Interactive comment on "ESD Ideas: Propagation of high-frequency forcing to ice age dynamics" *by* Mikhail Verbitsky *et al*.

M. Verbitsky

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Gerrit Lohmann, Received and published: 8 December 2018

Dear Professor Lohmann,

Thank you for your insightful review and practical suggestions. The following is our response to your specific comments:

Comment: "The wording "non-linear system response". Due to the Northern Hemisphere summer forcing (Berger, 1978), the system receives already a strong nonlinearity. Please clarify this somewhere"
Answer: We concede that the phrasing was unfortunate. When we say "This non-linear system response has a dramatic effect on ice-age dynamics", we mean that the system responds to a millennial-period sinusoid by a shift of the time-mean ice-sheet area and temperature, and that this shift depends non-linearly on the amplitude of the forcing.

Action: We will articulate this thinking more clearly - **p.2 line 7**

Comment: "Reference to the Π-theorem (Buckingham, 1914), the idea goes even further back with Bertrand (1878). Though the Barenblatt (2003) book is a potential reference, I suggest using the older literature here"
Answer: We agree.

Action: We will reference π-theorem to Buckingham (1914) – **p. 2 line 30**

Comment: "The sentence "This observation makes centennial, decennial, and maybe even annual variations potentially able to contaminate the spectrum throughout the millennial and multi-millennial range and perturb ice age dynamics." (lines 36 ff.) is essential for the conclusions. I would ask to substantiate it more with physics."
Answer: We agree that this observation deserves more discussion.

Action: We will articulate our conclusion with more details. Specifically we will emphasize that (a) centennial and millennial oscillations shift the mean state of the system, and (b) the sensitivity of ice sheets to the astronomical forcing depends on the system state. Taken together, these two observations show how centennial and millennial variability can perturb ice-age dynamics and, hence, contaminate the spectrum of variability. – **p. 3 lines 6-8**

Comment: "The authors proposed that their deterministic approach has advantages to show that the forcing propagates upscale. I find the deterministic forcing a little arbitrary. I cannot follow the sentence "... presents the advantage of using the non-linear character of ice sheet dynamics, which was derived naturally from the conservation laws ...". Given the stochastic nature of the millennial variability (Ditlevsen, 1999), the paper would benefit from an additional stochastic analysis which could be added. You mention that "the dynamics at the centennial, millennial, and astronomical time scales should not be considered separately"

Answer: It is correct that several authors (including one of us) have adopted noise models to express the effects of chaotic fluctuations --- with, generally, a reference to Hasselman's theory. Stochastic forcing do indeed provide a potentially fruitful approach to explain the background spectrum, with the reservation that we still have no good theory to determine how to quantify this forcing. A deterministic forcing provides other benefits which we wanted to take advantage of here. First, it allows us clearly to identify the non-linear origin of the cascade ---- while a stochastic forcing may simply be integrated linearly. Second, millennial variability can legitimately be modelled as a deterministic mode, which allows us to come up with a specific explanation of how this variability may influence ice age dynamics.

Action: We will add more discussion about deterministic versus stochastic approaches – **p. 3 lines 17-18, 30-33**

Comment: "Related to the last point: The single 5-ky high-amplitude sinusoid, moving the system into the phase-plane domain of higher temperatures and lower ice volume, is not motivated. Known modes are in the 2.5, 0.9, and 0.5 ky-bands (e.g., Dima and Lohmann, 2008). The periods spread between 3 ky and 9 ky are not really motivated. You may also mention that the mechanism you found is probably different from "noise-induced transitions" where the stochastic forcing generates new equilibria, which do not have a deterministic counterpart"

Answer: The 5-ky sinusoid and multiple sinusoids of periods spread between 3-ky and 9-ky are, indeed, arbitrary. We used them to demonstrate the "hijacking" phenomenon as well as ability of high frequencies to form a beating modulated by a low-frequency envelope and the model's capacity to demodulate a beating signal and to respond to its modulating envelope. This said, perhaps we should also acknowledge that identifying precise mode frequencies from time series analysis is not straightforward either. Time series analysis often relies on assumptions of stationarity and Gaussianity which are not always well verified. This motivates our original choice of using generic frequencies to represent the phenomenon of millennial variability. Hence, we understand the reviewer's concern. To address this concern, we used scaling arguments to anticipate the disruptive effect of forcing at other periods.

Action: We will complement our research with additional experiments with known modes of millennial variability and describe the results in the text – **p. 2 lines 35-40**

Comment: "The implications are only roughly sketched. Would your result imply that we need high-resolution data to understand the variations on orbital time scales? This is, of course, difficult because of the limited space and references allowed here"
Answer: One implication of our study is to formulate a realistic, physically justified alternative to the notion that the background spectrum is merely linearly-integrated noise. In doing so, we are alerting the reader to the potential pitfalls of the classical time-scale separation hypothesis, used to justify delivering separate explanations for DO-oscillations and ice ages.

Action: We will add to implications discussions – **p.3 lines 18-33**

Comment: "Please check the internal consistency of notations, e.g. ky and ka"
Answer: Thank you for noticing.

Action: Done

Interactive comment on "ESD Ideas: Propagation of high-frequency forcing to ice age dynamics" *by* Mikhail Verbitsky et al.

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Niklas Boers, Received and published: 13 February 2019

Dear Dr. Boers,

Thank you for your detailed review and constructive suggestions. The following is our response to your Questions and Comments. We believe that all requested clarifications can be done within ESD Ideas format limit.

Question or Comment: p1 l43: mid-June insolation at which latitude?
Answer: at 65^{o}N

Action: This will be clarified - **p.1 lines 46, 50**

Question or Comment: p2 l5: I think it is not explained why, in your model, the strong millennial-scale forcing leads to this specific change in the fixed point (in particular, warmer and less ice). Could you elaborate on the underlying mechanism? l11ff: I'm not sure if I understand what you write under part D: First, do you mean that the original frequencies of low amplitude sinusoids are preserved by the model? Second, I thought that this case would refer to only periodic (and no astronomical) forcing, how can precession and obliquity be overlapped? See also the corresponding figure panel D.
Answer: Because of the system's non-linearity, the response trajectory to negative forcing is not symmetric to the trajectory generated by a positive forcing. This leads to a shift of the time-mean ice-sheet area and temperature (a "hijacking" effect). When the system is "hijacked" by several sinusoids, millennial forcing is capable of making combined periods that are close to the orbital periods, e.g., periods of precession and obliquity. Specifically, millennial frequencies form a beating modulated by a low-frequency envelope. The model then has the capacity to demodulate a beating signal and respond to its modulating envelope. For example, 41-ky mode in Fig. 1D, which one might be tempted to attribute to obliquity, is in fact the demodulation of the envelope generated by the interplay of the 6-ky and 7-ky forcing sinusoids: $1/41 \approx 1/6-1/7$. Thus two millennial frequencies have created a low frequency forcing with a period similar to the orbital period, i.e. obliquity. The idea of combined periods is not new, this notion of

'combination of harmonics' was emphasized by Le Treut and Ghil (1983), who suggested that the 100-ky period characterizing the late Pleistocene glaciation came from the precession beating ($1/100 \approx 1/19 - 1/23$). We demonstrated here (and this is, indeed, new finding) that the periods of the orbital domain (or better to say, of the domain traditionally “reserved” for orbital periods) can be produced by the millennial forcing. On Figure 1D, you can see both original millennial periods and demodulated-envelope periods.

Action: We will add this discussion into the text – **p.2 lines 17-22**

Question or Comment: Could you elaborate how you use the Buckingham theorem to obtain this specific scaling relation? In particular, why does the amplitude have units km/kyr? You say previously that the amplitude of the periodic forcing is of similar amplitude as the insolation. Also, it would be good to carry out at a level of detail that allows everyone to understand why the exponents are fixed to -2, because this is crucial later on. Ideally, there would be a plot showing (from simulation data) that \Delta S is a quadratic function of (epsilon T).
Answer: We describe here our reasoning as it is applied to the variable S. Similar rationale can be also applied to the disruptive potential ΔS.

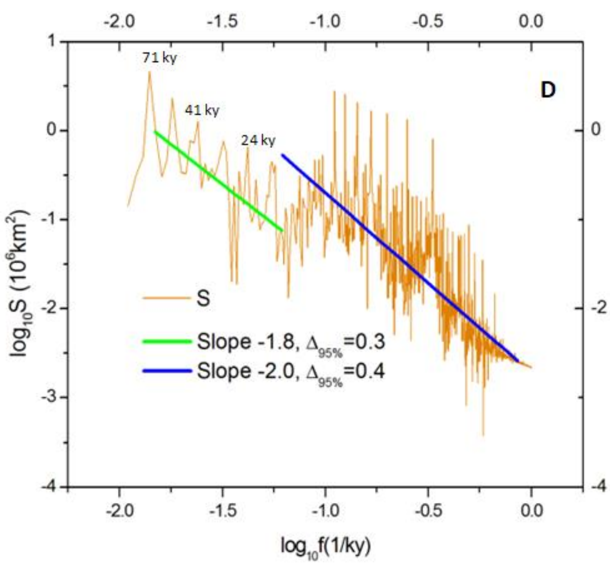
The statement $S = \phi (V, \varepsilon, T)$ is not, indeed, an exact solution of the system of differential equations but a hypothesis that has been inspired by a significant number of numerical experiments we conducted with our model (“numerical observations”, so to say). It provides a starting point for reasoning and also allows estimating the order of magnitude of scaling relationships in the model. Indeed, the V-number is a dimensionless combination of 8 model parameters. On the other hand, the external (astronomical or millennial) forcing of amplitude ε is introduced in our model as a component of the ice sheet surface mass balance and, therefore, it has the same dimension as ice accumulation/ablation rate: km/ky (Verbitsky et al, 2018; equations 18, 19). T is the forcing period, in ky. If the statement $S = \phi (V, \varepsilon, T)$ is true then, according to π-theorem, $S/(\varepsilon^2T^2) = F (V)$. While the F(V) function needs to be determined experimentally, it definitely doesn’t depend on T. Hence, the frequency slope of the amplitude spectrum of the system response (in terms of S-variable) should be close enough to “-2”.

In summary, the hypothesis $S = \phi (V, \varepsilon, T)$ needs indeed to be tested. Your next question (and our corresponding answer) concerns this test.

Action: (a) We will clarify the dimension of the amplitude of the astronomical and millennial forcing, and (b) we will conduct additional experiments to illustrate that ΔS is indeed proportional to $(\varepsilon T)^2$ – **p.2 lines 28-29, 35-40**

Question or Comment: l29: I'm not sure the 'brown' (red?) amplitude spectrum really has slope -2. Have you tried to make a linear fit for comparison?

Answer: In the modified Figure 1D below, we present amplitude spectrum of the system response to millennial forcing made of seven sinusoids of the same amplitude and periods of 3-, 4-, 5-, 6-, 7-, 8-, and 9-ky. The linear fit shows a "-1.8" slope in the orbital frequency domain (though, again, all peaks in this domain are, in fact, created by the millennial forcing) and a "-2" slope for the millennial domain. We consider this result as a remarkable test in favor of the above hypothesis, i.e. S = ϕ (V, ε, T).



Action: See new Fig. 1D

Question or Comment: l30: I don't understand where the different exponents come from; in particular, how does the "-5 to -1" range exactly relate to exponents given in the lines above?

Answer: Ice thickness H is proportional to the glaciation area as S^(1/4) (Verbitsky et al, 2018; equation 5). Accordingly, ice volume, HS, is proportional to S^(5/4). If a component of the climate system depends on the glaciation area as S^α and S is proportional to f^(-2), then the amplitude spectrum of this variable will be proportional to f^(-2α), and the power spectrum will be proportional to f^(-4α). For α=5/4 (responding to volume) it gives a frequency slope of "-5" and for α=1/4 (responding to height) it gives a frequency slope of "-1".

Action: We will clarify this in the text – **p.3 line 1**

Question or Comment: l34: I would suggest to make the frequency dependency of epsilon explicit

Answer: We agree that it would be helpful.

Action: See new Equation (2)

Question or Comment: l35: Can you explain your interpretation of Eq (2), please? In principle, high frequencies are damped by the fˆ-2 term. Your main conclusion, that ice age dynamics can be affected by centennial time scales (in your model), is evident from Fig.1A, but I find it hard to infer this from Eq. (2) alone. It (is) clear that there is interaction between the slow and the fast scales, but it's not clear how strong, because here it really depends on epsilon.
Answer: Two physical mechanisms can be of particular importance for propagating high frequencies upscale: (a) centennial and millennial oscillations shift the mean state of the system, and (b) the sensitivity of the ice sheets to the astronomical forcing depends on the system state. Your observation that amplitudes of high-frequency variability may compensate for frequency damping (f^{-2}) is correct.

Action: We will provide additional discussion in the text – **p 3 lines 3-8**

Question or Comment: Figure: - Would it be possible to provide them in higher resolution? - I would suggest to interchange panels B and C - remove the word "Section" from the caption - i think it would be better to use the same axes for panels A and C
Answer: All of the above can be done.

Action: Done, see new Figure 1.

Question or Comment: Technical corrections: p1 l33: ... state of rest of the climate,...? l37: ... of the positive climate feedback to the negative glaciation feedback l40: ... to an - even modest - increase ... l42: ..., following
standard practise, ...? l44: ...with sinosoids of 5-ky periodicity and variable amplitude.-> Could you be more specific regarding the ' variable' amplitude? l45: ... 5yr-periodic sinosoids of amplitude about ... p2 l17: ... given a periodic forcing ... l21: according to the π-theorem l44 ... forcing is large enough ...
Answer: Thank you for noticing

Action: Done

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ESD Ideas: Propagation of high-frequency forcing to ice age dynamics

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Abstract. Palaeoclimate records display a continuous background of variability connecting centennial to 100-ky periods. Hence, the dynamics at the centennial, millennial, and astronomical time scales should not be treated apart. Here, we show that the non-linear character of ice sheet dynamics, which was derived naturally from the ice-flow conservation laws, provides the scaling constraints to explain the structure of the observed spectrum of variability.

Introduction. Most theories of Quaternary climates consider that glacial-interglacial cycles emerge from components of the climate system interacting with each other, and responding to the forcing generated by the variations of summer insolation caused by climatic precession, the changes in obliquity, and in eccentricity. A common approach is to represent these interactions and response by ordinary differential equations. In a low-order dynamical system, the state vector only includes a handful of variables, which vary on roughly the same time scales as the forcing. Barry Saltzman has long promoted this approach, and his models state variables represented the volume of continental ice sheets, deep ocean temperature, carbon dioxide concentration, and in some models the lithospheric depression (e.g., Saltzman and Verbitsky, 1993). Similar models featuring other mechanisms were published more recently (e.g., Omta et al., 2016). The purpose of these models is to explain the temporal structure of ice age cycles, but the spectrum of variability at centennial and millennial time scales is generally ignored. This approach is commonly justified by a hypothesis of separation of time scales, as formulated by Saltzman (1990). However, this hypothesis is questionable. Indeed, the observational records display a continuous background of variability connecting centennial to 100-ky periods (Huybers and Curry, 2006). For this reason, the dynamics at the centennial, millennial, and astronomical time scales should not be considered separately. Here, we address this concern and show that the ice dynamics is an effective vehicle for propagating high-frequency forcing upscale.

Methods. To make this case, we use the dynamical model previously presented in Verbitsky et al. (2018). This non-linear dynamical system was derived from scaled conservation equations of ice flow, combined with an equation describing the evolution of a variable synthesizing the state of rest of the climate, called "climate temperature". The three variables are thus the area of glaciation, ice sheet basal temperature, and climate temperature. Without astronomical forcing, the system evolves to an equilibrium. When the astronomical forcing is present, the system exhibits different modes of non-linearity leading to different periods of ice-age rhythmicity. Specifically, when the ratio of positive climate feedback to negative glaciation feedback (quantified by the *V*-number) is about 0.75, the system displays glacial-interglacial cycles of a period of roughly 100-ky. In effect, the response doubles the obliquity period. For this mechanism to operate, ice needs to survive through a first maximum of insolation, and then grow to a level at which it is vulnerable to an - even modest - increase in insolation. In the Verbitsky et al. (2018) model with reference parameters, the threshold corresponds to a glaciation area S of roughly $20\text{x}10^6$ km^2.

In the reference experiment presented in Verbitsky et al.(2018) the system is driven, following standard practice, by mid-June insolation at 65^{o}N (Berger and Loutre, 1991). The output of three additional experiments is shown here. In the first experiment, the mid-June insolation is replaced with a sinusoid of 5-ky period and variable amplitude (first, about the same amplitude as of insolation forcing, and then increased tenfold). In the second experiment, this tenfold increased 5-ky-period sinusoid is combined with mid-June insolation at 65^{o}N. In the third experiment, the forcing is represented by several sinusoids of smaller amplitudes (~2.5 of the insolation forcing amplitude) and periods spread between 3 ky and 9 ky. The results demonstrate the following:

A. When our system is forced by a pure 5-ky sinusoid of small amplitude, the system remains in the vicinity of its equilibrium point, with glaciation area of 15×10^6 km^2 and climate temperature of 2^oC (cf. Fig 1(A)). When the amplitude of the sinusoid is increased tenfold, the effects of the negative phases of the forcing no longer symmetric to those of the positive phases, because of the system's non-linearity. As a consequence, the system moves to a different phase-plane domain, around 6×10^6 km^2 of glaciation area and climate temperature of 4.6^oC (Fig 1(A)).

B. This shift of the time-mean glaciation area and temperature has a dramatic effect on ice-age dynamics. When insolation forcing is combined with strong millennial forcing, the latter moves the system into the domain where obliquity-period doubling no longer occurs, because ice no longer grows to the level needed to enable the strong positive deglaciation feedback. Consequently, the 100-ky variability almost vanishes - Fig. 1(B). We term "hijacking", this suppression of ice age variability by millennial variability. This result by itself invalidates the classical time-scale separation hypothesis: we see here that increased *millennial* variability causes the *ice age* cycles to fade.

C. Millennial forcings can be aggregated: Several sinusoids of smaller amplitudes and of different millennial periods create the same "hijacking" effect as a single 5-ky high-amplitude sinusoid, moving the system into the phase-plane domain of higher temperatures and lower ice volume (Fig.1(C)).

D. Acting alone, low-amplitude millennial sinusoids preserve their original frequencies. However, several components may generate low-frequency beatings, which are then demodulated by the system. Through this mechanism, millennial forcing may induce responses at periods close to the orbital periods, e.g., periods of precession and obliquity (Fig. 1(D)). For example, 41-ky mode in Fig. 1D, which one might be tempted to attribute to obliquity, is in fact the demodulation of the envelope generated by the interplay of the 6-ky and 7-ky forcing sinusoids: $1/41 \approx 1/6 - 1/7$.

It is possible to anticipate the disruptive effect of forcing at other periods. Indeed, let us measure this disruption potential as the distance ΔS (km^2) on the phase plane, between the system's equilibrium point with zero forcing, and the time-mean ice-sheet area expected given a periodic forcing of amplitude ε and period T (Fig. 1 (A, C)). In Verbitsky et al. (2018), we have shown that the dynamical properties of the system are largely determined by the V-number. We therefore may expect $\Delta S = \varphi\ (V, \varepsilon, T)$. Since V is dimensionless, and since the dimensions of ε (km/ky) and T (ky) are independent (in our model, the forcing is introduced as a component of ice sheet mass balance and therefore ε has the same dimension as ice ablation rate, km/ky), the π-theorem (Buckingham, 1914) tells us that $\Delta S/(\varepsilon^2 T^2) = \Phi(V)$. We determined experimentally that ΔS=0 if V=0, and that $\Phi(V)$ can be approximated as a linear function. The scaling argument finally brings us to:

$$\Delta S \approx -\mu V \varepsilon^2 T^2 = -\mu V \varepsilon^2 f^{-2} \tag{1}$$

where f=1/T is the frequency, and μ is a constant that has to be determined experimentally. We thus see that $\Delta S \sim f^{-2}$. The "-2" frequency slope of ΔS has been confirmed in additional numerical experiments (not shown here) for forcing periods between 2 ky and 20 ky. The 5-ky-period sinusoids and multiple sinusoids of periods spread between 3-ky and 9-ky are arbitrary choices used to illustrate the "hijacking" and beating effects. The phenomena can be replicated with other modes of millennial activity such as, e.g., 6.5-ky, 2.5-ky, 0.9-ky, and 0.5-ky periods identified by Dima and Lohmann (2009). For example, we confirmed that, as it is implied by equation (1), the "hijacking" effect of a 6.5-ky sinusoid is the same as of a 5-ky sinusoid if the ratio of the corresponding amplitudes is 0.77.

Similar scaling arguments can be applied to the amplitude of S-variable, i.e. amplitude of the glacial area over a glacial cycle. This amplitude $\bar{S}$ has the same dimension as ΔS, i.e., $\bar{S} = \psi\ (V, \varepsilon, T)$ and $\bar{S} \sim \varepsilon^2 T^2$, where T is the period of the system response. Depending on the value of the V-number, the system response may feature periods of the external forcing, or multiples of forcing periods, or combinations of them. Accordingly, Fig. 1D shows a "-1.9" slope in the orbital frequency domain (though, again, all peaks in this domain are, in fact, created by the millennial forcing) and a "-2" slope for the millennial domain. We consider this result as a remarkable test in favor of the above hypothesis, i.e. $\bar{S} = \psi\ (V, \varepsilon, T)$. Different aspects of glacial geometry such as area (S), ice thickness ($H \sim S^{1/4}$, Verbitsky, et al., 2018), glaciation horizontal span ($\sim S^{1/2}$), or ice volume ($HS \sim S^{5/4}$), may play a role in shaping climate conditions in a specific geographical point. Thus, corresponding power spectra of empirical records may have frequency slopes ranging from "-5" ($f^{(-2)*5/4*2}$) to "-1" ($f^{(-2)*1/4*2}$). For multi-period forcing in a frequency domain Δf, the aggregate "hijacking" potential ΔS_A can be estimated as:

$$\Delta S_A = \frac{1}{\Delta f}\int \Delta S df = -\mu V \Delta f^{-1} \int \varepsilon(f)^2 f^{-2} df \tag{2}$$

Accordingly, since amplitudes of high-frequency variability, $\varepsilon(f)$, may compensate for the frequency damping (f^{-2}), centennial, decennial, and perhaps even annual variations potentially may contaminate the spectrum throughout the millennial and multi-millennial range, and perturb ice age dynamics via two physical mechanisms : (a) centennial and millennial oscillations shift the mean state of the system, and (b) the sensitivity of ice sheets to the astronomical forcing depends on the system state.

Discussion. To our knowledge, only Le Treut and Ghil (1983) have previously adopted a deterministic framework to model a background spectrum connecting millennial to astronomical time scales. Unfortunately, their model did not generate credible ice age time series. The more common route for simulating the centennial and millennial spectrum is to introduce a stochastic forcing (e.g. Wunsch, 2003, Ditlevsen and Crucifix, 2017). Such stochastic forcing may in principle be justified by the existence of chaotic or turbulent motion in the atmosphere-ocean continuum. However, whether such forcing is large enough to integrate all the way up to time scales of several tens of thousands of years is speculative. The deterministic theory proposed here presents the advantage of using the non-linear character of ice sheet dynamics, which was derived naturally from the conservation laws and therefore provides a clear physical interpretation of the non-linear origin of the cascade. Our approach is thus remarkably parsimonious, because it requires no more physics than the minimum needed to explain ice ages, plus the existence of centennial or millennium modes of motions. The latter may very plausibly arise as modes of ocean motion (Dijkstra and Ghil, 2005, Peltier and Vettoretti, 2014). Of course, stochastic forcing may still be added, and its cumulated effects would then be estimated by equation (2).

In summary, using deterministic non-linear dynamical model of the global climate, we demonstrated that astronomical time-scale variability cannot be considered separately from millennial phenomena and that the ice dynamics is an effective vehicle for propagating high-frequency forcing into the orbital time scale. This may imply that the knowledge of millennial and centennial variability is needed to fully understand and replicate ice-age history. As we have seen, *increased* millennial variability *decreases* the length of the ice age cycles. However, the reverse is also true. This state of affairs generates a new hypothesis for the middle-Pleistocene transition: a decrease in millennial variability may have caused the lengthening of ice ages. The millennial variability can legitimately be modelled as a deterministic mode, which would allow us to come up with a specific explanation of how this variability may influence ice age dynamics. Hence our completely deterministic approach makes a physically justified alternative to a popular notion that the background spectrum is merely linearly integrated noise.

Code and data availability. The MatLab R2015b code and data to calculate model response to astronomical and millennial forcing (Verbitsky et al., 2019) are available at http://doi.org/10.5281/zenodo.2628310 (last access: 4 April 2019)

Acknowledgements. MC is funded by the Belgian National Fund of Scientific Research. Dmitry Volobuev is funded in part by the Russian Foundation for Basic Research, grant 19-02-00088-a. We are grateful to our reviewers Gerrit Lohmann and Niklas Boers for their helpful comments.

Author contributions. MV conceived the research and developed the model. MV and MC wrote the paper. DV developed the numerical scheme and MatLab code.

Competing interests. The authors declare that they have no conflict of interest.

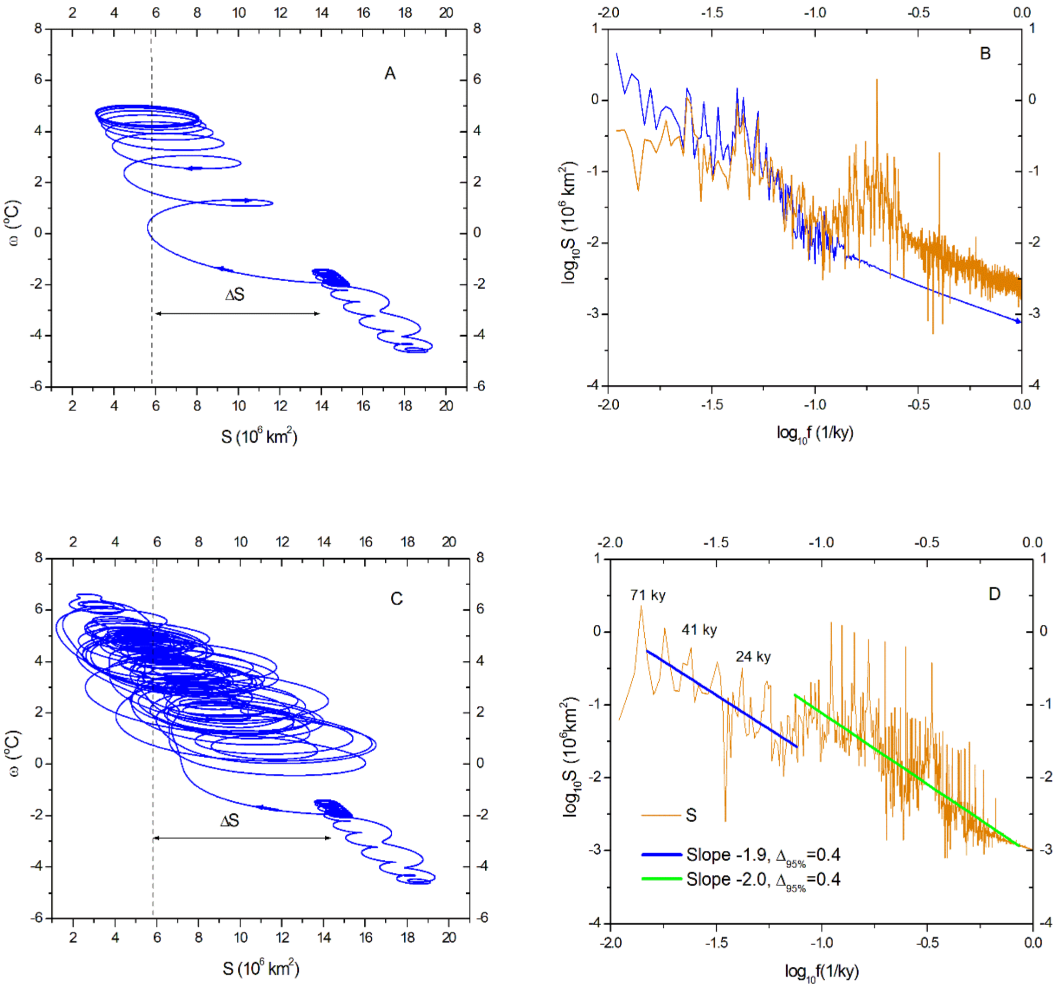

Fig. 1 A: The system response to pure 5-ky sinusoid of variable amplitude on a phase plane of glaciation area S (10^6 km^2) *vs.* climate temperature ω (^{o}C); ΔS-is disruption potential; **B**: Blue line represents reference system response to astronomical forcing (Verbitsky et al., 2018). Millennial forcing is absent here. Brown line shows the system response when the orbital forcing is combined with 5-ky sinusoid of the tenfold amplitude. The diagram is a linear amplitude spectrum on logarithmic scale; vertical axis measures the amplitude of glacial area variations, $\log_{10}[\overline{S}\,(10^6\ \mathrm{km}^2)]$; horizontal axis is $\log_{10}[f\,(1/\mathrm{ky})]$; **C**: Same as Section A but millennial forcing is formed by seven sinusoids of the same amplitude (~2.5 of the insolation forcing amplitude) and periods of 3-, 4-, 5-, 6-, 7-, 8-, and 9-ky. **D**: Same as Section B for multi- millennial forcing of Section C.

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