Earth Syst. Dynam. Discuss., https://doi.org/10.5194/esd-2018-77
Manuscript under review for journal Earth Syst. Dynam.
Discussion started: 9 November 2018

Earth System Dynamics Discussions Open Access

ESD Ideas: Propagation of high-frequency forcing to ice age dynamics

Mikhail Y. Verbitsky[1], Michel Crucifix[2], and Dmitry M. Volobuev[3]

[1]formerly at: Yale University, Department of Geology and Geophysics, New Haven CT, USA
[2]UCLouvain, Earth and Life Institute, Louvain-la-Neuve, Belgium
[3]The Central Astronomical Observatory of the Russian Academy of Sciences at Pulkovo, Saint Petersburg, Russia
Correspondence: Mikhail Verbitsky (verbitskys@gmail.com)

Abstract. The observational records display a continuous background of variability connecting centennial to 100-ka periods. Hence, the dynamics at the centennial, millennial, and astronomical time scales should not be treated apart. Here, we show that the non-linear character of ice sheet dynamics, which was derived naturally from the conservation laws, provides the scaling constraints to explain the structure of the observed spectrum of variability.

Introduction. Most theories of Quaternary climates consider that glacial-interglacial cycles emerge from components of the climate system interacting with each other, and responding to the forcing generated by the variations of summer insolation caused by climatic precession, the changes in obliquity, and in eccentricity. A common approach is to represent these interactions and response by ordinary differential equations. In a low-order dynamical system, the state vector only includes a handful of variables, which vary on roughly the same time scales as the forcing. Barry Saltzman has long promoted this approach, and his models state variables represented the volume of continental ice sheets, deep ocean temperature, carbon dioxide concentration, and in some models the lithospheric depression (e.g., Saltzman and Verbitsky, 1993). Similar models featuring other mechanisms were published more recently (e.g., Omta *et al.*, 2016). The purpose of these models is to explain the temporal structure of ice age cycles, and the spectrum of variability at centennial and millennial time scales is generally ignored. This approach is commonly justified by a hypothesis of separation of time-scales, as formulated by Saltzman (1990). However, this hypothesis is questionable. Indeed, the observational records display a continuous background of variability connecting centennial to 100-ka periods (Huybers and Curry, 2006). For this reason, the dynamics at the centennial, millennial, and astronomical time scales should not be considered separately. Here, we address this concern and show that the ice dynamics is an effective vehicle for propagating high-frequency forcing upscale.

Methods. To make this case, we use the dynamical model previously presented in Verbitsky *et al* (2018). This non-linear dynamical system was derived from scaled conservation equations of ice flow, combined with an equation describing the evolution of a variable synthesizing the state of the rest of the climate, called "climate temperature". The three variables are thus the area of glaciation, ice sheet basal temperature, and climate temperature. Without astronomical forcing, the system evolves to equilibrium. When it is astronomically forced, the system exhibits different modes of non-linearity leading to different periods of ice-age rhythmicity. Specifically, when the ratio of climate positive feedback to glaciation negative feedback (*V*-number) is about 0.75, the system evolves with a roughly 100-ky period. In effect, the response doubles the obliquity period. For this mechanism to operate, ice needs to grow to a level at which it is vulnerable to an increase - even modest - in insolation. In the Verbitsky *et al* (2018) model, the threshold corresponds to a glaciation area S of roughly 20×10^6 km^2.

In the reference experiment presented in Verbitsky *et al.* (2018) the system is driven, as standard practice, by mid-June insolation (Berger and Loutre, 1991). The output of three additional experiments is shown here. In the first experiment, the mid-June insolation is replaced with 5-ky period sinusoid of variable amplitude. In the second experiment, the mid-June insolation is combined with 5-ky period sinusoid of the amplitude about 10 times stronger than the insolation forcing amplitude. In the third experiment, the forcing is represented by several sinusoids of smaller amplitudes (~2.5 of the insolation forcing amplitude) and periods spread between 3 ky and 9 ky. The results demonstrate the following:

A. When our system is forced by a pure 5-ky sinusoid of small amplitude (about the same amplitude as of insolation forcing), the system evolves in the vicinity of its equilibrium point - 15×10^6 km^2 of the glaciation area and -2^{o}C of the climate temperature - Fig 1(A). When the amplitude of the sinusoid is

Earth Syst. Dynam. Discuss., https://doi.org/10.5194/esd-2018-77
Manuscript under review for journal Earth Syst. Dynam.
Discussion started: 9 November 2018

increased tenfold, the system moves into a different phase-plane domain: It evolves around $6x10^6$ km^2 of the glaciation area and 4.6°C of the climate temperature - Fig 1(A).

B. This non-linear system response has a dramatic effect on ice-age dynamics. When insolation forcing is combined with strong millennial forcing, the latter, as we described above, moves the system into the domain where obliquity-period doubling no longer occurs, because ice no longer grows to the level needed to enable the strong positive deglaciation feedback. Consequently, the 100-ky variability almost vanishes - Fig. 1(B).
C. Millennial forcings can be aggregated: Several sinusoids of smaller amplitudes and different millennial periods create the same "hijacking" effect as a single 5-ky high-amplitude sinusoid, moving the system into the phase-plane domain of higher temperatures and lower ice volume - Fig.1(C).
D. Acting alone, low-amplitude millennial sinusoids preserve their original frequencies. When the system is "hijacked" by several sinusoids and moved into new phase-plane domain, millennial forcing is period-multiplied and makes harmonics that overlap those of precession and obliquity - Fig. 1(D).

It turns out to be straightforward enough to anticipate the disruptive effect of forcing at other periods. Indeed, let us measure this disruption potential as the distance ΔS (in km^2) on the phase plane, between the system's equilibrium point with zero forcing, and the time-mean ice-sheet area expected given an periodic forcing of amplitude ε (in km/kyr) and period T (in kyr) - Fig. 1 (A, C). In Verbitsky *et al.* (2018), we have shown that the stability of the system is largely determined by the V-number. We therefore may expect ΔS to be a function $\varphi(V, \varepsilon, T)$. Since V is dimensionless, and dimensions of ε and T are independent, then according to Π-theorem (e.g., Barenblatt, 2003), $\Delta S/(\varepsilon^2 T^2) = \Phi(V)$. We determined experimentally that ΔS=0 if V=0 and that $\Phi(V)$ can be approximated as a linear function. Then the scaling argument finally brings us to:

$$\Delta S \approx -\mu V \varepsilon^2 T^2 = -\mu V \varepsilon^2 f^{-2} \quad (1)$$

where $f=1/T$ is the frequency, and μ is a constant that has to be determined experimentally. We thus see that $\Delta S \sim f^{-2}$. The "-2" frequency slope of ΔS has been confirmed in additional numerical experiments (not shown here) for forcing periods between 2 ky and 20 ky. Obviously, the same scaling arguments can be applied to another system variable S, glacial area, having the same dimension as ΔS. Accordingly, the amplitude spectrum of glacial area variations (Fig. 1, D) exhibits the "-2" frequency slope as well. Different qualities of glacial geometry such as area (S), ice thickness ($\sim S^{1/4}$), glaciation horizontal span ($\sim S^{1/2}$), or ice volume ($\sim S^{5/4}$), may play a role in shaping climate conditions in a specific geographical point. Accordingly, corresponding power spectra of empirical records may have frequency slopes ranging from "-5" to "-1". For multi-period forcing in a frequency domain Δf, the aggregate potential ΔS_A is:

$$\Delta S_A = \frac{1}{\Delta f} \int \Delta S df = -\mu V \Delta f^{-1} \int \varepsilon^2 f^{-2} df \quad (2)$$

Equation (2) implies that amplitude ε does not have to be substantially strong to affect ice ages if the increase of activity is spread over considerable frequency domain. This observation makes centennial, decennial, and may be even annual variations potentially able to contaminate the spectrum throughout the millennial and multi-millennial range, and perturb ice age dynamics.

Discussion. To our knowledge, only Le Treut and Ghil (1983) have previously adopted a deterministic framework to model a background spectrum connecting millennial to astronomical time scales. Unfortunately their model did not generate credible ice age time series. The more common route for simulating the centennial and millennial spectrum is to introduce a stochastic forcing (e.g. Wunsch, 2003, Ditlevsen and Crucifix, 2017). Such stochastic forcing may in principle be justified by the existence of chaotic or turbulent motion in the atmosphere-ocean continuum. However, whether such forcing is great enough to integrate all the way up to time scales of several tens of thousands of years is speculative. In that sense, the deterministic theory proposed here presents the advantage of using the non-linear character of ice sheet dynamics, which was derived naturally from the conservation laws, as an effective means for propagating the forcing upscale. Our approach is thus remarkably parsimonious, because it requires no more physics than the minimum needed to explain ice ages, plus the existence of centennial or millennium modes of motions. The latter may very plausibly arise as modes of ocean motion (Dijkstra and Ghil, 2005, Peltier and Vettoretti, 2014). Of course, stochastic forcing may still be added, and its cumulated effects would then be estimated by equation (2).

Finally, the above arguments come up with an unexpected bonus. As we have seen that *increased* millennial variability *decreases* the length of the ice age cycles. The reverse is also true. This state of affair generates a new hypothesis for the middle-Pleistocene transition: a decrease in millennial variability (which would however need to be explained) may have caused the lengthening of ice ages.

Author contributions. MV conceived the research and developed the model. MV and MC wrote the paper. DV developed the numerical scheme.

Competing interests. The authors declare that they have no conflict of interest.

References

Barenblatt, G. I.: Scaling. Cambridge University Press, 2003

Berger, A. and Loutre, M. F.: Insolation values for the climate of the last 10 million years, Quaternary Science Reviews, 10(4), 297-317, 1991

Dijkstra, H. A. and Ghil, M.: Low-frequency variability of the large-scale ocean circulation: A dynamical systems approach, Rev. Geophys., 43, RG3002, 2005

Ditlevsen, P. and Crucifix, M.: On the importance of centennial variability for ice ages. Past Global Changes Magazine, 25, 152, 2017

Huybers, P. and Curry, W.: Links between annual, Milankovitch and continuum temperature variability, Nature, 441, 329-332, doi:10.1038/nature04745, 2006

Le Treut, H. and Ghil, M.: Orbital forcing, climatic interactions and glaciation cycles, Journal of Geophysical Research, 88, 5167-5190, doi:10.1029/JC088iC09p05167, 1983

Omta, A. W., Kooi, B. W., Voorn, G. A. K., Rickaby, R. E. M. and Follows, M. J.: Inherent characteristics of sawtooth cycles can explain different glacial periodicities, Climate Dynamics, 46, 557-569, doi:10.1007/s00382-015-2598-x, 2016

Peltier, R. W. and Vettoretti, G.: Dansgaard-Oeschger oscillations predicted in a comprehensive model of glacial climate: A ``kicked" salt oscillator in the Atlantic, Geophysical Research Letters, 41, 7306–7313, doi:10.1002/2014GL061413, 2014

Saltzman, B.: Three basic problems of paleoclimatic modeling: A personal perspective and review. Climate Dynamics, 5(2), 67-78, 1990

Saltzman, B. and Verbitsky, M. Y.: Multiple instabilities and modes of glacial rhythmicity in the Plio-Pleistocene: a general theory of late Cenozoic climatic change, Clim. Dynam., 9, 1–15, 1993.

Verbitsky, M. Y., Crucifix, M., and Volobuev, D. M.: A theory of Pleistocene glacial rhythmicity, Earth Syst. Dynam., 9, 1025-1043, https://doi.org/10.5194/esd-9-1025-2018, 2018.

Wunsch, C.: The spectral description of climate change including the 100ky energy, Climate Dynamics, (20) 353-363 doi:10.1007:S00382-002-0279-z, 2003

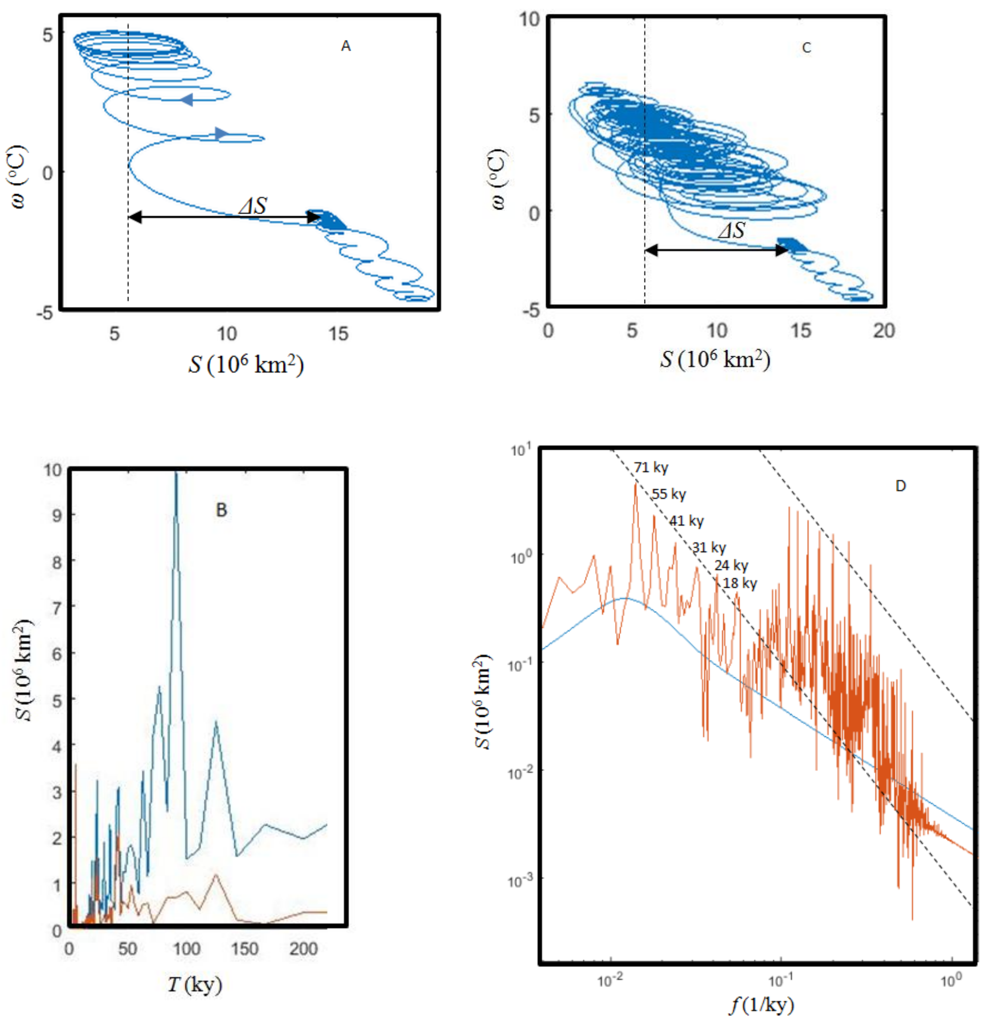

Fig. 1: The system response to millennial forcing. **Section A**: System response to pure 5-ky sinusoid of variable amplitude on a phase plane of glaciation area S (10^6 km^2) *vs.* climate temperature ω (^{o}C); ΔS-is disruption potential; **Section B**: Blue line represents reference system response to mid-June insolation (Verbitsky *et al.*, 2018). Millennial forcing is absent here. Brown line shows system response when mid-June insolation forcing is combined with 5-ky sinusoid of the tenfold amplitude. The diagram is a linear amplitude spectrum; vertical axis measures the amplitude of glacial area variations S (10^6 km^2); horizontal axis is T (ky); **Section C**: Same as Section A but millennial forcing is formed by seven sinusoids of the same amplitude (~2.5 of the insolation forcing amplitude) and periods of 3-, 4-, 5-, 6-, 7-, 8-, and 9-ky. **Section D**: Brown line is system response to millennial forcing made of seven sinusoids of the same amplitude and periods of 3-, 4-, 5-, 6-, 7-, 8-, and 9-ky. Blue line corresponds to the system's evolution to equilibrium (neither astronomical, nor millennial forcing has been applied). Dotted lines are reference "-2" slopes. The diagram is a linear amplitude spectrum on logarithmic scale; vertical axis measures the amplitude of glacial area variations S (10^6 km^2); horizontal axis is f (1/ky).