Interactive comment on "ESD Ideas: Propagation of high-frequency forcing to ice age dynamics" *by* Mikhail Verbitsky *et al*.

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Gerrit Lohmann, Received and published: 8 December 2018

Dear Professor Lohmann,

Thank you for your insightful review and practical suggestions. The following is our response to your specific comments:

Comment: "The wording "non-linear system response". Due to the Northern Hemisphere summer forcing (Berger, 1978), the system receives already a strong nonlinearity. Please clarify this somewhere"
Answer: We concede that the phrasing was unfortunate. When we say "This non-linear system response has a dramatic effect on ice-age dynamics", we mean that the system responds to a millennial-period sinusoid by a shift of the time-mean ice-sheet area and temperature, and that this shift depends non-linearly on the amplitude of the forcing.

Action: We will articulate this thinking more clearly.

Comment: "Reference to the Π-theorem (Buckingham, 1914), the idea goes even further back with Bertrand (1878). Though the Barenblatt (2003) book is a potential reference, I suggest using the older literature here"
Answer: We agree.

Action: We will reference π-theorem to Buckingham (1914)

Comment: "The sentence "This observation makes centennial, decennial, and maybe even annual variations potentially able to contaminate the spectrum throughout the millennial and multi-millennial range and perturb ice age dynamics." (lines 36 ff.) is essential for the conclusions. I would ask to substantiate it more with physics."
Answer: We agree that this observation deserves more discussion.

Action: We will articulate our conclusion with more details. Specifically we will emphasize that (a) centennial and millennial oscillations shift the mean state of the system, and (b) the sensitivity of ice sheets to the astronomical forcing depends on the system state. Taken

together, these two observations show how centennial and millennial variability can perturb ice-age dynamics and, hence, contaminate the spectrum of variability.

Comment: "The authors proposed that their deterministic approach has advantages to show that the forcing propagates upscale. I find the deterministic forcing a little arbitrary. I cannot follow the sentence "... presents the advantage of using the non-linear character of ice sheet dynamics, which was derived naturally from the conservation laws ...". Given the stochastic nature of the millennial variability (Ditlevsen, 1999), the paper would benefit from an additional stochastic analysis which could be added. You mention that "the dynamics at the centennial, millennial, and astronomical time scales should not be considered separately"

Answer: It is correct that several authors (including one of us) have adopted noise models to express the effects of chaotic fluctuations --- with, generally, a reference to Hasselman's theory. Stochastic forcing do indeed provide a potentially fruitful approach to explain the background spectrum, with the reservation that we still have no good theory to determine how to quantify this forcing. A deterministic forcing provides other benefits which we wanted to take advantage of here. First, it allows us clearly to identify the non-linear origin of the cascade ---- while a stochastic forcing may simply be integrated linearly. Second, millennial variability can legitimately be modelled as a deterministic mode, which allows us to come up with a specific explanation of how this variability may influence ice age dynamics.

Action: We will add more discussion about deterministic versus stochastic approaches

Comment: "Related to the last point: The single 5-ky high-amplitude sinusoid, moving the system into the phase-plane domain of higher temperatures and lower ice volume, is not motivated. Known modes are in the 2.5, 0.9, and 0.5 ky-bands (e.g., Dima and Lohmann, 2008). The periods spread between 3 ky and 9 ky are not really motivated. You may also mention that the mechanism you found is probably different from "noise-induced transitions" where the stochastic forcing generates new equilibria, which do not have a deterministic counterpart"

Answer: The 5-ky sinusoid and multiple sinusoids of periods spread between 3-ky and 9-ky are, indeed, arbitrary. We used them to demonstrate the "hijacking" phenomenon as well as ability of high frequencies to form a beating modulated by a low-frequency envelope and the model's capacity to demodulate a beating signal and to respond to its modulating envelope. This said, perhaps we should also acknowledge that identifying precise mode frequencies from time series analysis is not straightforward either. Time series analysis often relies on assumptions of stationarity and Gaussianity which are not always well verified. This motivates our original choice of using generic frequencies to represent the phenomenon of millennial variability. Hence, we understand the reviewer's concern. To address this concern, we used scaling arguments to anticipate the disruptive effect of forcing at other periods.

Action: We will complement our research with additional experiments with known modes of millennial variability and describe the results in the text.

Comment: "The implications are only roughly sketched. Would your result imply that we need high-resolution data to understand the variations on orbital time scales? This is, of course, difficult because of the limited space and references allowed here"
Answer: One implication of our study is to formulate a realistic, physically justified alternative to the notion that the background spectrum is merely linearly-integrated noise. In doing so, we are alerting the reader to the potential pitfalls of the classical time-scale separation hypothesis, used to justify delivering separate explanations for DO-oscillations and ice ages.

Action: We will add to implications discussions

Comment: "Please check the internal consistency of notations, e.g. ky and ka"
Answer: Thank you for noticing.

Action: This will be fixed.