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Earth System Dynamics Discussions

Interactive comment on "Temperatures from Energy Balance Models: the effective heat capacity matters" *by* Gerrit Lohmann

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Thanks for the constructive critics in the *Interactive comment on Earth Syst. Dynam. Discuss., https://doi.org/10.5194/esd-2019-35, 2019.* by referee #1. In the following, I will repeat and answer to these comments. Furthermore, possible action is proposed.

Comment

This manuscript revisits the relationship between the (global mean) surface temperature of the Earth and its radiation budget as is frequently used in Energy balance models (EBMs). The main point is, that the effective heat capacity (and its temporal variation over the daily/seasonal cycle) needs to be taken into account when estimat-

ing surface temperature from the energy budget. The results of this exercise together with coupled ocean-atmosphere GCM simulations lets the author suggest a potential mechanism for the relatively low equator-to-pole temperature gradient in past warm climates that has been observed in proxy data, but remains difficult to reproduce with GCMs. The paper includes a very useful discussion about general properties of the energy balance of the Earth and this certainly justifies publication in ESD. However, I have two main comments to be improved on before I can recommend publication.

Comment 1a)

The theoretical arguments should be much better explained. This holds in particular for sections 2 and 3. For example, after or before eq. (4), it should be very explicitly explained which variables become lat-lon dependent, and which not. Otherwise eq. (4) and the analysis that follows is very hard to understand (or reproduce). In my view, if you consider the local energy balance, temperature T, emissivity and albedo α, should be spatially dependent and therefore this should have consequences for the following integration. If they are not spatially dependent, then it should be clearly stated why not.

Answer/Action

Indeed, eq. (4) can be better explained. The left hand side of (4) as well as the right hand side are latitude φ- and longitute Θ-dependent.

$$\epsilon\sigma T^4(\Theta,\varphi) \quad = \quad (1-\alpha)S \quad \cos\varphi \quad \cos\Theta \quad \times \quad 1_{[-\pi/2<\Theta<\pi/2]}(\Theta)$$

The incoming radiation goes with the cosinus of latitude and longitude, and there is only sunshine during the day. This is noted as the $1_{[-\pi/2<\Theta<\pi/2]}(\Theta)$ function which is zero outside the interval $[-\pi/2 < \Theta < \pi/2]$. The beauty of this formulation is that we ignore the Earth orbital parameters for a while which makes an analytical calculation possible. A temperature-dependent formulation of the albedo is used later because of simplicity (α as a function of T).

If we now calculate the zonal mean of the temperature by integration at the latitudinal cycles we have

$$
\begin{aligned}
T(\varphi) &= \frac{1}{2\pi} \int\limits_{-\pi/2}^{-\pi/2} \sqrt[4]{\frac{(1-\alpha)S\cos\varphi\cos\Theta}{\epsilon\sigma}} d\Theta \\
&= \frac{\sqrt{2}}{2\pi} \underbrace{\int\limits_{-\pi/2}^{\pi/2} (\cos\Theta)^{1/4} d\Theta}_{=\sqrt{\pi}\Gamma(5/8)/\Gamma(9/8)} \quad \sqrt[4]{\frac{(1-\alpha)S}{4\epsilon\sigma}} \quad (\cos\varphi)^{1/4} \\
&= \frac{1}{\sqrt{2\pi}} \frac{\Gamma(5/8)}{\Gamma(9/8)} \cdot \sqrt[4]{\frac{(1-\alpha)S}{4\epsilon\sigma}} \quad (\cos\varphi)^{1/4} \\
&= 0.608 \cdot \sqrt[4]{\frac{(1-\alpha)S}{4\epsilon\sigma}} \quad (\cos\varphi)^{1/4} \qquad (1)
\end{aligned}
$$

as a function on latitude. Γ is Euler's Gamma function with $\Gamma(x+1) = x\Gamma(x)$. When we integrate this over the latitudes, we obtain

$$
\begin{aligned}
\overline{T} &= \frac{1}{2} \int\limits_{-\pi/2}^{\pi/2} T(\varphi) \cos\varphi \, d\varphi \\
&= \frac{1}{2} \frac{\Gamma(5/8)}{\sqrt{2\pi}\,\Gamma(9/8)} \cdot \sqrt[4]{\frac{(1-\alpha)S}{4\epsilon\sigma}} \quad \underbrace{\int\limits_{-\pi/2}^{\pi/2} (\cos\varphi)^{5/4} d\varphi}_{=\sqrt{\pi}\Gamma(9/8)/\Gamma(13/8)}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{1}{2} \frac{1}{\sqrt{2}} \frac{\Gamma(5/8)}{\Gamma(13/8)} \sqrt[4]{\frac{(1-\alpha)S}{4\epsilon\sigma}} = \frac{\sqrt{2}}{4} \frac{8}{5} \sqrt[4]{\frac{(1-\alpha)S}{4\epsilon\sigma}} \\
&= 0.4\sqrt{2} \sqrt[4]{\frac{(1-\alpha)S}{4\epsilon\sigma}} = 0.566 \sqrt[4]{\frac{(1-\alpha)S}{4\epsilon\sigma}} \qquad (2)
\end{aligned}
$$

Comment 1b)

I find it very puzzling that the heat capacity C_p does not explicitly appear in eq. (11), although I clearly see how you get there. A few words of explanation would be very useful to the (less-expert) reader.

Answer/Action

Indeed, the equilibrium solution is calculated from (10) as there is no time-dependence due to the integration over longitude and day. I will be more explicit in the revised version in explaining the $1_{[-\pi/2<\Theta<\pi/2]}(\Theta)$ function.

Comment 1c)

Then, after eq. (12) the reference heat capacity is chosen as the atmospheric heat capacity. Why is that? Above in the text you have said that the heat capacity is manly given by the ocean, so why do you use the atmospheric heat capacity here?

Answer/Action

Yes, the effective heat capacity is time-scale dependent. For the day and night cycle values in the order of the atmospheric heat capacity are realistic for our Earth with 24 h rotation. In the revised version, I will be more explicit here and could include a figure to show how the temperature would change in a slowly rotating planet.

Comment 1d)

A bit more explanation and motivation should also enter the fact that in one case in Fig. 5 you use a latitudinal dependent heat capacity (in the text just after eq. (12)). How exactly? And what is the motivation for that?

Answer/Action

Yes, this can be better motivated. The high latitudes have a much higher effective heat capacity due to the deeper mixed layer.

Comment 1e)

On page 6, line 18, the temperatures T1 and T2 remain unexplained!

Answer/Action

In the revised version, these temperatures are better explained. It is a back-on-the-envelope calculation to see the main argument.

Comment 2)

The second point relates to the vertical mixing in the ocean. It is interesting to see how the vertical mixing in the ocean obviously can affect the equator-to-pole surface temperature gradient. However, why should the vertical mixing be so different in the Palaeogene/Neogene before 3 Ma? Tidal dissipation can play a role, but also bathymetry and probably also the number and specific geometry of the ocean gate-

ways. But so far, this remains very speculative and unmotivated in the manuscript. For example, how does the factor 25 in the vertical mixing coefficient that is used in the GCM simulations relate to expected changes in vertical mixing due to tides and bathymetry?

Answer/Action

The manuscript is admittedly a little vague at this point. A more explicit statement about a more explicit calculation of the vertical mixing is beyond the scope of the present paper. I will add more literature dealing with bathymetry, tides, and geometry of the ocean in the revised version. stressed out that The effective heat capacity is not an intrinsic property of the climate system but is reflective of the rate of penetration of heat energy into the ocean in response to the particular pattern of forcing and the background state (Schwartz, 2007). I will also explore the relevance for climate warming scenarios.

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