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Interactive comment on "Temperatures from Energy Balance Models: the effective heat capacity matters" *by* Gerrit Lohmann

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Thanks for the detailed critics on the *Interactive comment on Earth Syst. Dynam. Discuss., https://doi.org/10.5194/esd-2019-35, 2019.* by referee #2. The comments will be helpful in improving the readability of the manuscript. In the following, I will repeat and answer to these comments. Furthermore, possible action is proposed.

Comment 1

Eq. (4)

$$\epsilon\sigma T^4(\Theta,\varphi) \quad = \quad (1-\alpha)S \quad \cos\varphi \quad \cos\Theta \quad \times \quad 1_{[-\pi/2<\Theta<\pi/2]}(\Theta)$$

is not a good description of the energy balance on the surface of the earth. A description of the spatial distribution of energy requires an introduction of energy redistribution by ocean currents, eddies, etc. Further, sun's declination angle should be taken into account in order to describe reasonable spatial distribution of energy at any specific time of the year. What is described here is, at best, an energy balance in an annual-mean sense when there is no physical mechanism for redistribution of strong energy surplus in the equatorial region and strong energy deficit in the polar region.

Answer/Action

The left-hand side of (4) as well as the right-hand side are latitude φ- and longitute Θ-dependent. The incoming radiation goes with the cosine of latitude and longitude, and there is only sunshine during the day. This is noted as the $1_{[-\pi/2<\Theta<\pi/2]}(\Theta)$ function which is zero outside the interval $[-\pi/2 < \Theta < \pi/2]$. The global mean temperature is not affected by the obliquity and precession (Berger and Loutre 1991; 1997; Laepple and Lohmann 2009). Therefore, we ignore the Earth orbital parameters for a while which makes an analytical calculation possible. Later in the numerical treatment, the full seasonal cycle is taken into account (equation (16) in the manuscript). Indeed, eq. (4) can be better explained and motivated.

Comment 2

Eq. (8): This is a strange derivation. Let us consider outgoing longwave radiation and incoming solar radiation in the form

$$\epsilon\sigma T^4(\phi,\Theta) \quad = \quad (1-\alpha)S \quad \cos\phi \quad \cos\Theta \quad \times \quad 1_{[-\pi/2<\Theta<\pi/2]}(\phi)$$

where ϕ is longitute and Θ is latitude. Equation (1) defines energy per unit time per unit area as the dimension of $\sigma = 5.670373 \times 10^{-8} Wm^{-2}K^{-4}$ indicates. Thus, total incoming radiation can be written as . . .

$$(1-\alpha)S\ \pi\ R^2$$

Similarly, total outgoing radiation can be written as . . .

$$\epsilon\sigma\overline{T}^4\ 4\pi\ R^2$$

Thus, we arrive at

$$\overline{T} = \sqrt[4]{\frac{(1-\alpha)S}{4\epsilon\sigma}}$$

Answer/Action

Your equation above (in your review eq. (1)), is basically the same as the one I used. The difference is that you defined ϕ as longitude and Θ as latitude, whereas I do it in the other way round. (By the way: This is the same approach you criticized in your comment #1.)

The calculation you presented is also more or less the same as mine, with one **fundamental** difference. In my derivation by integration over the Earth surface

$$\epsilon\sigma 4\pi\overline{T^4} = (1-\alpha)S\ \pi\ .$$

In this formula, the average $\overline{T^4}$ is calculated, not $\overline{T}^4$. Therefore,

$$\overline{T} = \sqrt[4]{\frac{(1-\alpha)S}{4\epsilon\sigma}}$$

is not correct. This argument to exchange the $\overline{T^4}$ by $\overline{T}^4$ was one of the motivation to write down the global energy balance in a correct form. In order to $\overline{T}$, one has a

more lengthy calculation: If we now calculate the zonal mean of the temperature by integration at the latitudinal cycles we have

$$\begin{aligned}
T(\varphi) &= \frac{1}{2\pi}\int_{-\pi/2}^{-\pi/2}\sqrt[4]{\frac{(1-\alpha)S\cos\varphi\cos\Theta}{\epsilon\sigma}}d\Theta \\
&= \frac{\sqrt{2}}{2\pi}\underbrace{\int_{-\pi/2}^{\pi/2}(\cos\Theta)^{1/4}d\Theta}_{=\sqrt{\pi}\Gamma(5/8)/\Gamma(9/8)}\quad\sqrt[4]{\frac{(1-\alpha)S}{4\epsilon\sigma}}\quad(\cos\varphi)^{1/4} \\
&= \frac{1}{\sqrt{2\pi}}\frac{\Gamma(5/8)}{\Gamma(9/8)}\cdot\sqrt[4]{\frac{(1-\alpha)S}{4\epsilon\sigma}}\quad(\cos\varphi)^{1/4} \\
&= 0.608\cdot\sqrt[4]{\frac{(1-\alpha)S}{4\epsilon\sigma}}\quad(\cos\varphi)^{1/4}
\end{aligned}$$

as a function of latitude. Γ is Euler's Gamma function with $\Gamma(x+1) = x\Gamma(x)$. When we integrate this over the latitudes, we obtain

$$\begin{aligned}
\overline{T} &= \frac{1}{2}\int_{-\pi/2}^{\pi/2}T(\varphi)\ \cos\varphi\, d\varphi \\
&= \frac{1}{2}\frac{\Gamma(5/8)}{\sqrt{2\pi}\,\Gamma(9/8)}\cdot\sqrt[4]{\frac{(1-\alpha)S}{4\epsilon\sigma}}\quad\underbrace{\int_{-\pi/2}^{\pi/2}(\cos\varphi)^{5/4}d\varphi}_{=\sqrt{\pi}\Gamma(9/8)/\Gamma(13/8)}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{1}{2}\frac{1}{\sqrt{2}}\frac{\Gamma(5/8)}{\Gamma(13/8)}\sqrt[4]{\frac{(1-\alpha)S}{4\epsilon\sigma}} = \frac{\sqrt{2}}{4}\frac{8}{5}\sqrt[4]{\frac{(1-\alpha)S}{4\epsilon\sigma}} \\
&= 0.4\sqrt{2}\sqrt[4]{\frac{(1-\alpha)S}{4\epsilon\sigma}} = 0.566\sqrt[4]{\frac{(1-\alpha)S}{4\epsilon\sigma}}
\end{aligned}
$$

As a side remark: The fact that $\sqrt[4]{\overline{T^4}}$ is higher than $\overline{T}$ is consistent with Hölder's inequality (Rodgers, 1888; Hölder 1889; Hardy et al., 1934, Kuptsov, 2001).

Comment 3

Specific heat is not needed to reproduce the reasonable spatial distribution of temperature. For example, 1D energy balance model with meridional heat flux can be written as (see North and Kim, 2017; p123-134)

$$
-\frac{d}{d\mu}\left(D(1-\mu^2)\frac{dT(\mu)}{d\mu}\right) + A + BT(\mu,t) = Qa(\mu)s(\mu)
$$

...

with a realistic insolation distribution function and a realistic albedo (see North and Kim, 2017 for details), we obtain a solution as in Figure 1. The model solution is fairly similar to the observational data. Further, the diffusive heat transport in a zonal mean sense looks very reasonable compared to that derived from satellite observations (see Figure 2).

Answer/Action

Thanks for your comment and hinting to the important work of North and Kim (2017). As I mention in the manuscript: "The linearization of the long wave radiation in several models (North et al., 1975a, b; Chen et al., 1995) implicitly assumes the above

heat capacity and fast rotation arguments. " Indeed, the linearized version can give a reasonable zonal mean climate.

In the revised version, I can stress out this more clearly. My point is that we need a rapidly rotating object with significant heat capacity. Without these effects, the global mean temperature would be lower. I will explicitly show that these effects are important for the radiation balance, other processes - like horizontal transport processes - are only of secondary importance for the globally averaged temperature. The finding is furthermore important to see that the effective heat capacity (which is time-scale dependent) has a direct influence on the global (and regional) temperatures). In the revised version, I will include a figure to show how the temperature would change in a slowly rotating planet.

Comment 4

"The atmospheric circulation provides an efficient way to propagate heat along latitudes which is ignored and is a second order effect (not shown)." This statement is erroneous. As demonstrated in Comment #3 above, a reasonable temperature distribution on the surface of the earth is reproduced by using diffusive heat transport. Heat capacity is not even used in this calculation of equilibrium temperature.

Answer/Action

Yes, this can be better motivated. Indeed, the energy input is time-dependent. Therefore, the mean and latitudinal temperature depends on the time derivative. See also my comments and action points in answer to Comment #3. As seen in the manuscript, the temperatures do depend on the effective heat capacity.

Comment 5

Eq. (12): The author introduced diurnal cycle of temperature and determined the global average of the averaged diurnal cycle of temperature. This discussion is erroneous. We can write diurnal temperature change as ...

Further, incoming solar radiation has the same form as in (1). Thus, we arrive at the same conclusion as in Comment #2.

Answer/Action

Thanks for making this hint. Indeed, I agree to your basic equation (1), but not with the conclusion. See my answer to your Comment #2.

Comment 6

A time-dependent 1D EBM can be written as ...

Equation (12) is equivalent to (6) as far as the annual mean temperature is concerned. Obviously, an adequate explanation is needed in terms of how Figure 3 is produced.

Answer/Action

I agree for the linearized EBM. See my answer to your Comment #3. I will explicitly state that in the linearized EBM, the heat capacity argument is implicitly included. Furthermore, parameter study of Figure 3 will be better explained. The model that I use is the non-linear model with the T^4-term and a time-dependent forcing.

Comment 7

Figure 3: The main effect of heat capacity in the original EBM is in the context of the amplitude of the annual and semi-annual cycles (see North and Kim, p152). The annual and semi-annual cycles are seriously affected by the choice of heat capacity,

whereas the annual mean component is not. The author should demonstrate that not only annual-mean temperature distribution but also the annual and semi-annual cycles of temperature is reproduced reasonably by their choice of heat capacity (see, for example, Fig. 6.8 of North and Kim).

Answer/Action

Thanks again for your comment and for hinting at the important work of North and Kim (2017). Yes, I agree for the linearized EBM. See my answer to your Comment #3 and #6. In the non-linear model, the temperature is affected by the heat capacity. The effective heat capacity is important as it is reflective of the rate of penetration of heat energy into the ocean in response to the particular pattern of forcing and the background state (Schwartz, 2007). I will also emphasize the relevance of this for climate warming scenarios. In the revised version, I will explicitly show the results of the linearized EBM.

Comment 8

P8 L20: What does the first law of thermodynamics have anything to do with incoming radiation = outgoing radiation? Is the author referring to the zeroth law of thermodynamics?

Answer/Action

Thanks. This is indeed not well formulated. In the revised version, I will explicitly state this point.

Comment 9

As already demonstrated in Comment #2, global average temperature is close to the observed value without the effect of heat capacity. Further, by using diffusive heat transport, zonally average temperature is reproduced close to actual observation in Comment #3.

Answer/Action

Thanks. I disagree and point to my answers to Comments #2 and #3.

Comment 10

It is difficult to review the entire manuscript until my earlier comments are fully addressed. In particular, the author needs to explain clearly how the solutions in each figure are computed (with appropriate equation if possible) and demonstrate clearly that his full solution (with diurnal and annual cycles) matches reasonably with the observations for his choice of heat capacity.

Answer/Action

I agree that it seems a misunderstanding. My answers to Comments #2 and #3 shall clarify the mistake in the calculation of the temperature. The time dependence will be more explicitly stated. Furthermore, the non-linearity in the outgoing radiation makes this model different from your EBMs. The equations for the EBMs used are in the manuscript (9, 15, 16). For the equations, it is important to explicitly spell out the assumptions made. See my answer to your Comments #3 and #6. I will explicitly state that in the linearized EBM, the heat capacity argument is implicitly included. The model that I use is the non-linear model with the T^4-term and a time-dependent forcing: my equations (9, 15, 16).

In (16), I wrote that we include an explicit seasonal cycle into the EBM:

$$C_p \, \partial_t T \; = \; \nabla \cdot HT \; + \; (1-\alpha) S(\varphi, t) - \; \epsilon \sigma T^4 \quad .$$

with $S(\varphi, t)$ being calculated daily (Berger and Loutre, 1991; 1997).

In (9), the daily cycle is resolved through the time-dependence of the $1_{[-\pi/2<\Theta<\pi/2]}(\Theta)$-term in

$$C_p \, \partial_t T \; = \; (1-\alpha) S \cos\varphi \cos\Theta \quad \times 1_{[-\pi/2<\Theta<\pi/2]}(\Theta) \; - \; \epsilon \sigma T^4$$

This time-dependence will be stated more explicitly.

Furthermore, I will explicitly show the results of the linearized EBM in the revised version. Indeed, the linear model has some advantages because it directly indicated the climate sensitivity. However, for a rigorous derivation of the global and regional effects, the non-linear version is an advantage. Finally, it is necessary to see that $\overline{T^4}$ is not $\overline{T}^4$. Simplified and conceptual models can be used to study long-term climate (Hasselmann, 1976; Lemke, 1977; Timmermann and Lohmann, 2000; Lohmann, 2018). As pointed out in the manuscript, the effective heat capacity is important to understand past and potential future climate.

Technical Comment 1

There is, in general, lack of explanation for variables used in the equations.

Answer/Action

Thanks. I will go through all equations in the manuscript to avoid misunderstandings. In the revised manuscript, it will be clearer that I use the non-linear EBM (as in my equations (9, 15, 16), and not the linear (North et al., 1975a, b; Chen et al., 1995; Lohmann and Gerdes, 1998; North and Kim, 2017).

Technical Comment 2

"shown in Fig. 2 as the (read) red line with the mean . . ."

Answer/Action

Thanks, this will be corrected.

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