



1 **An emergent transition time-scale in the atmosphere and its implications to global-**
2 **averaged precipitation control mechanisms, time-series reconstruction and**
3 **stochastic downscaling**

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11 **Abstract**

12 Detrended Cross-Correlation Analysis (DCCA) revealed an emergent transition in non-
13 periodic (deseasonalized) atmospheric variability at time-scales ~ 1 -year. At multi-year
14 time-scales (i) $\rho_{SST, T_{land}} \sim 0.6$ (i.e. the correlation between global-averaged sea surface
15 temperature, SST, and 2-meter air temperature averaged over global-land, T_{land}); (ii)
16 Clausius-Clapeyron relationship becomes the dominant control of global-averaged
17 precipitable water vapor (W), with $\rho_{W, T_{2m}} \sim \rho_{W, SST} \sim 0.9$; (iii) atmospheric radiative fluxes,
18 specifically the surface downwelling longwave radiative flux (DLR), become a key
19 constraint for global-mean precipitation (P) variability ($\rho_{P, R_{atm}} \sim \rho_{P, DLR} \sim -0.8$); (iv) cloud
20 effects are negligible in (iii), and clear-sky DLR becomes a dominant P constraint; and
21 (v) $\rho_{P, T_{2m}}$ and $\rho_{P, SST}$ displayed significant multi-year correlations, although with large
22 spread amongst different datasets (~ 0.4 to ~ 0.7). Result (v) provides a new perspective
23 into the well-known uncertainties climate models associated with the dynamical
24 component of precipitation. At sub-yearly time-scales all correlations underlying these
25 five results decrease abruptly towards negligible values.

26 The relevance and validity of this multi-scale structure is demonstrated by three
27 reconstructed P time-series at 2-year resolution, two relying on clear-sky DLR constraints
28 and one based on P -SST correlation. These simple models, particularly one based on
29 clear-sky DLR, were able to reproduce observed P anomaly time-series with similar
30 accuracy to a (uncoupled) atmospheric model (ERA-20CM) and two climate reanalysis
31 (ERA-20C and 20CR). The idealized models aren't applicable at sub-yearly time-scales,
32 where the underlying correlations become negligible. However, monthly P probability
33 density functions (PDFs) were derived by stochastic downscaling of reconstructed P ,



34 leveraging on scale-invariant properties, outperforming the statistics simulated by ERA-
35 20C, 20CR and ERA-20CM.

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38 1. Introduction

39 The precipitation response to changes in increased concentrations of greenhouse gases is
40 a central topic for the climate science community. Although its regional variability is
41 essential to determine the societal impacts, global-averaged precipitation (P) is an
42 important first-order climate indicator, and a measure of the global water cycle, that must
43 be accurately simulated if robust climate projections are to be obtained across a wide
44 range of spatial and temporal scales. However, even the long-term P response is still
45 poorly understood, constrained and simulated (Collins et al., 2013; Allan et al., 2014;
46 Hegerl et al., 2015), largely due to the limited knowledge on the complex interactions
47 between the key components of the atmospheric branch of the water cycle and its forcing
48 mechanisms. This problem is tackled here by employing a multi-scale analysis framework
49 to study the variability of P, and its relation to two key governing mechanisms: the
50 Clausius-Clapeyron (C-C) relationship and the constraints imposed by the atmospheric
51 energy balance.

52 The C-C relationship is a well-known mechanism controlling the variability of the global
53 water cycle. Assuming constant relative humidity, it implies that fractional changes in
54 global-averaged precipitable water vapor ($\Delta W/W$) are linearly related to fluctuations of
55 global-averaged near-surface (e.g. 2-meter) air temperature (ΔT_{2m}) (e.g. Held & Soden,
56 2006; Schneider et al., 2010):

$$57 \frac{\Delta W}{W} \approx \alpha_{W,T_{2m}} \Delta T_{2m}, \quad (1)$$

58 where $\alpha_{W,T_{2m}} \approx 0.07 \text{ K}^{-1}$ at temperatures typical of the lower troposphere. Numerous
59 studies have provided a robust confirmation for C-C at multi-decadal to centennial time-
60 scales, while also reporting an analogous linear response of ΔP to ΔT (see e.g. Schneider
61 et al., 2010; Trenberth, 2011; O’Gorman et al., 2012; and Allan et al., 2014 for reviews).
62 In general, these previous investigations agree on the $\sim 7\%/K$ sensitivity coefficient for
63 W. However, there is large spread on the P sensitivity coefficient estimates, typically in
64 the 1%/K to 3%/K range.

65 A widely recognized explanation for the sub-C-C sensitivity of P to temperature
66 fluctuations at long temporal scales comes from the atmospheric energy balance (Allen



67 & Ingram, 2002; Stephens & Ellis, 2008; Stephens & Hu, 2010). Specifically, averaging
68 over the global atmosphere, the latent heat flux associated with precipitation formation
69 ($L_V P$, L_V being the latent heat of vaporization) must be in balance with the net atmospheric
70 radiative flux (R_{atm}) and the surface sensible flux (F_{SH}):

$$71 \quad L_V P + R_{atm} + F_{SH} \approx 0, \quad (2)$$

72 Equation (2) represents a general state of radiative convective equilibrium (Pauluis &
73 Held, 2002), with energy fluxes defined positive for atmospheric gain, and negative
74 otherwise.

75 If the C-C relationship was the dominant mechanism controlling the response of
76 atmospheric moisture content and the global water cycle to temperature fluctuations, then
77 W and P could be expected to be strongly correlated to surface temperature. Previously
78 Gu and Adler (2011, 2012) found strong correlations between the inter-annual variability
79 of W and global-averaged surface temperature, in tight agreement with the C-C
80 relationship. However, they found weaker (but significant) correlations between the inter-
81 annual variability of P and global-averaged surface temperature, suggesting that C-C
82 might not be directly extendable to global precipitation. But these results focusing on a
83 single temporal scale might not represent the entire picture. In fact, it is now a well-
84 established fact that precipitation and other relevant atmospheric variables (including
85 temperature, atmospheric moisture, wind, etc.) display a complex statistical structure,
86 with significant variability over a wide range of temporal scales, and with the possibility
87 of different mechanisms governing variability at different time-scales (see e.g. Lovejoy
88 & Schertzer, 2013 for a comprehensive review). Furthermore, it has been shown that this
89 complex multiscale structure plays a role (at least) as important and the large amplitude
90 periodic components, namely diurnal and seasonal cycles (Lovejoy, 2015; Nogueira,
91 2017a). However, our understanding of the underlying governing mechanisms at different
92 time-scales remains largely elusive, representing a central problem for future
93 improvements to climate simulation and projection.

94 Recently, Nogueira (2018) analyzed satellite-based observational datasets, a long Global
95 Climate Model simulation and reanalysis products and found a tight correlation (~ 0.8)
96 between anomaly (deseasonalized) time-series of W and global-averaged surface
97 temperature, which emerged at time-scales larger than ~ 1 -2 years. In contrast, at smaller
98 time-scales the correlation decreased rapidly towards negligible values (< 0.3). In other
99 words, the C-C relationship is the dominant mechanism of deseasonalized W anomalies
100 at multi-year time-scales, but not at sub-yearly time-scales. Nogueira (2018) also found



101 that the magnitude of the correlations between anomaly time-series for P and global-
102 averaged surface temperature was negligible at sub-yearly time-scales, while at multi-
103 year time-scales the results showed large spread amongst different data-sets, ranging
104 between negligible (<0.3) and strong (~ 0.8) correlation values. Building on this previous
105 study, here the multi-scale analysis of the mechanisms governing P variability is
106 extended, including the energetic constraints on P represented in Equation (2).
107 Additionally, a simple stochastic model is proposed to reconstruct P time-series based on
108 the strong correlations found at multi-year time-scales, while monthly statistics are
109 reproduced by employing a stochastic downscaling algorithm based on scale-invariant
110 symmetries of P. The manuscript is organized as follows: section 2 describes the
111 considered datasets and the multi-scale analysis framework; the results of multi-scale
112 correlation analysis on P variability are presented and discussed in section 3; in section 4
113 a simple idealized model is proposed for reconstruction of P variability; and finally the
114 main conclusions are summarized and discussed in section 5.

115

116 **2. Data and Methodology**

117 **2.1. Data sets**

118 Observations of P were obtained from the Global Precipitation Climatology Project
119 (GPCP) version 2.3 monthly precipitation dataset (Adler et al., 2003), which covers the
120 full globe at 2.5° resolution from 1979 to present. Gridded datasets of monthly average
121 surface temperatures were obtained from the Goddard Institute for Space Studies
122 (GISSTEMP) analysis (Hansen et al., 2010), which covers the globe at 2° resolution from
123 1880 to present, with the values provided as anomalies relative to the 1951-1980 reference
124 period. GISSTEMP blends near-surface air temperature measurements from
125 meteorological stations (including Antarctic stations) with a reconstructed SST dataset
126 over oceans. Observations of atmospheric radiative fluxes were obtained from the
127 National Aeronautics and Space Administration (NASA) Clouds and the Earth's Radiant
128 Energy System, Energy Balanced and Filled (CERES-EBAF) Edition 4.0 (Loeb et al.,
129 2009), a monthly dataset covering the full globe at 1° resolution from March/2000 to
130 June/2017.

131 Two state-of-the-art reanalyses of the twentieth-century were considered in the present
132 study. One was the National Oceanic and Atmospheric Administration Cooperative
133 institute for Research in Environmental Sciences (NOAA-CIRES) twentieth-century
134 reanalysis (20CR) version 2c (Compo et al., 2011), which covers the full globe at 2°



135 resolution, spanning from 1851 to 2014. Only surface pressure observations and reports
136 are assimilated in this reanalysis. SST boundary conditions are obtained from 18 members
137 of pentad Simple Ocean Data Assimilation with Sparse Input (SODAsi) version 2, with
138 the high latitudes corrected to the Centennial in Situ Observation-Based Estimates of the
139 Variability of SST and Marine Meteorological Variables, version 2 (COBE-SST2). Here,
140 global-mean time-series of P, W, SST, T_{2m} , DLR and OLR are obtained from 20CR at
141 daily resolution for the 1900-2010 period. R_{atm} cannot be obtained the incoming solar
142 radiation at TOA is not available for the 20CR dataset, due to an error with output
143 processing.

144 The other reanalysis considered in the present study was the European Centre for
145 Medium-Range Weather Forecasts (ECMWF) twentieth-century reanalysis (ERA-20C,
146 Poli et al., 2015), which covers the full globe at 1° resolution spanning from 1900-2010.
147 It assimilates marine surface winds from the International Comprehensive Ocean-
148 Atmosphere Data Set version 2.5.1 (ICOADSv2.5.1) and surface and mean-sea-level
149 pressure from the International Surface Pressure Databank version 3.2.6 (ISPDv3.2.6)
150 and from ICOADSv2.5.1. SST boundary conditions are obtained from the Hadley Centre
151 Sea Ice and Sea Surface Temperature data set version 2.1 (HadISST2.1). Global-mean
152 time-series of P, W, SST, T_{2m} , R_{atm} , DLR and OLR are obtained from ERA-20C at daily
153 resolution for the 1900-2010 period.

154 Finally, the uncoupled ECMWF twentieth-century ensemble of ten atmospheric model
155 integrations (ERA-20CM, Hersbach et al., 2015) was considered, which uses the same
156 model, grid, initial conditions, radiative and aerosol forcings as ERA-20C. However, no
157 observations are assimilated, the simulation is integrated continuously over the full 1900-
158 2010 period, and SST is prescribed by an ensemble of realizations from HadISST2.1,
159 including one control simulation and nine simulations with perturbed SST and sea-ice
160 concentration. A 10-member ensemble of global-mean time-series of P, W, SST, T_{2m} ,
161 R_{atm} , DLR and OLR were obtained from ERA-20CM at monthly resolution for the 1900-
162 2010 period. Considering ERA-20CM allowed to test the sensitivity of the multi-scale
163 correlation structure derived from ERA-20C to data assimilation, but different
164 atmospheric evolutions associated with perturbations to the forcing fields (particularly to
165 SST).

166 Notice that the clear-sky radiative fluxes considered here obtained from ECMWF datasets
167 are computed for the same atmospheric conditions of temperature, humidity, ozone, trace
168 gases and aerosol, but assuming that the clouds are not there. Clear-sky estimates from



169 ERA-20C and ERA-20CM cover the full globe area and not just the cloud free regions at
 170 each time instant. However, they are available for net radiative fluxes, but not for
 171 downwelling or upwelling radiation fluxes.

172 2.2. Detrended Cross-Correlation Analysis (DCCA)

173 DCCA allows to accurately quantify power-law correlations between two different time-
 174 series over wide ranges of time-scales (Podobnik & Stanley, 2008). Consider two time-
 175 series, y and y' , with N data points each. Due to the strong yearly cycle present in the
 176 considered time-series, the periodic seasonal trend is first eliminated by subtracting the
 177 long-term average (over all the years in the record) of each calendar day (or month,
 178 depending on temporal resolution):

$$179 \Delta y(i) = y(i) - \langle y \rangle_d, \quad (3)$$

180 Then two integrated signals, R and R' , are constructed from the deseasonalized anomaly
 181 time-series, Δy and $\Delta y'$:

$$182 R_k = \sum_{i=1}^k [\Delta y(i) - \langle y_{ds} \rangle], \quad (4)$$

183 Where $k=1, \dots, N$ and $\langle \cdot \rangle$ indicates temporal averaging. The integrated signals are
 184 divided into $N - n$ overlapping segments, each containing $n + 1$ values. For each
 185 segment from each integrated signal, the “local trend” is estimated using a first-order
 186 polynomial. The detrended integrated signal is then defined as the difference between the
 187 original integrated signal and the local trend ($R_v - \widetilde{R}_v$), where \widetilde{R}_v is the fitting first-order
 188 polynomial to the v th segment R_v . Next, the covariance of the residuals in each segment
 189 is calculated as:

$$190 f_{R,R'}^2(n, i) = \frac{1}{n+1} \sum_{k=i}^{i+n} [(R_v - \widetilde{R}_v)(R_v' - \widetilde{R}_v')], \quad (5)$$

191 The detrended covariance is estimated by summing over all overlapping $N-n$ segments:

$$192 F_{R,R'}^2(n) = \frac{1}{N-n} \sum_{i=1}^{N-n} f_{R,R'}^2(n, i), \quad (6)$$

193 Finally, the DCCA cross-correlation coefficient at time-scale n , $\rho_{y,y'}(n)$, is defined as
 194 the ratio between the detrended covariance function and the product of the square-rooted
 195 detrended variance function for each time-series:

$$196 \rho_{y,y'}(n) = \frac{F_{R,R'}^2(n)}{\sqrt{F_{R,R}^2(n) \times F_{R',R'}^2(n)}}, \quad (7)$$

197 The values of $\rho_{y,y'}(n)$ range between -1 and 1 (for perfect negatively and positively
 198 correlated signals, respectively). It has been previously shown that critical points for the
 199 95% significance level of $|\rho_{DCCA}|$ can vary between values below 0.1 and up to about 0.4,



200 depending on the time series length, the considered time-scale, and the power law
201 exponents of both time-series (Podobnik et al., 2011). Here it is assumed that $|\rho_{DCCA}|$
202 values below 0.3 are nonsignificant, and that $|\rho_{DCCA}|$ values in the 0.3 to 0.4 range should
203 be interpreted with care.

204

205 **3. DCCA analysis of the mechanisms governing P variability across time-scales**

206 **3.1. Multi-scale structure of the atmospheric water cycle response to surface** 207 **temperature fluctuations**

208 DCCA reveals strong correlations (~ 0.9) between deseasonalized anomaly time-series for
209 W and T_{2m} or SST at multi-year time-scales (Fig. 1a). However, as the time-scale
210 decreases there is a transition in the correlation structure, and negligible correlations
211 (< 0.3) emerge at sub-yearly time-scales. This result suggested that the C-C relationship
212 in Equation (1) holds to a very good approximation at multi-year time-scales, but not at
213 sub-yearly time-scales. Lovejoy et al. (2018) employed multi-scale analysis framework
214 based on Haar wavelets to GISSTEMP and found a similar transition in the multi-scale
215 correlation structure of SST against global-averaged surface temperature, between low-
216 correlations at time-scales below a few months and strong correlations (~ 0.8) at multi-
217 year time-scales. These strong correlations weren't surprising, since SST was a major
218 component in their definition of global-averaged surface temperature (also considering
219 SST over the ocean pixels and 2-meter air temperature over land pixels). But their results
220 also showed a transition in the correlation coefficients between SST and near-surface air
221 temperature over global-land (T_{land}), with maximum correlation values ~ 0.6 at multi-year
222 time-scales. The transition in $\rho_{SST, T_{land}}$ was confirmed here by employing DCCA to
223 ERA-20C, ERA-20CM, 20CR and GISSTEMP (Fig. 1b). Thus, the present results
224 support Lovejoy et al. (2018) argument that these abrupt correlation changes correspond
225 to a fundamental behavioral transition, where the atmosphere and the oceans start to act
226 as a single coupled system. Furthermore, the results presented here suggest that W
227 anomalies at multi-year resolution can be derived, to a very good approximation, from
228 SST alone.

229 Nogueira (2018) also reported a transition in the multi-scale correlation structure between
230 deseasonalized anomaly time-series of P and global-averaged surface temperature
231 (considering SST over the oceans and T_{2m} over land), with negligible values at sub-yearly
232 time-scales, but with large spread in the magnitude of the multi-year correlations, ranging



233 between values ~ 0.3 to ~ 0.8 . Here, a similar result was found for $\rho_{P,T_{2m}}$ and $\rho_{P,SST}$ (Fig.
234 1c), with large spread in correlation magnitude at multi-year time-scales (~ 0.7 in ERA-
235 20C and ERA-20CM, ~ 0.6 in 20CR, and < 0.4 in observations). This large spread and the
236 relatively low correlations obtained from observational datasets confirmed the
237 uncertainty on the extension of C-C relationship as the dominant control of P variability.
238 Notice that the large spread in $\rho_{P,T_{2m}}$ and $\rho_{P,SST}$ represents a different perspective, under
239 a multi-scale analysis framework, on a previously established fact: there are large
240 uncertainties in climate simulations associated with the role of the non-thermodynamical
241 (circulation) component of precipitation response to climate change (see e.g. Shepherd,
242 2014).

243 3.2. Multi-scales structure of the energetic constraints to P variability

244 A study of the circulation component of the P response to temperature fluctuations
245 requires a detailed representation of several spatially heterogeneous variables and their
246 nonlinear interactions. An alternative path towards understanding P variability was taken
247 in the present investigation, focusing on the constraints imposed by the atmospheric
248 energy balance represented in Equation (2). Fig. 2a (solid lines) shows that the estimated
249 DCCA correlation coefficients between the deseasonalized anomaly time-series for P and
250 R_{atm} were strongly (negatively) correlated at multi-year time-scales ($\rho_{P,R_{atm}} \sim -0.8$ in
251 ERA-20C, ERA-20CM and observations), in agreement with the balance in Equation (2).
252 The same wasn't true at sub-yearly time-scales, where the correlation magnitude
253 decreased rapidly, changed sign around monthly time-scales, and reached values ~ 0.4 at
254 time-scales below about 10 days.

255 Considering the effect of F_{SH} in Equation (2) (i.e. $\rho_{P,R_{atm}+F_{SH}}$) slightly increased the
256 (positive) correlations at sub-monthly time-scales (Fig. 2a, dashed lines), although the
257 absolute changes are essentially below 0.1 and $\rho_{P,R_{atm}+F_{SH}}$ at sub-monthly time-scales
258 (which is only available for the ERA-20C dataset). More importantly, the change between
259 $\rho_{P,R_{atm}}$ and $\rho_{P,R_{atm}+F_{SH}}$ at multi-year time-scales was negligible. Indeed,
260 $\rho_{P,R_{F_{SH}}}$ displayed values up to about 0.5 at sub-monthly time-scales, but essentially < 0.4
261 at multi-year time-scales (Fig. 2a, dot-dashed lines). Given the results in Fig. 1a, the
262 following linear relation was hypothesized: $L_V \Delta P \approx c_1 \times (-\Delta R_{atm}) + c_2$, where c_1 and
263 c_2 are arbitrary constants, and Δ represents fluctuations taken as deseasonalized
264 anomalies at multi-year resolutions. At sub-yearly time-scales this simplification is not



265 adequate, since $\rho_{P,R_{atm}}$ becomes negligible and, thus, the energy balance represented in
266 Equation (2) doesn't represent the dominant constraint on P variability, most likely due
267 to non-negligible changes in atmospheric heat storage.

268 The analysis was extended by decomposing R_{atm} into its net atmospheric longwave and
269 shortwave radiative flux components, i.e. $R_{atm} = R_{LW,net} + R_{SW,net}$. On the one hand,
270 $\rho_{P,R_{atm}} \approx \rho_{P,R_{LW,net}}$ over the full range of time-scales considered (Fig. 2b). On the other
271 hand, $\rho_{P,R_{SW,net}}$ also displays significant values (~ 0.6) at multi-year time-scales, but the
272 latter magnitude was nearly 0.2 lower than $\rho_{P,R_{atm}}$ and $\rho_{P,R_{LW,net}}$ (Fig. 2b). Consequently,
273 the above linear relationship for multi-scale P anomalies was further refined as $L_V \Delta P \approx$
274 $c_1 \times (-\Delta R_{atm}) + c_2 \approx c_3 \times (-\Delta R_{LW,net}) + c_4$, where c_3 and c_4 are arbitrary constants.
275 Subsequently, $R_{LW,net}$ was further decomposed into the top-of-atmosphere (TOA) and
276 surface net longwave fluxes, i.e. $R_{LW,net} = R_{LW,TOA} + R_{LW,SFC}$. At multi-year time-
277 scales, $\rho_{P,R_{atm}} \approx \rho_{P,R_{LW,SFC}}$ (Fig. 2c). $\rho_{P,R_{LW,TOA}}$ also displayed significant values at
278 multi-year time-scales, up to ~ 0.6 in ERA-20C and ERA-20CM datasets. Notice that
279 20CR displayed values $|\rho_{P,R_{LW,TOA}}| < 0.4$ at multi-year time-scales. But ECMWF
280 datasets were in better agreement with observations, suggesting that significant (negative)
281 correlations existed between P and $R_{LW,TOA}$ anomalies at multi-year time-scales.
282 Nonetheless, even for ECMWF and observational products, the magnitude of $\rho_{P,R_{LW,TOA}}$
283 at multi-year time-scales was nearly 0.2 lower than for $\rho_{P,R_{LW,SFC}}$. Consequently, a further
284 approximation was considered on the linear model for P fluctuations at multi-year time-
285 scales:

$$L_V \Delta P \approx c_1 \times (-\Delta R_{atm}) + c_2 \approx c_3 \times (-\Delta R_{LW,net}) + c_4 \approx c_5 \times (-\Delta R_{LW,SFC}) + c_6.$$

287 Finally, $R_{LW,SFC}$ can be further decomposed into its upwelling ($R_{LW,SFC,UP}$) and
288 downwelling ($R_{LW,SFC,DOWN}$, henceforth denoted downwelling longwave radiation, DLR)
289 components. Fig. 2d shows that, at multi-year time-scales, the differences between
290 $\rho_{P,R_{DLR}}$ and $\rho_{P,R_{atm}}$ were within 0.1 in observations, ERA-20C and ERA-20CM (R_{atm} is
291 unavailable for 20CR). Thus, at multi-year time-scales, the fluctuations in downwelling
292 surface longwave radiative fluxes are, to a good approximation, linearly related to P
293 fluctuations: $L_V \Delta P \approx c_7 \times (-\Delta DLR) + c_8$. Notice that the differences between
294 $\rho_{P,R_{LW,SFC,UP}}$ and $\rho_{P,R_{atm}}$ are identically low in observations, but these differences are
295 somewhat higher (~ 0.2) in ERA-20CM and ERA-20C. Thus, a similar linear relationship



296 between ΔP and $\Delta R_{LW,SFC,UP}$ might also hold to a good approximation, although the
297 correlations are less robust than for ΔP against ΔDLR .

298 The correlation between global-mean clear-sky net radiative atmospheric heating and P,
299 i.e. $\rho_{P,R_{atm,cs}}$, was nearly identical to $\rho_{P,R_{atm}}$ at multi-year time-scales (Fig. 3a). This
300 suggested that the cloud effects on the multi-year linear dependence between P variability
301 and net atmospheric radiative fluxes may be neglected. But the same isn't true at time-
302 scales below a few months, where significant differences emerge between $\rho_{P,R_{atm,cs}}$ and
303 $\rho_{P,R_{atm}}$. This clear-sky approximation holds at multi-year time-scales for correlations of
304 P against global-averaged net atmospheric longwave radiative fluxes and, also, and
305 against the global-averaged net surface longwave fluxes (Fig. 3b). Based on these results,
306 it was further hypothesized that cloud effects are also negligible for the correlation
307 between P and DLR at multi-year temporal scales. This hypothesis could not be tested
308 directly because clear-sky DLR time-series were not available for the ECMWF datasets.
309 Nonetheless, the results in Section 4 based on an empirical algorithm for DLR estimation
310 under a clear-sky approximation provided support for this hypothesis.

311 In summary, DCCA suggested that P variability at multi-year time-scales is linearly
312 related to the net atmospheric radiative fluxes. Furthermore, this linear relationship is
313 dominated by its longwave component and, more specifically, by the surface longwave
314 radiative fluxes, particularly DLR. DCCA also suggests that clouds play a negligible
315 effect in these linear correlations at multi-years scales. The hypothesized tight correlation
316 between P and clear-sky DLR fluxes at multi-year time-scales was particularly
317 interesting, since clear-sky DLR may be estimated directly from atmospheric water vapor
318 content and surface temperature (e.g. Stephens et al., 2012b). This fact will be further
319 explored below, in Section 4.

320 Finally, notice that the results in Fig. 2c showed that P variability was best correlated to
321 $R_{LW,TOA}$ variability at sub-monthly time-scales, reaching positive values $\sim 0.5-0.6$. This
322 corresponds to a well-known relation between convective rainfall and the outgoing
323 longwave radiation at TOA, often denoted OLR (e.g. Xie & Arkin, 1998). However, this
324 result provided no further simplification in the sense that, unlike for clear-sky DLR at
325 multi-year resolution, it is equally difficult to model and predict P and OLR (including
326 cloud effects) at sub-monthly time-scales.

327 At this point, it is important to notice that the existence of strong correlations does not
328 necessarily imply causality between two variables. However, the atmospheric energy



329 balance in Equation (2) provides a physical basis for the obtained strong (negative)
330 correlations values between P and atmospheric radiative fluxes. In fact, the importance
331 of energetic constraints to global precipitation, the dominant role of surface longwave
332 fluxes, namely DLR, and the negligible cloud effects in these relations has been pointed
333 out by previous investigations (e.g., Stephens and Hu, 2010; Stephens et al., 2012a,b).
334 The DCCA presented here provided further robustness to these results. More importantly,
335 a clear transition emerged between robust correlations at multi-year time-scales and
336 negligible correlations at sub-yearly time-scales, which was found for P against R_{atm} (or
337 DLR), for W against $T_{2\text{m}}$ (and SST), for SST against T_{land} and, less robustly, for P against
338 $T_{2\text{m}}$ (or SST). Given the interdependence between these variables, these transitions are
339 likely to be interrelated, representing a more fundamental transition in the atmosphere.
340 Notice that these results also contribute to sharpen the picture from previous studies
341 reporting a ‘fast’ P response at sub-monthly time-scales, where P is suggested respond
342 directly to the radiative effects of increasing CO_2 ; and a ‘slow’ response where P increases
343 due to increasing surface temperature (Allen & Ingram, 2002; Bala et al., 2010; Andrews
344 et al., 2010; O’Gorman et al., 2012; Allan et al., 2014).

345

346 **4. Stochastic model for global-mean precipitation**

347 **4.1. Reconstruction of P time-series at multi-year resolution**

348 Here a very simple model for P response to climate change is proposed aiming to
349 demonstrate the robustness of the tight correlation between P and clear-sky DLR (DLR_{CS})
350 at multi-year time-scales presented in Section 3. The rationale is that the correlation
351 between P and DLR_{CS} at multi-year time-scales is significantly more robust than between
352 P and $T_{2\text{m}}$ (or SST). Additionally, DLR_{CS} can be derived, to a good approximation, from
353 the global averaged near-surface temperature alone (e.g. Stephens et al., 2012b).
354 Furthermore, given the tight coupling between T_{land} and SST at multi-year time-scales
355 (Fig. 1b), it is hypothesized that DLR_{CS} variability could be obtained, to a good
356 approximation directly from the SST forcing. This hypothesis is also supported by the
357 nearly identical correlations between W and $T_{2\text{m}}$ or SST (Fig. 1a).

358 Here two different algorithms to estimate DLR_{CS} are tested: the Dilley-O’Brien model
359 (Dilley & O’Brien, 1998), and the Prata model (Prata, 1996). In the Dilley-O’Brien
360 model:

$$361 \quad \text{DLR}_{2y,DO} = a_1 + a_2 \left(\frac{\text{SST}_{2y}}{\text{SST}_c} \right)^6 + a_3 \left(\frac{\Delta W_{2y} + W_c}{W_c} \right)^{1/2}, \quad (8)$$



362 Where $a_1 = 59.38 \text{ Wm}^{-2}$, $a_2 = 113.7 \text{ Wm}^{-2}$ and $a_3 = 96.96 \text{ Wm}^{-2}$ are the model parameters,
 363 and $W_c = 22.5 \text{ kg m}^{-2}$ is the climatological value for W . The subscript ‘2y’ (e.g. DLR_{2y})
 364 indicates a time-series at 2-year resolution. The fluctuations Δ represent anomaly time-
 365 series relative to a climatological time-series, for example $\Delta DLR_{2y,DO} = DLR_{2y,DO} -$
 366 $DLR_{c,DO}$. Notice that for multi-year resolution time-series, this yields the same result as
 367 first deseasonalizing the time-series (using Equation (3)) and then coarse-graining it to 2-
 368 year resolution. $DLR_{c,DO} = a_1 + a_2 + a_3$ is obtained by replacing the climatological
 369 values of W and SST in Equation (8).

370 The Prata model for $\Delta DLR_{2y,Pr}$ is based on the Stefan-Boltzmann equation:

$$371 \quad DLR_{2y,Pr} = \varepsilon_{ctr} \sigma_{SB} SST_{2y}^4 \quad (9)$$

372 Where $\sigma_{SB} = 5.67 \times 10^{-8} \text{ Wm}^{-2}\text{K}^{-4}$ is the Stefan-Boltzmann constant and:

$$373 \quad \varepsilon_{ctr} = 1 - (1 + W_{2y}) \exp(-(1.2 + 3W_{2y})^{1/2}) \quad (10)$$

374 The anomaly-time series is computed from $\Delta DLR_{2y,Pr} = DLR_{2y,Pr} - DLR_{c,Pr}$, where
 375 $DLR_{c,Pr}$ is obtained by replacing the climatological values of W and SST in Equations
 376 (9) and (10).

377 The high values of $\rho_{W,SST} (\approx \rho_{W,T_{2m}})$ at multi-year time-scales (Section 3.1) allowed to
 378 remove the W dependence in Equations (8) and (11), by replacing $W_{2y} \approx$
 379 $\alpha_{W,SST} \Delta SST_{2y} W_c + W_c$. The forcing ΔSST_{2y} time-series were obtained by coarse-
 380 graining the deseasonalized (using Equation (3)) global-averaged SST obtained from
 381 GISSTEMP dataset. The sensitivity coefficient, $\alpha_{W,SST} \approx 0.08 \text{ K}^{-1}$ was estimated by
 382 least-square regression of $\Delta W_{2y}/W_c$ against ΔSST_{2y} , pooling together all datasets (ERA-
 383 20C, ERA-20CM and 20CR). The $\alpha_{W,SST}$ estimates are summarized in Table 1, including
 384 for each individual dataset, ranging between 0.07 and 0.10 K^{-1} . Notice that the obtained
 385 values are close to the typical 0.07 K^{-1} value.

386 The results from Section 3.2 suggested a linear relation between P and DLR_{CS} variability
 387 at multi-year time-scales, which can be written as $P_{2y} \approx \alpha_{P,DLR} (-\Delta DLR_{CS,2y}) P_c + P_c$. In
 388 this way, two reconstructed anomaly time-series for P were obtained, $P_{2y,DO}$ and $P_{2y,Pr}$,
 389 respectively by replacing $\Delta DLR_{CS,2y}$ with $\Delta DLR_{2y,DO}$ and $\Delta DLR_{2y,Pr}$. The coefficient
 390 $P_c \approx 2.7 \text{ mm/day}$ was estimated from GPCP dataset. The sensitivity coefficient $\alpha_{P,DLR} \approx$
 391 $0.004 (\text{W/m}^2)^{-1}$ was estimated by least-square regression of $\Delta P_{2y}/P_c$ against ΔDLR_{2y} ,
 392 pooling together all available datasets (ERA-20C, ERA-20CM, 20CR and GPCP against
 393 CERES-EBAF). Notice that, in estimating $\alpha_{P,DLR}$, clear-sky DLR time-series were used



394 where available (i.e. for ERA-20C and ERA-20CM) datasets, but they were replaced by
395 (full-sky) DLR otherwise (i.e. for 20CR and CERES-EBAF). The $\alpha_{P,DLR}$ estimates are
396 summarized in Table 2, including values obtained from each dataset (no estimate was
397 made for GPCP against CERES-EBAF due to the limited duration of the latter), ranging
398 between $0.003 \text{ (W/m}^2\text{)}^{-1}$ and $0.005 \text{ (W/m}^2\text{)}^{-1}$.

399 Another simple linear model for reconstruction of multi-year P anomaly time-series was
400 tested, based on the direct response (correlations) of P to SST fluctuations, i.e. $P_{2y,SST} \approx$
401 $\alpha_{P,SST} \Delta SST_{2y} P_c + P_c$. Again, the ΔSST_{2y} was obtained from GISSTEMP dataset. The
402 sensitivity coefficient, $\alpha_{P,SST} \approx 0.02 \text{ K}^{-1}$ was estimated by least-square regression of
403 $\Delta P_{2y}/P_c$ against ΔSST_{2y} , pooling together all datasets (ERA-20C, ERA-20CM, 20CR and
404 GPCP against GISSTEMP). The $\alpha_{P,SST}$ estimates are summarized in Table 3, including
405 for each individual dataset, ranging between 0.02 and 0.04 K^{-1} . Notice that the obtained
406 values are close to the 0.01 to 0.03 K^{-1} range reported in the relevant literature (e.g.
407 Schneider et al., 2010; Trenberth, 2011; O’Gorman et al., 2012; and Allan et al., 2014).

408 When compared against ΔP_{2y} directly derived from GPCP for the 1979 to 2010 period,
409 the errors in the proposed linear ΔP_{2y} reconstructions were generally close to those for
410 atmospheric model-based products (Fig. 4). $\Delta P_{2y,Pr}$ displays the highest mean bias,
411 somewhat higher than for atmospheric model-based datasets, but also higher than the
412 mean bias for $\Delta P_{2y,DO}$ and $\Delta P_{2y,SST}$ (Fig. 4a). Notice that all atmospheric model-based
413 products considered here also display a positive bias. While this may be due a negative
414 bias of GPCP (e.g. Gehne et al., 2015), this observational dataset represents the longest
415 reliable dataset for global precipitation studies and thus was considered here as ‘the truth’.
416 More importantly, the mean bias is easy to correct, simply by subtracting its value from
417 the time-series. This correction was implemented here for all atmospheric model-based
418 and linear-model based ΔP_{2y} time-series. Figure 4c shows that the normalized standard
419 deviation ($\sigma_n = \sigma_{2y,model}/\sigma_{2y,obs}$) estimated from $\Delta P_{2y,DO}$ (~ 0.4) and, particularly, from
420 $\Delta P_{2y,SST}$ (~ 0.3) were lower than the values estimated from atmospheric model-based
421 products (~ 0.5 - 0.9). In contrast, σ_n estimated from $\Delta P_{2y,Pr}$ was nearly 0.8 , which was
422 higher than 20CR and most members in the ERA-20CM ensemble, only outperformed by
423 ERA-20C dataset. The root-mean squared error after bias-correction ($RMSE_{bc}$) estimated
424 from $\Delta P_{2y,Pr}$ and $\Delta P_{2y,DO}$ were well within the range of the values obtained from
425 atmospheric model-based products (Fig. 4b), with the Prata model slightly



426 overperforming the Dilley-O'Brien model. $RMSE_{bc}$ estimated from $\Delta P_{2y,SST}$ was on the
427 high-end of the atmospheric model-based range of values, and somewhat worse than for
428 the DLR-based linear models. Finally, the Pearson correlation coefficient between models
429 and observations (Fig. 4d) was similar amongst all linear-based models and well within
430 the range of values estimated from the atmospheric model-based products. The latter
431 result was expected since all linear models were forced by the same SST time-series.

432 Overall, these results suggested that $\Delta P_{2y,Pr}$ (after bias correction) reproduced the
433 observations with similar accuracy to atmospheric model-based products, including
434 similar $RMSE_{bc}$, variability amplitude and phase of the signal. $\Delta P_{2y,DO}$ displayed similar
435 performance for $RMSE_{bc}$ and for the phase, but not for the variability amplitude. Finally,
436 $\Delta P_{2y,SST}$ had the worst performance concerning $RMSE_{bc}$, but also in capturing the
437 variability amplitude, while it displayed similar ability to the other linear models in
438 reproducing the phase. The overall weakest performance of $\Delta P_{2y,SST}$ was coherent with
439 the less robust correlations underlying this model. Additionally, the results indicate that
440 the non-linear transformations on SST employed in the Prata and the Dilley-O'Brien
441 algorithms improved the linear models.

442 **4.2. Stochastic reproducing of P monthly PDFs**

443 At sub-yearly time-scales, the magnitude of $\rho_{P,W}$, $\rho_{P,SST}$ and $\rho_{P,DLR}$ decreased abruptly
444 to negligible values (Section 3). Thus, at these time-scales, the C-C relationship is no
445 longer the dominant control of W (nor P) variability, and the longwave radiative fluxes
446 are no longer the main constraints for P. Additionally, the cloud-effects on P variability
447 become non-negligible (Fig. 3). Consequently, the linear relationships underlying the
448 above P reconstruction at 2-year resolution are no longer appropriate at sub-yearly time-
449 scales. Building on the strong scale-invariant symmetries present in the variability of
450 global and regional rainfall across wide ranges of time-scales (e.g. Lovejoy and Schertzer,
451 2013; Nogueira et al., 2013; Nogueira and Barros, 2014, 2015; Nogueira, 2017, 2018), an
452 algorithm was proposed here to derive the sub-yearly statistics from the multi-year
453 information alone. The physical basis for this algorithm is that while the atmosphere is
454 governed by continuum mechanics and thermodynamics, it simultaneously obeys
455 statistical turbulence cascade laws (e.g., Lovejoy & Schertzer, 2013; Lovejoy et al.,
456 2018).

457 Conveniently, precipitation (and many other atmospheric variables) is characterized by
458 low spectral slopes $\beta < 1$, with quasi-Gaussian and quasi-non-intermittent statistics, at



459 time-scales between ~10 days and a few decades (Lovejoy & Schertzer, 2013; de Lima
460 & Lovejoy, 2015; Lovejoy et al., 2015, 2018; Nogueira, 2017b, 2018). Grounded by these
461 scale-invariant properties, fractional Gaussian noise was used here to generate multiple
462 realizations of downscaled ΔP at monthly resolution from each member of each ΔP_{2y}
463 time-series:

$$464 \quad \Delta P_{1m}(t) = fGn_{1m}(t) \frac{\Delta P_{2y}(t)}{fGn_{2y}(t)} \quad (11)$$

465 where fGn_{1m} is a fractional Gaussian noise, which was computed by first generating a
466 random Gaussian noise (g), then taking its Fourier transform (\tilde{g}), multiplying by $k^{-\beta/2}$,
467 and finally taking the inverse transform to obtain fGn_{1m} . The mean of fGn_{1m} was forced
468 to be equal to the number of data-points of ΔP_{2y} . Then fGn_{2y} was obtained by coarse-
469 graining fGn_{1m} using 24-point (i.e. 2 years) length boxes. In this way, $\Delta P_{1m,DO}$, $\Delta P_{1m,Pr}$,
470 $\Delta P_{1m,SST}$ ensembles are generated respectively from the bias-corrected $\Delta P_{2y,DO}$, $\Delta P_{2y,Pr}$
471 and $\Delta P_{2y,SST}$ time-series. One hundred plausible realizations are generated for each
472 ensemble, corresponding to one hundred different realizations of fGn_{1m} . Based on recent
473 investigations on P scale-invariance properties, a spectral exponent $\beta \approx 0.3$ is assumed
474 (de Lima & Lovejoy, 2015; Nogueira, 2018). In Equation (11), the 2-year resolution time-
475 series were assumed to have a constant value for every month inside each 2-years period.
476 Notice that a resolution limit should exist to the proposed stochastic downscaling
477 algorithm, namely at time-scales below ~10 days where a fundamental transition occurs
478 in the scaling behavior of most atmospheric fields (including P, see e.g. Lovejoy &
479 Schertzer, 2013; Lovejoy, 2015; de Lima & Lovejoy, 2015; Nogueira, 2017a,b, 2018). At
480 faster time-scales intermittency becomes non-negligible and the quasi-Gaussian
481 approximation to the statistics is no longer robust.

482 The proposed downscaling methodology corresponds to treating the sub-yearly
483 frequencies as random ‘weather noise’, which is characterized, to a good approximation,
484 by scale-invariant behavior with quasi-Gaussian statistics (Vallis, 2009; Lovejoy et al.,
485 2015). A similar downscaling methodology has been previously demonstrated to
486 reproduce the spatial sub-grid scale variability of topographic height (Bindlish & Barros,
487 1996), rainfall (Bindlish & Barros, 2000; Reborá et al., 2006; Nogueira & Barros, 2015)
488 and clouds (Nogueira & Barros, 2014).

489 Figure 5a showed that the PDFs computed from $\Delta P_{1m,DO}$, $\Delta P_{1m,Pr}$ and $\Delta P_{1m,SST}$ were in
490 remarkable agreement with GPCP PDFs for the 1979-2010 period, representing a
491 significant improvement compared to all atmospheric model-based products. This



492 improved PDF accuracy was quantified using the Perkins skill score, S-Score (Perkins et
493 al., 2007), defined as:

$$494 \text{ S-Score} = 100 \times \sum_{i=1}^M \min[f_{mod}(i), f_{obs}(i)] \quad (12)$$

495 where $f_{mod}(i)$ and $f_{obs}(i)$ are respectively the frequency of the modeled and observed P
496 anomaly values in bin i , M is the number of bins used to compute the PDF (here $M=15$),
497 and $\min[x,y]$ is the minimum between the two values. The S-Score is a measure of
498 similarity between modeled and observed PDFs, such that if a model reproduces the
499 observed PDF perfectly then S-Score=100%.

500 The linear-based models showed S-Score values around 95%, which were significantly
501 higher than then ~80% found for the atmospheric model-based products (Fig. 6).
502 Furthermore, the stochastic model captured the change in the PDFs between two separate
503 periods (1979-1990 and 1999-2010, Fig. 5b), while preserving the remarkably high
504 ($\geq 90\%$) S-Scores (Fig. 6, blue and red markers). Indeed, the S-Scores for linear-based
505 were consistently better than the S-Scores obtained from atmospheric model-based
506 products (~80%). Despite some differences between PDFs obtained from $\Delta P_{1m,DO}$,
507 $\Delta P_{1m,Pr}$ and $\Delta P_{1m,SST}$, their similar performance in reproducing observations was
508 somewhat unexpected, given the distinct performances in reproducing the observed time-
509 series at multi-year resolutions. While the error analysis here was based on a limited
510 sample (mainly due to short duration of the satellite-period), these results suggested that
511 the proposed stochastic downscaling mechanism is quite robust in reproducing the
512 monthly P statistics, with only moderate sensitivity to the coarse resolution forcing.

513

514 **5. Conclusions**

515 Atmospheric variables display significant variability over a wide range of temporal
516 scales, both due changes in external forcings (including surface fluxes, changes to
517 greenhouse gases and aerosol concentrations, solar forcing, and volcanic eruptions), but
518 also due to intrinsic modes of atmospheric variability. Additionally, external and internal
519 atmospheric processes interact nonlinearly amongst themselves, resulting in an intricate
520 multi-scale structure, which is still ill understood and responsible for significant
521 uncertainties in climate models. Here a multi-scale analysis framework was employed,
522 aiming to disentangle the complex structure of global-averaged precipitation variability.
523 A critical transition emerges from DCCA at time-scales ~1-year, revealing a change in
524 the control mechanisms of the P and W, but also in the strength of the atmosphere-ocean



525 coupling. At multi-year time-scales W becomes tightly correlated to T_{2m} and to SST
526 (~ 0.9), while at sub-yearly time-scales this correlation decreases abruptly towards
527 negligible values (~ 0.2). A sensitivity coefficient for W close to the typically estimated
528 $0.07\%/K$ was found for multi-year time-scales. In other words, the C-C relationship is the
529 dominant mechanism of W at multi-year time-scale, but not at sub-year time-scales.
530 Furthermore, at time-scales >1 -2 years SST becomes tightly correlated to T_{land} , pointing
531 to a fundamental behavioral transition, where the atmosphere and the oceans start to act
532 as a single coupled system at multi-year time-scales, as previously suggested by Lovejoy
533 et al. (2018).

534 A similar transition was also found for $\rho_{P,T_{2m}}$ and $\rho_{P,SST}$, with negligible correlations and
535 sub-year time-scales, which tend increase at multi-year time-scales, although the latter
536 displayed significant spread amongst different datasets (ranging between ~ 0.4 to ~ 0.7).
537 More robust correlations were obtained for the P response to the energetic constraints
538 imposed by a simple atmospheric energy balance. DCCA showed that P variability is
539 tightly (negatively) coupled to the net atmospheric radiative balance at multi-year time-
540 scales (with $\rho_{P,R_{atm}} \lesssim -0.8$). The transition between multi-year and sub-yearly time-
541 scales also emerged for $\rho_{P,R_{atm}}$, with the correlation magnitude decreased rapidly at sub-
542 yearly time-scales, changing signal, and reached values ~ 0.4 at sub-monthly time-scales.
543 Additionally, DCCA revealed that the positive sub-monthly correlations are dominated
544 by the TOA OLR, while the multi-year correlations were dominated by surface longwave
545 fluxes, particularly by DLR. Furthermore, DCCA suggested that cloud effects play a
546 negligible on the multi-year correlations, but they are important for the sub-monthly
547 $\rho_{P,R_{atm}}$ values. Notice that the sensitivity coefficients of P to SST estimated here were in
548 the 2 - $4\%/K$ range, close to the typical 1 - $3\%/K$ values (for P against T_S) obtained from
549 energetic constraints on global rainfall.

550 The robustness and relevance of this emergent multi-scale correlation structure is
551 demonstrated by proposing simple models for reconstruction of P at multi-year time-
552 scales. Anomaly time-series for P at 2-year resolution were derived from SST
553 observations alone, either directly based on $\rho_{P,SST}$, or by combining $\rho_{R,DLR_{CS}}$, empirical
554 algorithms for clear-sky DLR estimation, and the C-C relationship. After correcting for
555 their systematic mean bias, the highly-idealized model for ΔP_{2y} based on clear-sky DLR
556 estimated from the Prata algorithm displayed similar accuracy in reproducing
557 observations as atmospheric model-based products, as measured by $RMSE_{bc}$, Pearson



558 correlation coefficient and normalized standard deviation. The simple model based on the
559 Dilley-O'Brien algorithm for clear-sky DLR estimation showed a somewhat poorer
560 performance, particularly in reproducing the observed variability amplitude. Finally, the
561 model based on P-SST correlation showed the weakest performance, which agrees with
562 the less robust correlations underlying this idealized model.

563 The proposed linear models cannot be extended to sub-yearly the time-scales because all
564 the correlations upon which they rely become negligible. This abrupt transition in the
565 multi-scale correlation structure implies that at sub-yearly time-scales the tight linear
566 coupling between atmospheric and ocean temperature, the Clausius-Clapeyron
567 relationship, and the atmospheric energy balance are no longer dominant linear
568 constraints for P. Nonetheless, the multi-scale analysis framework provides another path
569 for reconstruction of the P statistics at sub-yearly resolution. A stochastic downscaling
570 algorithm based on scale-invariant symmetries of P was applied to ΔP_{2y} reconstructed
571 time-series, resulting in monthly P PDFs. Remarkably, the reconstructed PDFs of P at
572 monthly resolution showed better accuracy in reproducing GPCP statistics than
573 atmospheric model-based products, as measured by S-Score computed over decadal and
574 30-year periods. Interestingly, the PDFs obtained by downscaling the three algorithms
575 proposed for multi-year P reconstruction showed similar performance, suggesting a weak
576 sensitivity of this algorithm to the accuracy of the 2-year resolution forcing time-series.

577 The present investigation highlights the complex multi-scale structure of the water cycle
578 variability and its governing mechanisms. Finally, it is hypothesized that the path for
579 stochastic regional precipitation simulation may be opened by leveraging on the widely
580 reported scale-invariant properties of precipitation in the spatial domain (e.g. Lovejoy &
581 Schertzer, 2013; Nogueira & Barros, 2014, 2015), and exploring control mechanisms for
582 slow variability of regional precipitation, such as the El-Niño Southern Oscillation and
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584

585

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597 https://eosweb.larc.nasa.gov/project/ceres/ebaf_surface_table

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- 734
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736 **Table 1** Linear regression coefficient $\alpha_{W,SST}$ estimated from $\Delta W/W_c$ against ΔSST at 2-
 737 year resolution, assuming a relationship as given by Equation (1). The respective
 738 coefficient of determination is also provided. The $\alpha_{W,SST}$ are computed for ERA-20C,
 739 20CR, and for the ensemble of ERA-20CM simulations. Additionally, the coefficient is
 740 estimated by pooling together ERA-20C, ERA-20CM (ensemble) and 20CR datasets.

Dataset	$\alpha_{W,SST} [K^{-1}]$	R^2
ERA-20C	0.09	0.97
20CR	0.10	0.92
E20CM (Ensemble)	0.07	0.92
All Datasets	0.08	0.91

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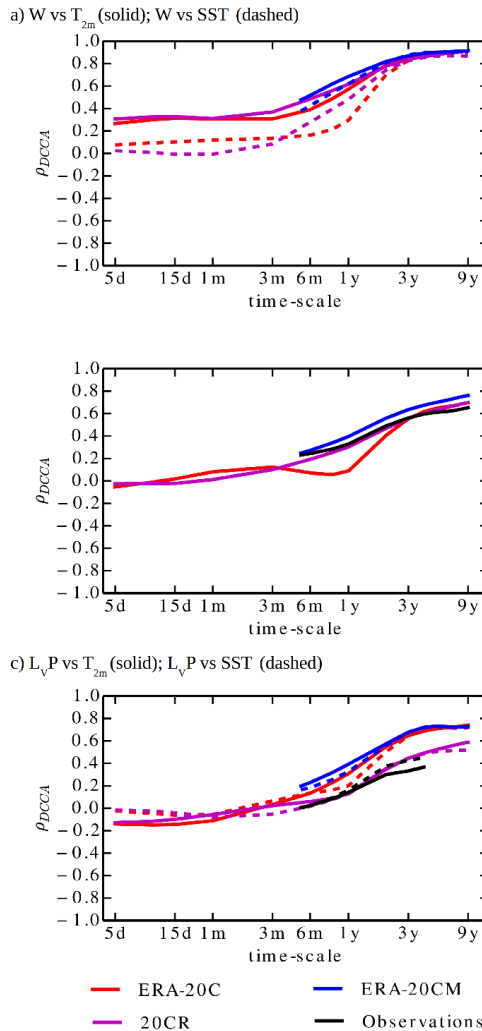
743 **Table 2.** Linear regression coefficient $\alpha_{P,DLR}$ estimated from $\Delta P/P_c$ against ΔDLR at 2-
 744 year resolution, assuming a relationship as given by Equation (11). The respective
 745 coefficients of determination are also provided. The $\alpha_{P,DLR}$ values are computed for ERA-
 746 20C, 20CR, and for the ensemble of ERA-20CM simulations. Additionally, the
 747 coefficient is estimated by pooling together all datasets, including GPCP observations
 748 against DLR from CERES-EBAF.

Dataset	$\alpha_{P,DLR} [(Wm^{-2})^{-1}]$	R^2
ERA-20C	0.005	0.88
20CR	0.003	0.51
E20CM (Ensemble)	0.004	0.81
All datasets (includes observations)	0.004	0.70

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751 **Table 3.** Linear regression coefficient $\alpha_{P,SST}$ estimated from $\Delta P/P_c$ against ΔSST at 2-
 752 year resolution. The respective coefficients of determination are also provided. The $\alpha_{P,SST}$
 753 values are computed for ERA-20C, 20CR, for the ensemble of ERA-20CM simulations,
 754 and for GPCP against ERA-20CM control SST forcing. Additionally, the coefficient is
 755 estimated by pooling together all datasets.

Dataset	$\alpha_{P,SST} [K^{-1}]$	R^2
ERA-20C	0.04	0.89
20CR	0.02	0.35
E20CM (Ensemble)	0.02	0.73
GPCP	0.04	0.42
All datasets (includes observations)	0.02	0.53



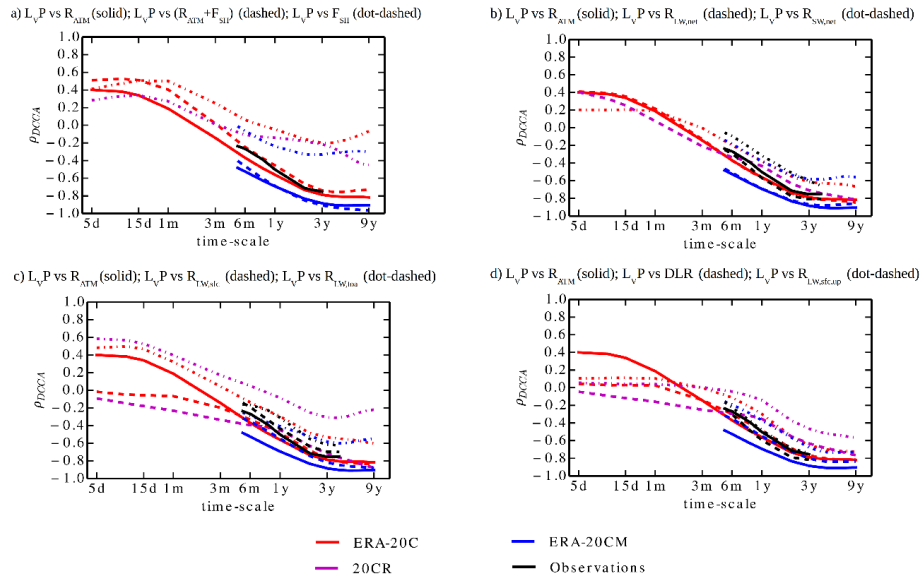
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757 **Figure 1.** DCCA cross-correlation coefficients against temporal scale computed for
 758 global-mean time-series of a) W vs T_{2m} (solid) and W vs SST (dashed); b) SST vs T_{land} ;
 759 and c) L_vP vs T_{2m} (solid) and L_vP vs SST (dashed). Red lines represent results from
 760 ERA-20C, blue lines are from ERA-20CM, pink lines are from 20CR and black lines are
 761 estimated from observational products. Notice that R_{atm} is not available from 20CR
 762 dataset, and that observational-based estimates of ρ_{P,T_S} (and $\rho_{P,SST}$) are only computed
 763 up to 4-year time-scales due to the limited duration of GPCP dataset.

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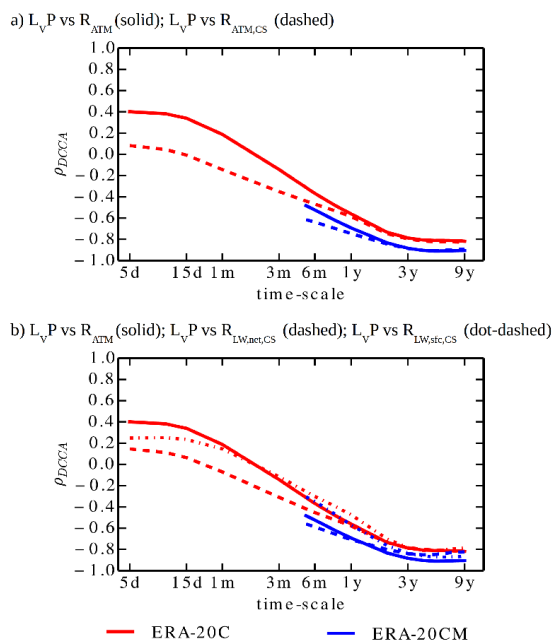
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768 **Figure 2.** DCCA cross-correlation coefficients against temporal scale computed for a)
 769 L_vP vs R_{atm} (solid), L_vP vs $(R_{atm} + F_{SH})$ (dashed) and L_vP vs F_{SH} (dot-dashed); b) L_vP
 770 vs R_{atm} (solid), L_vP vs $R_{LW,net}$ (dashed), and L_vP vs $R_{SW,net}$ (dot-dashed); c) L_vP vs
 771 R_{atm} (solid), L_vP vs $R_{LW,SFC}$ (dashed), and L_vP vs $R_{LW,TOA}$ (dot-dashed); and d) L_vP vs
 772 R_{atm} (solid), L_vP vs DLR (dashed), and L_vP vs $R_{LW,SFC,UP}$ (dot-dashed). Red lines are
 773 computed from ERA-20C, blue lines are from ERA-20CM, pink lines are from 20CR and
 774 black lines are computed from GPCP and CERES-EBAF observational products. Notice
 775 that R_{atm} and $R_{SW,net}$ are not available from 20CR, and that correlation coefficients
 776 estimated from observational products are only computed up to 4-year time-scales due to
 777 the limited duration of GPCP dataset.



778

779 **Figure 3.** DCCA cross-correlation coefficients against temporal scale computed for a)

780 $L_v P$ vs R_{atm} (solid) and $L_v P$ vs $R_{atm,CS}$ (dashed); b) $L_v P$ vs $R_{LW,SFC}$ (solid) and $L_v P$ vs

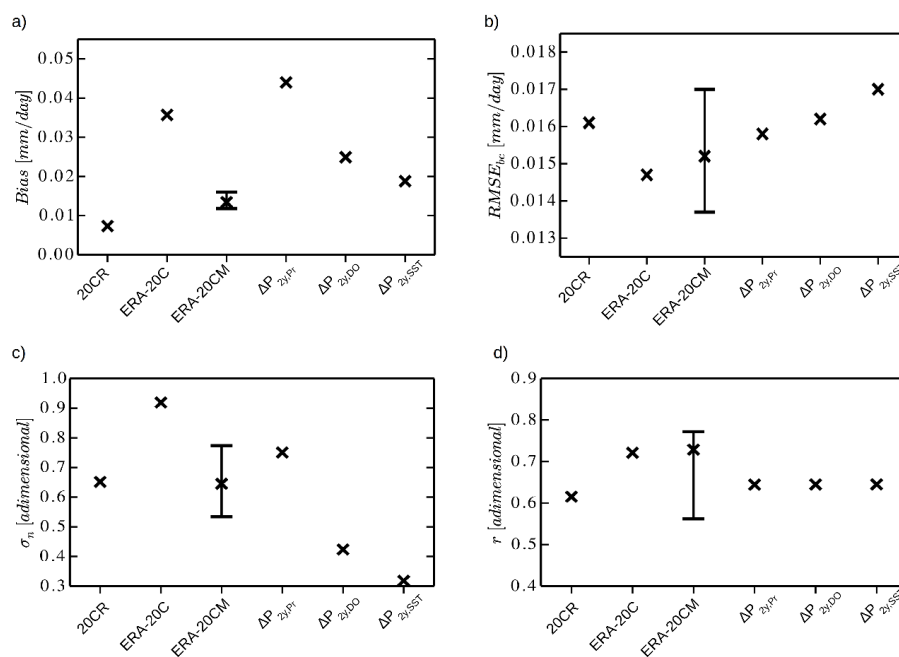
781 $R_{LW,SFC,CS}$ (dashed). Red lines are computed from ERA-20C and blue lines are from

782 ERA-20CM.

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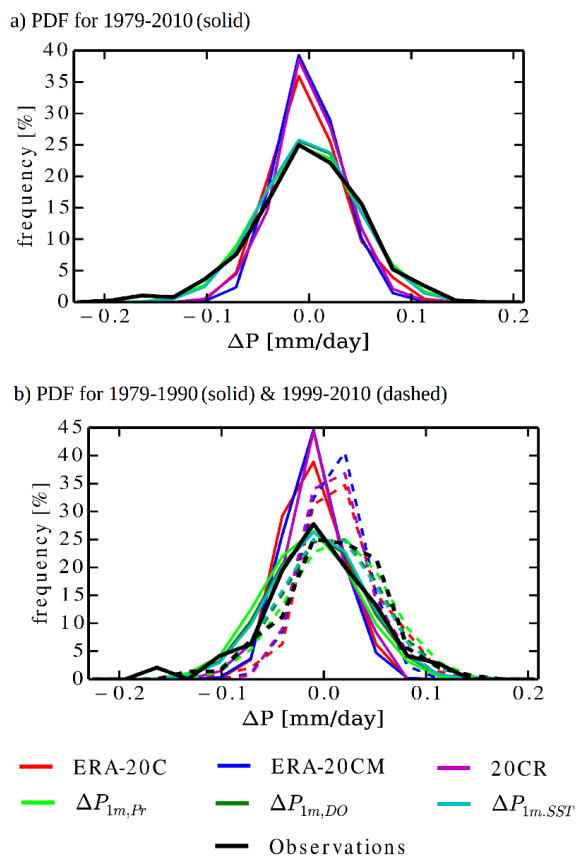
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786 **Figure 4.** Error estimates from simulated anomaly time-series for P at 2-year resolution
 787 against GPCP, computed for the 1979-2010 period, including a) mean bias (Bias); b) root-
 788 mean-square error after bias correction (RMSE_{bc}); c) model standard deviation
 789 normalized by observed standard deviation (σ_n); and d) Pearson correlation coefficient
 790 (r). For ERA-20CM dataset the range for all ensemble members is shown, while ‘x’ marks
 791 their mean value. The p-value for all correlations shown in panel (d) are <0.05.

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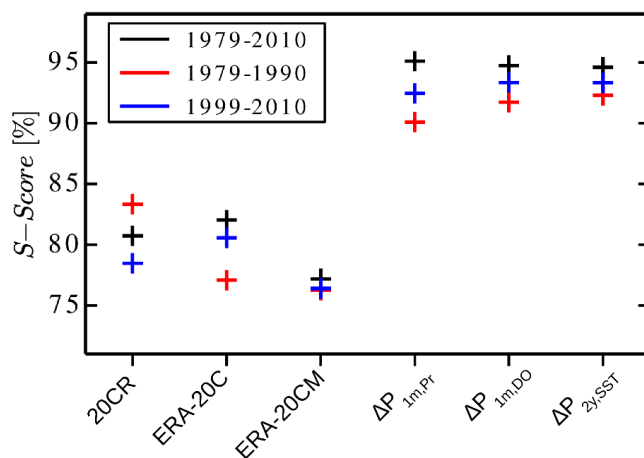


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794 **Figure 5.** PDFs estimated from monthly anomaly time-series for P from ERA-20C (red),
 795 ERA-20CM (dark blue), 20CR (pink), GPCP (black), $\Delta P_{1m,DO}$ (dark green), $\Delta P_{1m,Pr}$
 796 (light green), and $\Delta P_{1m,SST}$ (light blue). In panel a) the PDFs are estimated for the 1979-
 797 2010 period, and in panel b) the PDFs are estimated for the 1979-1990 period (solid) and
 798 the 1999-2010 period (dashed).

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802 **Figure 6.** S-Score computed from the different P simulations against GPCP. The values
803 estimated for the full satellite period (1979-2010) are presented in black, for the 1979-
804 1990 period are presented in red, and for 1990-2010 period are presented in blue. For
805 ERA-20CM dataset, the S-Score is estimated from the 10-member ensemble PDF.