Interactive comment on “ESD Ideas: Propagation of high-frequency forcing to ice age dynamics” *by* Mikhail Verbitsky et al.

M. Verbitsky

verbitskys@gmail.com

Niklas Boers, Received and published: 13 February 2019

Dear Dr. Boers,

Thank you for your detailed review and constructive suggestions. The following is our response to your Questions and Comments. We believe that all requested clarifications can be done within ESD Ideas format limit.

Question or Comment: p1 l43: mid-June insolation at which latitude?
Answer: at 65^{o}N

Action: This will be clarified

Question or Comment: p2 l5: I think it is not explained why, in your model, the strong millennial-scale forcing leads to this specific change in the fixed point (in particular, warmer and less ice). Could you elaborate on the underlying mechanism? l11ff: I’m not sure if I understand what you write under part D: First, do you mean that the original frequencies of low amplitude sinusoids are preserved by the model? Second, I thought that this case would refer to only periodic (and no astronomical) forcing, how can precession and obliquity be overlapped? See also the corresponding figure panel D.
Answer: Because of the system’s non-linearity, the response trajectory to negative forcing is not symmetric to the trajectory generated by a positive forcing. This leads to a shift of the time-mean ice-sheet area and temperature (a “hijacking” effect). When the system is “hijacked” by several sinusoids, millennial forcing is capable of making combined periods that are close to the orbital periods, e.g., periods of precession and obliquity. Specifically, millennial frequencies form a beating modulated by a low-frequency envelope. The model then has the capacity to demodulate a beating signal and respond to its modulating envelope. For example, 41-ky mode in Fig. 1D, which one might be tempted to attribute to obliquity, is in fact the demodulation of the envelope generated by the interplay of the 6-ky and 7-ky forcing sinusoids: 1/41≈1/6-1/7. Thus two millennial frequencies have created a low frequency forcing with a period similar to the orbital period, i.e. obliquity. The idea of combined periods is not new, this notion of 'combination of harmonics' was emphasized by Le Treut and Ghil (1983), who suggested that

the 100-ky period characterizing the late Pleistocene glaciation came from the precession beating (1/100 ≈ 1/19 - 1/23). We demonstrated here (and this is, indeed, new finding) that the periods of the orbital domain (or better to say, of the domain traditionally "reserved" for orbital periods) can be produced by the millennial forcing. On Figure 1D, you can see both original millennial periods and demodulated-envelope periods.

Action: We will add this discussion into the text.

Question or Comment: Could you elaborate how you use the Buckingham theorem to obtain this specific scaling relation? In particular, why does the amplitude have units km/kyr? You say previously that the amplitude of the periodic forcing is of similar amplitude as the insolation. Also, it would be good to carry out at a level of detail that allows everyone to understand why the exponents are fixed to -2, because this is crucial later on. Ideally, there would be a plot showing (from simulation data) that \Delta S is a quadratic function of (epsilon T).

Answer: We describe here our reasoning as it is applied to the variable S. Similar rationale can be also applied to the disruptive potential ΔS.

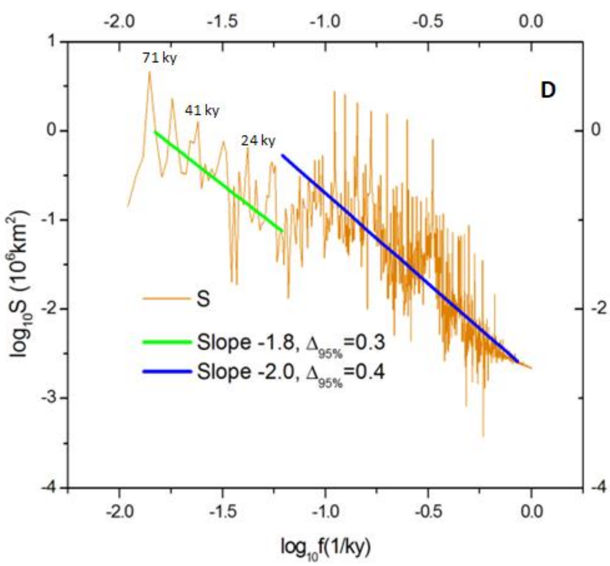
The statement S = ϕ (V, ε, T) is not, indeed, an exact solution of the system of differential equations but a hypothesis that has been inspired by a significant number of numerical experiments we conducted with our model ("numerical observations", so to say). It provides a starting point for reasoning and also allows estimating the order of magnitude of scaling relationships in the model. Indeed, the V-number is a dimensionless combination of 8 model parameters. On the other hand, the external (astronomical or millennial) forcing of amplitude ε is introduced in our model as a component of the ice sheet surface mass balance and, therefore, it has the same dimension as ice accumulation/ablation rate: km/ky (Verbitsky et al, 2018; equations 18, 19). T is the forcing period, in ky. If the statement S = ϕ (V, ε, T) is true then, according to π-theorem, $S/(\varepsilon^2 T^2)$ = F (V). While the F(V) function needs to be determined experimentally, it definitely doesn't depend on T. Hence, the frequency slope of the amplitude spectrum of the system response (in terms of S-variable) should be close enough to "-2".

In summary, the hypothesis S = ϕ (V, ε, T) needs indeed to be tested. Your next question (and our corresponding answer) concerns this test.

Action: (a) We will clarify the dimension of the amplitude of the astronomical and millennial forcing, and (b) we will conduct additional experiments to illustrate that ΔS is indeed proportional to $(\varepsilon T)^2$.

Question or Comment: l29: I'm not sure the 'brown' (red?) amplitude spectrum really has slope -2. Have you tried to make a linear fit for comparison?

Answer: In the modified Figure 1D below, we present amplitude spectrum of the system response to millennial forcing made of seven sinusoids of the same amplitude and periods of 3-, 4-, 5-, 6-, 7-, 8-, and 9-ky. The linear fit shows a "-1.8" slope in the orbital frequency domain (though, again, all peaks in this domain are, in fact, created by the millennial forcing) and a "-2" slope for the millennial domain. We consider this result as a remarkable test in favor of the above hypothesis, i.e. S = ϕ (V, ε, T).



Action: We will update Fig. 1D to include linear fit as it is shown above.

Question or Comment: l30: I don't understand where the different exponents come from; in particular, how does the "-5 to -1" range exactly relate to exponents given in the lines above?

Answer: Ice thickness H is proportional to the glaciation area as S^(1/4) (Verbitsky et al, 2018; equation 5). Accordingly, ice volume, HS, is proportional to S^(5/4). If a component of the climate system depends on the glaciation area as S^α and S is proportional to f^(-2), then the amplitude spectrum of this variable will be proportional to f^(-2α), and the power spectrum will be proportional to f^(-4α). For α=5/4 (responding to volume) it gives a frequency slope of "-5" and for α=1/4 (responding to height) it gives a frequency slope of "-1".

Action: We will clarify this in the text

Question or Comment: l34: I would suggest to make the frequency dependency of epsilon explicit

Answer: We agree that it would be helpful.

Action: Equation (2) will be modified accordingly

Question or Comment: l35: Can you explain your interpretation of Eq (2), please? In principle, high frequencies are damped by the fˆ-2 term. Your main conclusion, that ice age dynamics can be affected by centennial time scales (in your model), is evident from Fig.1A, but I find it hard to infer this from Eq. (2) alone. It (is) clear that there is interaction between the slow and the fast scales, but it's not clear how strong, because here it really depends on epsilon.
Answer: Two physical mechanisms can be of particular importance for propagating high frequencies upscale: (a) centennial and millennial oscillations shift the mean state of the system, and (b) the sensitivity of the ice sheets to the astronomical forcing depends on the system state. Your observation that amplitudes of high-frequency variability may compensate for frequency damping (f^{-2}) is correct.

Action: We will provide additional discussion in the text

Question or Comment: Figure: - Would it be possible to provide them in higher resolution? - I would suggest to interchange panels B and C - remove the word "Section" from the caption - i think it would be better to use the same axes for panels A and C
Answer: All of the above can be done.

Action: We will provide higher quality pictures.

Question or Comment: Technical corrections: p1 l33: ... state of rest of the climate,...? l37: ... of the positive climate feedback to the negative glaciation feedback l40: ... to an - even modest - increase ... l42: ..., following
standard practise, ...? l44: ...with sinosoids of 5-ky periodicity and variable amplitude.-> Could you be more specific regarding the ' variable' amplitude? l45: ... 5yr-periodic sinosoids of amplitude about ... p2 l17: ... given a periodic forcing ... l21: according to the π-theorem l44 ... forcing is large enough ...
Answer: Thank you for noticing

Action: Everything will be fixed

References

Le Treut, H. and Ghil, M.: Orbital forcing, climatic interactions, and glaciation cycles. Journal of Geophysical Research: Oceans, 88(C9), 5167-5190, 1983

Verbitsky, M. Y., Crucifix, M., and Volobuev, D. M.: A theory of Pleistocene glacial rhythmicity, Earth Syst. Dynam., 9, 1025-1043, https://doi.org/10.5194/esd-9-1025-2018, 2018