Earth Syst. Dynam. Discuss.,
https://doi.org/10.5194/esd-2019-35-AC5, 2020

Earth System Dynamics Discussions

Interactive comment on "Temperatures from Energy Balance Models: the effective heat capacity matters" *by* Gerrit Lohmann

Gerrit Lohmann

gerrit.lohmann@awi.de

Received and published: 5 January 2020

Thanks for the constructive critics in the reviews. I realized two major common points that are addressed here.

1) The first is on the theoretical arguments, in particular, equation (4) in the paper. In the enclosed figure, I show the lat-lon dependence of the incoming short wave radiation. We assume an idealized geometry of the Earth. The global mean temperature is not affected by the obliquity and precession (Berger and Loutre 1991; 1997; Laepple and Lohmann 2009). The beauty of equation (4) is that we ignore the Earth orbital

parameters first (no obliquity and no precession) which makes an analytical calculation possible. The incoming radiation goes with the cosine of latitude and longitude, and there is only sunshine during the day.

To illustrate equation (4), I refer to Fig. R1. Panel a shows the latitudinal dependence. As we assume no tilt (this assumption is later relaxed), the latitudinal dependence is a function of latitude only: $\cos\varphi$. On the right-hand side, the function is shown. Panel b shows the latitudinal dependence is a function of longitude: $\cos\Theta$ for the sun-shining side of the Earth, and for the dark side of the Earth it is zero. For simplicity, we can define the angle Θ anti-clockwise on the for the sun-shining side between $-\pi/2$ and $\pi/2$. We define the maximal insolation always at $\Theta = 0$ which is moving in time. In the panel, the Earth's rotation is schematically sketched as the red arrow, and we see the time-dependence in the right-hand side. It is noted that the geographical longitude can be calculated by $mod(\Theta - 2\pi \cdot t/24, 2\pi)$ where t is measured in hours and mod is the modulo operation.

The motivation is that we may think of a climate system having a higher net heat capacity producing flat temperature gradients. The main point is, that the effective heat capacity (and its temporal variation over the daily/seasonal cycle) needs to be taken into account when estimating surface temperature from the energy budget.

Fig. 3 in the paper indicates that the temperature gradient is getting flatter for large heat capacities when analyzing the diurnal cycle. Fig. R2 shows explicitly the temperature dependence on heat capacity (and rotation rate). Later in the paper, an explicit seasonal cycle is included as

$$C_p \, \partial_t T \; = \; \nabla \cdot HT \, + \, (1-\alpha) S(\varphi, t) - \, \epsilon\sigma T^4 \quad .$$

with $S(\varphi, t)$ being calculated daily (Berger and Loutre, 1991; 1997). Figs. 5 and 6 in the paper indicate that the temperature gradient is getting flatter for large heat capacities associated with a reduced seasonal cycle. A similar feature is found in the circulation model (see my comments below).

2) The second comment is related to the statement that the linearization of the long-wave radiation in several energy balance models "implicitly assumes the above heat capacity and fast rotation arguments". Indeed the linearization of $\epsilon\sigma T^4$ is performed around $0°C$ (North et al., 1975a, b; Chen et al., 1995; Lohmann and Gerdes, 1998; North and Kim, 2017) and is formulated as $A + B \cdot T'$ with T' being measured in $°C$.

As the temperatures based on the local energy balance without a heat capacity would vary between $T_{min} = 0$ K and $T_{max} = \sqrt[4]{\frac{(1-\alpha)S}{\epsilon\sigma}} = \sqrt{2} \cdot \sqrt[4]{\frac{(1-\alpha)S}{4\epsilon\sigma}} = \sqrt{2} \cdot 288K = 407$ K, a linearization would be not permitted. If the assumptions of fast rotation rate and heat capacity are ignored, the global mean temperature would be much lower as calculated in equation (8) of the paper and shown in the figures (Figs. 2 and 3, Fig. R2).

There is also a second point of why linearization has to be used carefully. As seen in Figs. 5 and 6 in the paper, the temperature gradient is getting flatter for large heat capacities. A reduced seasonal cycle is responsible for significant warming. The larger the seasonal contrast, the colder is the climate. Let us define $\bar{\cdot}$ as the averaging over a time period (in our case the seasonal or diurnal cycle), then $\overline{T^4} > \overline{T}^4$ which is consistent with Hölder's inequality (Rodgers, 1888; Hölder 1889; Hardy et al., 1934, Kuptsov, 2001). This feature is missing in the linearized version with $A + B \cdot T'$.

The climate model experiment with enhanced mixing is admittedly highly idealized. Due to higher mixing, more heat can be taken by the climate system, the seasonal cycle is reduced (Fig. 7 in the paper), and especially the high latitudes warm-up (Fig. R3). (In this circulation model, the heat takeup goes to the deeper levels in the ocean but it is not finalized even after 1000 years (Fig. R4).) The pronounced winter warming provides an important non-linearity (cf. Hölder's inequality): Let us denote T_{summer} and T_{winter} for the local summer and winter temperatures, then $0.5 \cdot (T^4_{summer} + T^4_{winter})$ is much greater and therefore associated to a colder climate than a climate under a reduced seasonal cycle with the extreme case $(0.5 \cdot T_{summer} + 0.5 \cdot T_{winter})^4$.

References

Berger, A. L.: Long-term variations of daily insolation and Quaternary climatic changes, J. Atmos. Sci., 35, 2362-2367, 1978.

Berger, A., and Loutre, M.F.: Insolation values for the climate of the last 10 million years, Quat. Sci. Rev., 10(4), 297 - 317, 1991. doi:10.1016/0277-3791(91)90033-Q.

Berger, A., and Loutre, M.F.: Intertropical latitudes and precessional and half-precessional cycles, Science, 278(5342), 1476 - 1478, 1997. doi:10.1126/science.278.5342.1476.

Hardy, G. H.; Littlewood, J. E.; Pólya, G., 1934: Inequalities. Cambridge University Press, pp. XII+314, ISBN 0-521-35880-9, JFM 60.0169.01

Hölder, O., 1889: Ueber einen Mittelwertsatz. Nachrichten von der Königl. Gesellschaft der Wissenschaften und der Georg-Augusts-Universität zu Göttingen, Band (in German), (2): 38-47, JFM 21.0260.07.

Kuptsov, L. P., 2001: Hölder inequality. In Hazewinkel, Michiel (ed.), Encyclopedia of Mathematics, Springer Science+Business Media B.V. / Kluwer Academic Publishers, ISBN 978-1-55608-010-4.

Laepple, T., and Lohmann, G.: The seasonal cycle as template for climate variability on astronomical time scales. Paleoceanography, 24, PA4201, doi:10.1029/2008PA001674, 2009.

Lohmann, G., and Gerdes, R.: Sea ice effects on the Sensitivity of the Thermohaline Circulation in simplified atmosphere-ocean-sea ice models. J. Climate 11, 2789-2803, 1998.

North, G. R.: Analytical solution of a simple climate model with diffusive heat transport. J. Atm. Sci. 32, 1300-1307, 1975a

North, G. R.: Theory of energy-balance climate models. J. Atm. Sci. 32, 2033-2043, 1997b.

North, G. R., and Kim, K.-Y., 2017: Energy Balance Climate Models. Wiley, ISBN:9783527411320, DOI:10.1002/9783527698844

Rogers, L. J.: An extension of a certain theorem in inequalities. Messenger of Mathematics, New Series, XVII (10): 145–150, 1888. JFM 20.0254.02

Interactive comment on Earth Syst. Dynam. Discuss., https://doi.org/10.5194/esd-2019-35, 2019.

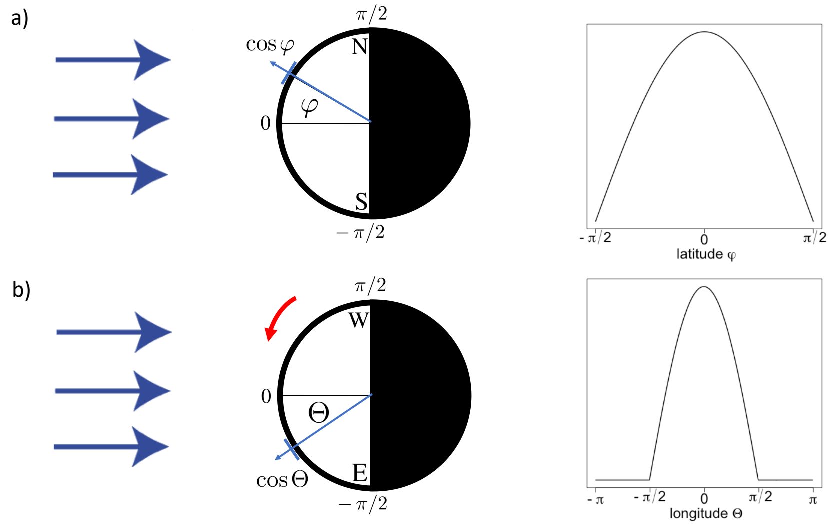

Fig. 1. Fig. R1. Latitudinal (a) and longitudinal (b) dependence of the incoming short wave radiation. See text for the details.

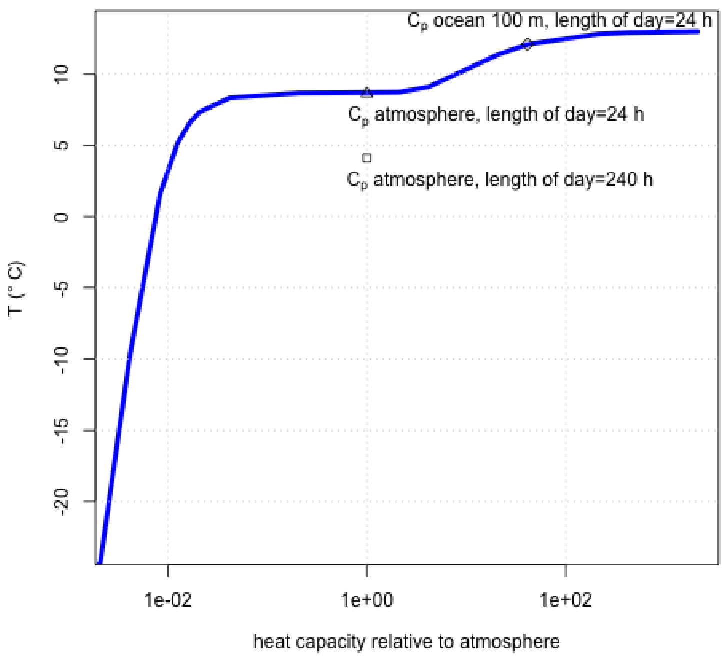

Fig. 2. Fig. R2. Temperature dependence on heat capacity (and rotation rate) when analyzing the diurnal cycle.

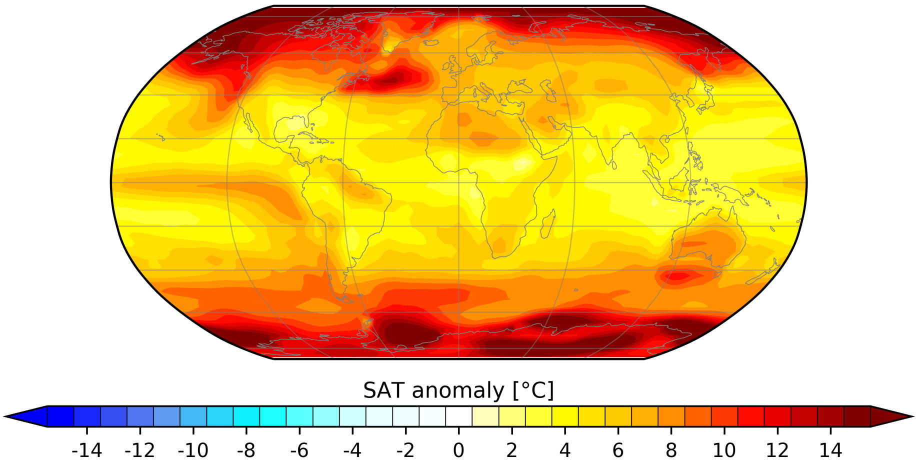

Fig. 3. Fig. R3. Temperature anomaly (annual mean, mean over the last 100 years of integration) using the Earth system model (see text for more details).

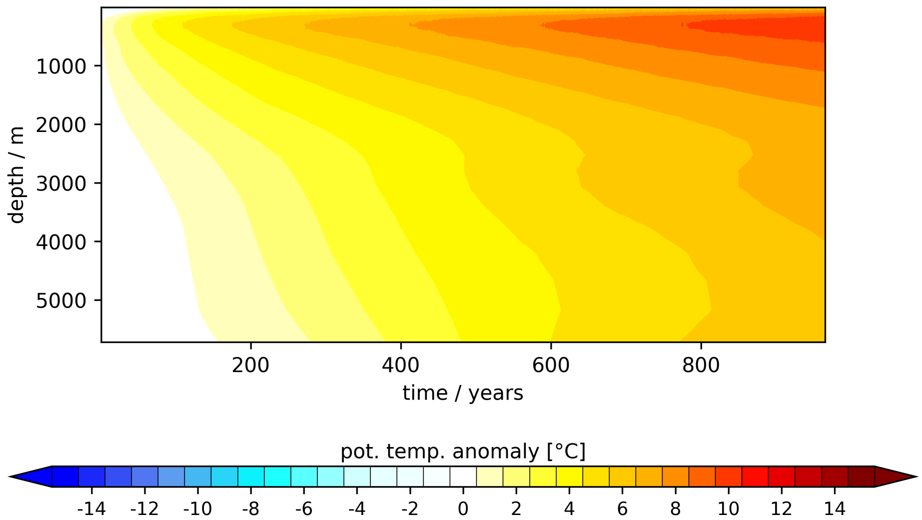

Fig. 4. Fig. R4. Hovmoeller-diagram of global mean potential ocean temperatures (enhanced vertical mixing in the ocean) minus control experiment (standard PI) over the time period from 1 to year 1000.