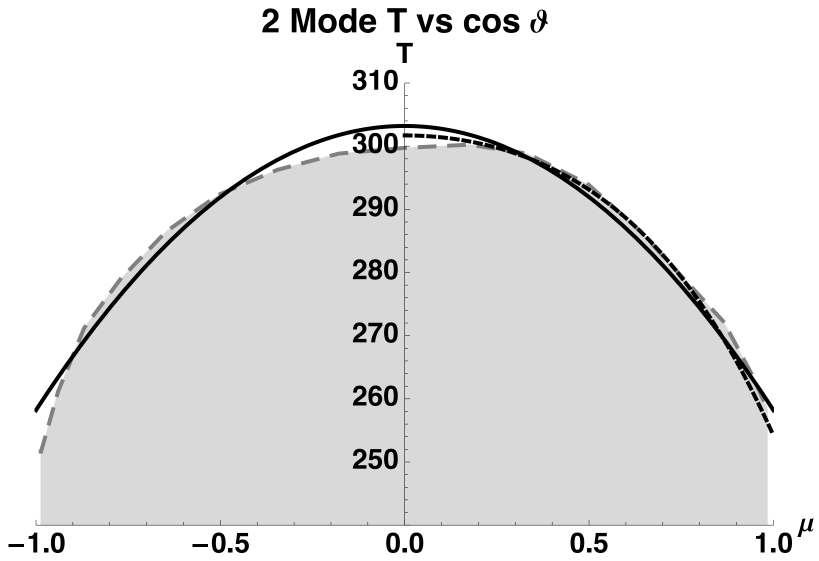

Figure 1. Illustration of the level of agreement at large-scale of the two-mode EBM with the observations. The solid line denotes the pole-to-pole solution of the two-model model-computed temperature (K) versus μ, where $\mu \equiv \cos\vartheta$ = sine(latitude). The dashed curve indicates zonally averaged Northern Hemisphere surface air temperature taken from data in Hartmann (1994). The black-dashed curve shows the temperature curve with the T_4 mode added. (copied from North and Kim, 2017)

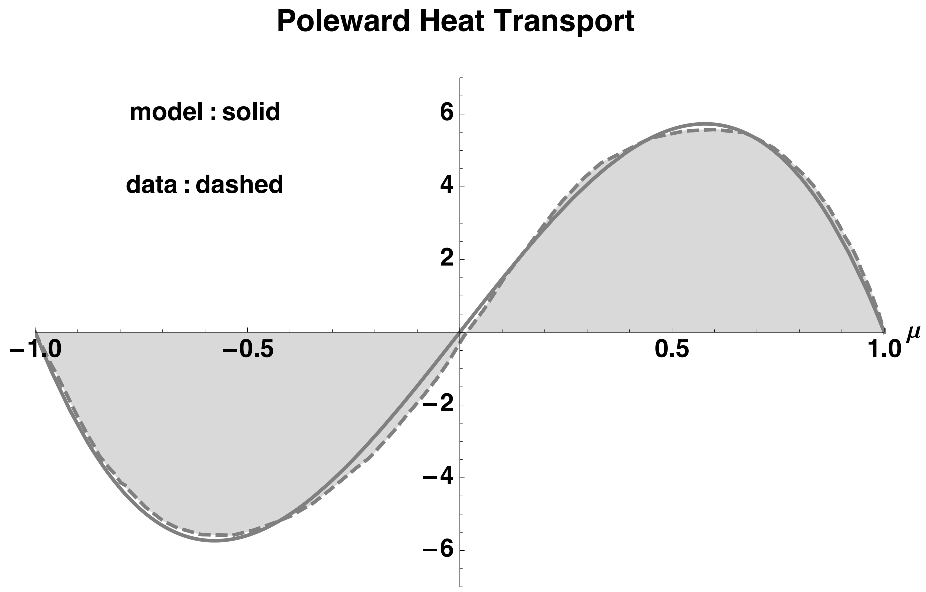

Figure 2. Illustration of the level of agreement in large-scale total poleward transport of heat from pole to pole for the two-mode EBM despite the huge differences in the geography of the two hemispheres. Poleward transport of heat derived from radiation budget data of Trenberth and Caron (2001) based on 4 years of ERBE data (1984-1988). The dashed curve represents the transport derived from the satellite observations and the solid line is based on the two-mode approximation to the surface temperature field with diffusive transport. (copied from North and Kim, 2017)

4. P4 L4: "The atmospheric circulation provides an efficient way to propagate heat along latitudes which is ignored and is a second order effect (not shown)." This statement is erroneous. As demonstrated in Comment #3 above, a reasonable temperature distribution on the surface of the earth is reproduced by using diffusive heat transport. Heat capacity is not even used in this calculation of equilibrium temperature.

5. Eq. (12): The author introduced diurnal cycle of temperature and determined the global average of the averaged diurnal cycle of temperature. This discussion is erroneous. We can write diurnal temperature change as

$$T(\phi,\theta,t)=T_0(\phi,\theta)+T_1(\phi,\theta)\cos\left(\frac{2\pi t}{24}+\varphi_1\right)+T_2(\phi,\theta)\cos\left(\frac{4\pi t}{24}+\varphi_2\right)+\cdots, \quad (8)$$

where $T_1(\phi,\theta)$ and $T_2(\phi,\theta)$ are respectively the amplitude of the diurnal cycle and of the semi-diurnal cycle with phase φ_1 and φ_2. If we average (8) over the period of 1 day, we have

$$\frac{1}{T_{\text{day}}}\int_0^{T_{\text{day}}} T(\phi,\theta,t)dt=T_0(\phi,\theta). \quad (9)$$

Further, incoming solar radiation has the same form as in (1). Thus, we arrive at the same conclusion as in Comment #2.

6. P4 L11: A time-dependent 1D EBM can be written as

$$C(\mu,t)\frac{\partial T(\mu,t)}{\partial t}-\frac{\partial}{\partial\mu}\left(D(1-\mu^2)\frac{\partial T(\mu,t)}{\partial\mu}\right)+A+BT=Qa(\mu,t)s(\mu,t), \quad (10)$$

where $C(\mu,t)$ is heat capacity (having different values over land, ice, and ocean). By expanding dependent variables as

$$T(\mu,t)=T_0(\mu)+T_1(\mu)e^{i\omega_1 t}+T_2(\mu)e^{i\omega_2 t}+\cdots, \quad (11a)$$

$$s(\mu,t)=s_0(\mu)+s_1(\mu)e^{i\omega_1 t}+s_2(\mu)e^{i\omega_2 t}+\cdots, \quad (11b)$$

where ω_1 and ω_2 are the frequencies of sinusoidal components. Inserting (11) into (10) (with the assumption that the model parameters are independent of time) and taking the zeroth component (mean) of the resulting equation, we have

$$-\frac{\partial}{\partial\mu}\left(D(1-\mu^2)\frac{\partial T_0(\mu)}{\partial\mu}\right)+A+BT_0=Qa(\mu)s_0(\mu). \quad (12)$$

Equation (12) is equivalent to (6) as far as the annual mean temperature is concerned. Obviously, an adequate explanation is needed in terms of how Figure 3 is produced.

7. Figure 3: The main effect of heat capacity in the original EBM is in the context of the amplitude of the annual and semi-annual cycles (see North and Kim, p152). The annual and semi-annual cycles are seriously affected by the choice of heat capacity, whereas the annual mean component is not. The author should demonstrate that not only annual-mean temperature distribution but also the annual and semi-annual cycles of temperature is reproduced reasonably by their choice of heat capacity (see, for example, Fig. 6.8 of North and Kim).

8. P8 L20: What does the first law of thermodynamics have anything to do with incoming radiation = outgoing radiation? Is the author referring to the zeroth law of thermodynamics?

9. P8 L22-24: As already demonstrated in Comment #2, global average temperature is close to the observed value without the effect of heat capacity. Further, by using diffusive heat transport, zonally average temperature is reproduced close to actual observation in Comment #3.

10. It is difficult to review the entire manuscript until my earlier comments are fully addressed. In particular, the author needs to explain clearly how the solutions in each figure are computed (with appropriate equation if possible) and demonstrate clearly that his full solution (with diurnal and annual cycles) matches reasonably with the observations for his choice of heat capacity.

Technical Comments

1. There is, in general, lack of explanation for variables used in the equations.

2. P4 L6?: "shown in Fig. 2 as the ~~read~~ red line with the mean …"