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A Radiative Convective Model based on constrained Maximum Entropy Production

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Abstract. The representation of atmospheric convection induced by radiative forcing is a longstanding question mainly because turbulence plays a key role in the transport of energy as sensible heat, geopotential and latent heat. Recent works have tried to use Maximum Entropy Production as a closure hypothesis in Simple Climate Models in order to compute implicitly temperatures and vertical energy flux. However, these models failed to compute realistic profiles. To solve this problem, we prescribe a simplified 1D mass scheme transport which ensures energy fluxes. The later appears as a mechanical constraint which imposes the direction and/or limits the amplitudes of energy fluxes. This leads to a different MEP steady state which depends on the considered energy transfers in the model. Results using such model are improved with respect to another model, not including such effect: temperature and energy flux are closer to the observations and we naturally reproduce stratification when we consider geopotential. Variations of the atmospheric composition, such as doubling of the carbon dioxide concentration, is also investigated.

1 Introduction

Climate system is complex and usually divided in different components: atmosphere, ocean, cryosphere, lithosphere and biosphere (Peixoto and Oort , 1992). There are different approach to climate modelling (Randall et al. , 2007). We can classify them in three main classes. Global Climate Models (GCMs) are the more sophisticated ones (see (Dufresne et al. , 2013) for an example). They represent explicitly the circulation of atmosphere and ocean. Earth Models of Intermediate Complexity (EMICs) simulate the all Earth system with more simplifications than GCMs (see (Goosse et al. , 2010) for an example). These simplifications allows simulations over larger time periods, which is useful to study past climates. Simple Climate Models (SCMs) use only a few key processes to answers specific questions (see (Paillard , 1998) for an example).

Both complex and simple models have different strengths and weaknesses, and applications. GCMs are largely used to make climate projections for the next centuries. Since the numerical resolution of dynamical equations from the micro-scale (of order $\simeq 10^{-3}$ m for dissipation) to the scale of interest ($\simeq 10^{4}$ m) is still impossible, GCMs however need to represent sub-grid

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processes such as turbulent flows, convection or cloud's formation. To do it, models usually express the intensity of fluxes due to unresolved phenomena as a function of model resolved-variables. This approach needs a "turbulent closure" which usually requires the introduction of empirical parameters such as turbulent master length scale, turbulent velocity diffusion terms, ... (Mellor and Yamada , 1982), or the use of different quantities as Convective Available Potential Energy (CAPE) or Convective Inhibition (CIN) to fix the convective intensity (Yano et al. , 2013). These parametrizations change from a model to another, resulting in different predictions (Stevens and Bony , 2013). They also require adjusting the numerous free parameters ("tuning") in order to track observations (Hourdin et al. , 2017). Consequently, SCMs appear as an interesting alternative to GCMs, when considering past or future climate study over a larger periods like glacial–interglacial cycles. Indeed, an SCMs is an ensemble of a reasonable number of equations and physical quantities, providing an easier assessment of the impact of some parameters of the models (like the concentration of greenhouse gases for atmospheric models). Many SCMs are based on the idea that the computation of all the microscopic details looks unnecessary if we are interested on quantities at larger spatio-temporal scales. They use classical approaches system with high number of degrees of freedom to apply thermodynamics (or more formally statistical physics) to the Earth system (Lucarini et al. , 2014).

A lot of SCMs describe the Earth simply with energetic considerations (North et al. , 1981). Those models are called Energy Balance Models (EBMs). The atmosphere is mainly driven by radiative forcing: solar radiations give energy to the Earth which emits infra-red radiations to Space. This heating is not homogeneous around the globe for geometric reasons. It is more important for tropics than poles. This lead to the latitudinal heat transport. For the vertical axis, the amount of radiative energy absorbed by the Earth naturally depend on the atmospheric components. The ground usually receives more solar radiations because of its opacity. As a result the atmosphere is heated from the bottom which may lead to unstable situations where the temperature gradient exceeds the adiabatic gradient. Then, this causes atmospheric motion and vertical energy transport named convection. Since EBMs are usually based only on the energy budget, it is necessary to find an expression or a relation for the energy fluxes. It is called a closure hypothesis. So EBMs mainly differs by the choice of the representation fluxes ensured by the fluids layers of the Earth (atmosphere and ocean). For example, horizontal fluxes are usually represented by a purely diffusion terms (North et al. , 1981). On the other hand, vertical fluxes has been modeled using different approaches like convective adjustment. The later consists in computing the temperature profile at radiative equilibrium for stable regions and adjusting it where critical gradient is exceeded (Manabe and Strickler , 1964). Representing both horizontal and vertical energy fluxes is an important issue for EBMs. This concerns the subject of the present paper.

Since the seventies (Paltridge , 1975), Maximum Entropy Production (Martyushev and Seleznev , 2006) (MEP) is also used as closure hypothesis. This conjecture stipulates that the climatic system (or one of its component) optimizes its entropy production due to internal heat transfers. The physical basis and applicability of MEP to climate modeling (or another scientific field) is still subject to debate. However, the idea of describing the Earth from a purely thermodynamical point of view is appealing. As a consequence, some EBM's are still using MEP. This is the case both for horizontal fluxes (O'Brien and Stephens , 1995; Lorenz et al. , 2001), and vertical fluxes (Ozawa and Ohmura , 1997; Pujol and Fort , 2002). Former MEP based Models (MEPMs) can be criticized for three main reasons. One is the absence of dynamics and/or the validity of MEP (Rodgers , 1976). The second criticism deals with extra parametrizations and/or assumptions used in MEPMs. Indeed, one may asked ourself if the successes of the models are really due to MEP, or to tuning and/or others ingredients (Goody , 2007). The final criticism concerns the usually simplified description of the radiative forcing in these models. Recently, a MEPM overcoming the last two criticisms has been built by Herbert et al. (Herbert , 2012). It includes a refined description of the radiative budget in the Net Exchange Formalism, and without extra assumptions. The only adjustable parameters of these models concern the radiative budget (such as the albedo), and not the atmosphere or ocean energy fluxes. The model provides a relatively good approximation for the temperature and horizontal energy fluxes (Herbert et al. , 2011b).

However, the vertical energy fluxes are still overestimated in such model (Herbert et al. , 2013) when we compared to observations and conventional Radiative Convective Models (RCMs) like (Manabe and Strickler , 1964). Furthermore, the energy fluxes are not always oriented against the energy gradient and it does not predict stratification in the upper atmosphere. This is not surprising because the geopotential was not taken into account. Yet, we know from fluids mechanics that the gravity plays a major role for natural convection (Rieutord , 2015). Gravity is also obviously responsible of stratification in the upper atmosphere. In this paper, we develop an MPEM that describes more properly the atmospheric convection. In the same spirit as the previous SCMs, we do not attempt to resolve all mechanisms responsible of convection. So the dynamics is still absent of the model and we add only some keys features. Two ingredients are introduced to represent vertical energy fluxes more correctly. The first one is to assume that a mass transport scheme is responsible of the energy transport. This constitutes an introduction of a "mechanical constraint" into a MEPM. The second one is to consider different energy terms: sensible heat, geo-potential, and latent heat. We show that this simplified description of the energy transport, combined to MEP closure hypothesis, can lead to relatively realistic results.

The outline of this paper is as follows. In a first part, we describe our model, presenting the transport scheme and deriving the formulation of the constrained optimization problem (part 2). Then, we compute temperature, energy content and energy fluxes profiles. We give a physical interpretation of the effect of mechanical constraints emerging from our prescribed transport scheme. The impact of different expressions for energy is discussed (part 3). A sensitivity test for concentration of O_3 and CO_2 is also performed. Finally, we discuss further works and objectives (part 4). The resolution of optimisation problem is described in (annexe A) with some details of computations in (annexe B).

2 Model

2.1 Vertical structure of the atmosphere

In our model, we divide the atmosphere into a column of N vertical layers. We work with prescribed pressure levels, and also fix the CO_2, O_3 and water vapour profiles according to (McClatchey , 1972). The ground is represented by a layer with a fixed surface albedo α. The atmosphere is supposed to be in hydrostatic equilibrium and is considered as an ideal gas. The energy per unit mass in layer i, of mean elevation z_i, temperature T_i and moisture q_i is the so called moist static energy

$$e_i = C_pT_i + gz_i + Lq_i, \quad (1)$$

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where $C_p = 1005$ J.kg^{-1}.K^{-1} is the heat capacity of the air, $g = 9.81$ m.s^{-2} is the terrestrial acceleration of gravity and $L = 2,5.10^6$ J.kg^{-1} is the latent heat of vaporization.

We assume that vertical energy fluxes in the atmosphere can be represented with a purely convective mass transport (Mihelich , 2015): we note m_i^+ the upward mass flux leaving the layer $i-1$ and m_i^- the downward mass flux coming to the layer $i-1$. Without sources or sinks of mass, the conservation law recursively imposes $m_i^- = m_i^+ = m_i$ (cf figure 1). The exchanges are therefore due to "mixing" between adjacent layers (i.e without a net transport of mass). Then, the net upward energy flux between layers i and $i-1$ is

$$F_i = m_i^+ \, e_{i-1} - m_i^- \, e_i = -m_i \left(e_i - e_{i-1}\right), \tag{2}$$

Taking into account the net radiative energy budget $\mathcal{R}_i$, the energy balance at the stationary state for the layer i reads

$$F_i - F_{i+1} + \mathcal{R}_i = 0, \tag{3}$$

We use the radiative code developed in (Herbert et al. , 2013) to compute the net radiative energy budget $\mathcal{R}_i$. This model

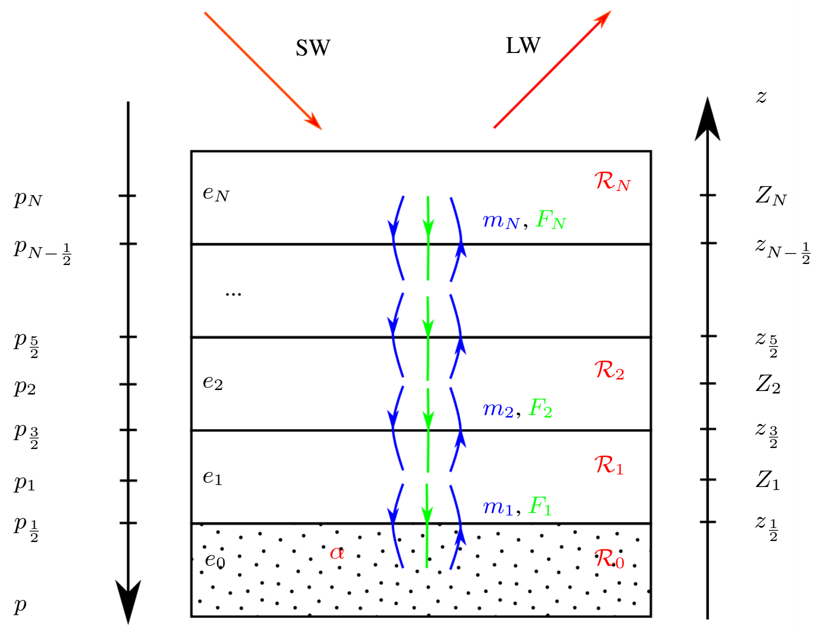

Figure 1. Discretization of an atmospheric column into N layers. The layer i, at temperature T_i, pressure P_i, elevation z_i and moisture q_i, has an energy per unit of mass $e_i = C_pT_i + gz_i + Lq_i$. m_i is the mass flux between layers $i-1$ and i which leads to the net energy flux F_i. $\mathcal{R}_i$ is the radiative flux convergence of energy between layers and space in the layer i. The ground is represented by layer 0 with fixed surface albedo α.

was developed to give a realistic description of the absorption properties of the more radiatively active constituents of the

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atmosphere while keeping a smooth dependence of the radiative flux with respect to temperature profile. As suggested by the authors, this last requirement is useful in the framework of a variational problem. The model is based on Net Exchange Formalism (Dufresne et al. , 2005), where the basic variables are the net exchange rates between each pair of layers instead of radiative fluxes. Radiative budget in layer i equal to solar short-wave radiations SW minus the long-wave radiations re-emitted LW:

$$\mathcal{R}_i\left(T,q,O_3,CO_2,\alpha\right) = SW_i\left(T,q,O_3,\alpha\right) - LW_i\left(T,q,CO_2\right). \tag{4}$$

It has a non local dependence with respect to the composition of the atmosphere and on the temperature profile. Clouds are not considered in the model. More details can be found in (Herbert et al. , 2013) and its supplementary material. In the previous equation and in the following F, T, q, O_3, CO_2 will refer to the all profiles (i.e $T = \{T_i\}_{i=0,\dots,N}$ etc). Given that q, O_3, CO_2 and α are fixed in our model, we will only indicate the T dependence.

2.2 Maximum Entropy Production with mechanical constraint

The principle of our model is to determine the fluxes F and temperatures T with the maximization of convective entropy production defined as

$$\sigma_s = \sum_{i=0}^{N} \frac{(F_i - F_{i+1})}{T_i}. \tag{5}$$

We can easily express the later with temperatures $\sigma_s(T)$ with the energy balance in stationary state $F_i - F_{i+1} + \mathcal{R}_i(T) = 0$. Then the problem is usually solved in term of temperature with a global constraint of energy conservation:

$$\left\{ \max_{T_0,\dots,T_N} \left(-\sum_{i=0}^{N} \frac{\mathcal{R}_i(T)}{T_i} \right) \quad \middle| \quad \sum_{i=0}^{N} \mathcal{R}_i(T) = 0 \right\}. \tag{6}$$

However, we now require that the optimisation of entropy production must respect constraints imposed by the mechanics. Namely, the energy transport must results from the mass motions. We thus impose the straightforward requirements of conservation of mass and the mass flux positivity $m \geq 0$. It is then natural to solve the problem in term of flux by expressing the entropy production $\sigma_s(F)$ and inequality constraints $m_i \geq 0$ in terms of energy flux. We assume that the relation $\mathcal{R}(T)$ is invertible (annexes A and B)

$$F_{i+1} - F_i = \mathcal{R}_i(T) \quad \Leftrightarrow \quad T_i = \mathcal{R}_i^{-1}(F). \tag{7}$$

Note that the conservation of mass and the energy balance are implicit since $m^+ = m^- = m$. The energy balance in stationary state can then be used to express σ_s in term of flux. This results in the following the optimisation problem with inequality constraints :

$$\left\{ \max_{F_1,\dots,F_N} \left(\sum_{i=0}^{N} \frac{F_i - F_{i+1}}{\mathcal{R}_i^{-1}(F)} \right) \quad \middle| \quad \exists \quad m_i \geq 0 \quad \text{with} \quad F_i = -m_i\left(e_i - e_{i-1}\right) \right\}, \tag{8}$$

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The condition $F_i = -m_i\,(e_i - e_{i-1})$ with $m_i \geq 0$ will be called mechanical constraint. The later simply imposes the energy fluxes to be opposed to the energy gradient. We stress that this is an implicit consequence of the choice of a conservative 1D mass scheme transport. It does not hold for more complex scheme.

3 Results

We have computed temperature, energy content and convective energy flux profiles for different prescribed atmospheric compositions (McClatchey , 1972) corresponding to tropical, mid-latitude summer, mid-latitude winter, sub-arctic summer and sub-arctic winter conditions. We work with fixed relative humidity profile (ratio of the partial pressure of water vapour to the equilibrium vapour pressure of water at a given temperature). Representative values of surface albedo are used: $\alpha = 0.1$ for tropical and mid-latitude conditions and $\alpha = 0.6$ for sub-arctic ones. The atmosphere is discretized in $N = 20$ vertical levels.

3.1 The effect of the constraint

We investigate the effect of the following energy terms on the mechanical constraint:

- Sensible heat C_pT;
- Geo-potential gz;
- Latent heat for a saturated moisture profile $Lq_s(T)$, where q_s is the absolute ratio moisture at saturation. It depends only on temperature (since pressure is prescribed). This is a first attempt to take into account the effect of humidity without an explicit derivation of the humidity profile and water cycle. However, the radiative budget is still computed using fixed standard relative humidity profile.

For illustration purpose, consider the case with only 2 layers. We note $F = m\ (e_1 - e_2)$ the net energy flux from the layer 1 to the layer 2, where m is the mass flux between layers (cf. figure 2). In this simple case, the entropy production writes $\sigma_s = F\,(1/T_2 - 1/T_1)$ and is limited by the constraint $m \geq 0 \Longleftrightarrow F\ (e_1 - e_2) \geq 0$. We interpret the different energy terms on this constraint as follow:

- $e = C_pT$: $F \geq 0$ if $T_1 \geq T_2$. The constraint simply imposes energy transport from hot to cold regions.
- $e = C_pT + gz$: $F \geq 0$ if $T_1 \geq T_2 + g\ (z_2 - z_1)/C_p$: the geopotential gz limits the upward energy flux. We predict a warmer air at the bottom and a colder air at the top compared to the model with only sensible heat.
- $e = C_pT + gz + Lq_s$: $F \geq 0$ if $T_1 \geq T_2 + [g\ (z_2 - z_1) + L\ (q_s(T_2) - q_s(T_1))]\,/C_p$. Since $q_s(T)$ is an increasing function, $q_s(T_2) - q_s(T_1)$ has the same sign as $T_2 - T_1$. The temperature gradient is usually negative (i.e $T_2 \leq T_1$). Adding the latent energy at saturation makes the transport of energy to the top easier. Consequently, the atmospheric temperature gradient weakens.

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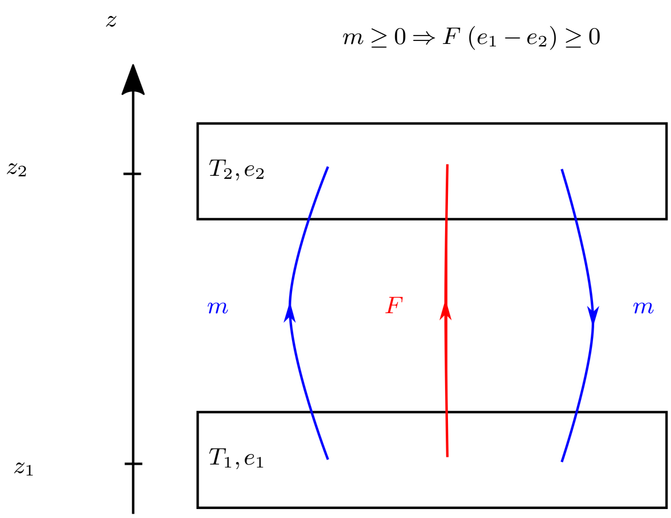

Figure 2. Convective energy exchanges between 2 layers of elevation z_1 and z_2 ($z_2 \geq z_1$), temperatures T_1 et T_2, and energy per unit mass e_1 and e_2. We note $F = m\,(e_1 - e_2)$ the net convective energy flux from the layer 1 to the layer 2, where m is the mass flux between the two layers.

3.2 General remarks

We now consider the general case with $N = 20$ vertical levels. Various profiles are shown for tropical (figure 3) and sub-arctic winter (figure 4) conditions. The "unconstrained" model of (Herbert et al. , 2013) is labelled as "NO" (rigorously speaking, the model is constrained by the global conservation of energy, but we will refer to it as "unconstrained" since no constraints are imposed on fluxes), and compared with constrained models with different energy terms ($e = C_pT$, $e = C_pT + gz$ and $e = C_pT + gz + Lq_s$). Reference temperature profiles (McClatchey , 1972) are also represented for comparison.

For the unconstrained model, the energy flux is positive for two conditions in all the column despite the energy gradient inversion in the upper layers of atmosphere. Therefore, the flux is opposed to the energy gradient in this region. This also corresponds to local negative entropy production. Consequently the upward flux is overestimated, and temperature gradient is weak. As discussed above (part 2.1), the addition of mechanical constraints imposes flux to be opposed to the energy gradient. Constrained model with all energy terms ($e = C_pT$, $e = C_pT + gz$ or $e = C_pT + gz + Lq_s$) indeed respect this condition. When we consider the geopotential term (i.e $e = C_pT + gz$ or $e = C_pT + gz + Lq_s$), we observe:

- An energy profile divided in 3 regions:

  1. An unstable surface layer with an decreasing energy profile;
  2. A neutral (slightly stable) mixed layer in the middle atmosphere with a vanishing energy gradient;

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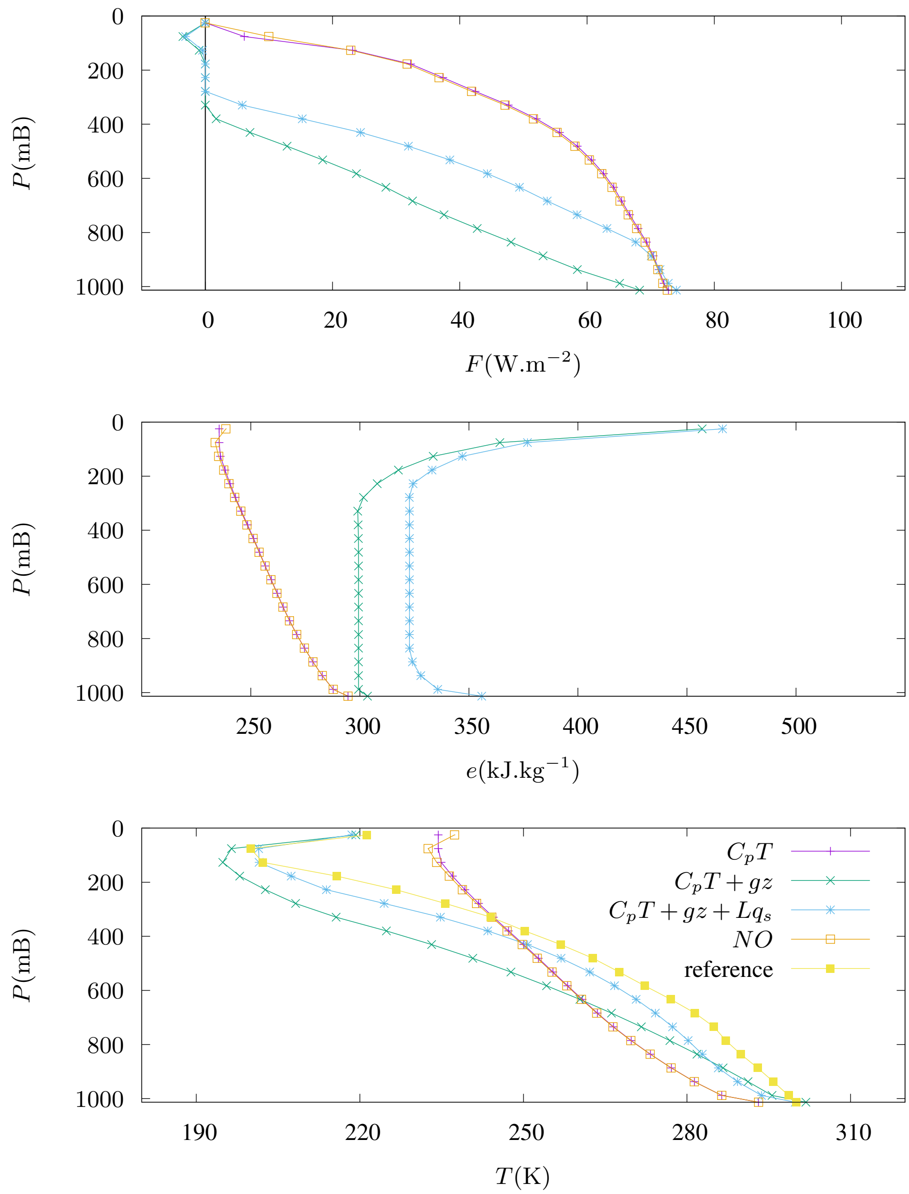

Figure 3. Energy per unit mass e, convective energy flux F and temperature T for tropical conditions and different expressions for energy on the mechanical constraint ($e = C_pT$, $e = C_pT + gz$ and $e = C_pT + gz + Lq_s$). The elevation is given with pressure level P. Results for the unconstrained model (Herbert et al. , 2013), labelled as "NO", are shown. We also give the reference temperature profile corresponding to meteorological observations (McClatchey , 1972) for qualitative comparison.

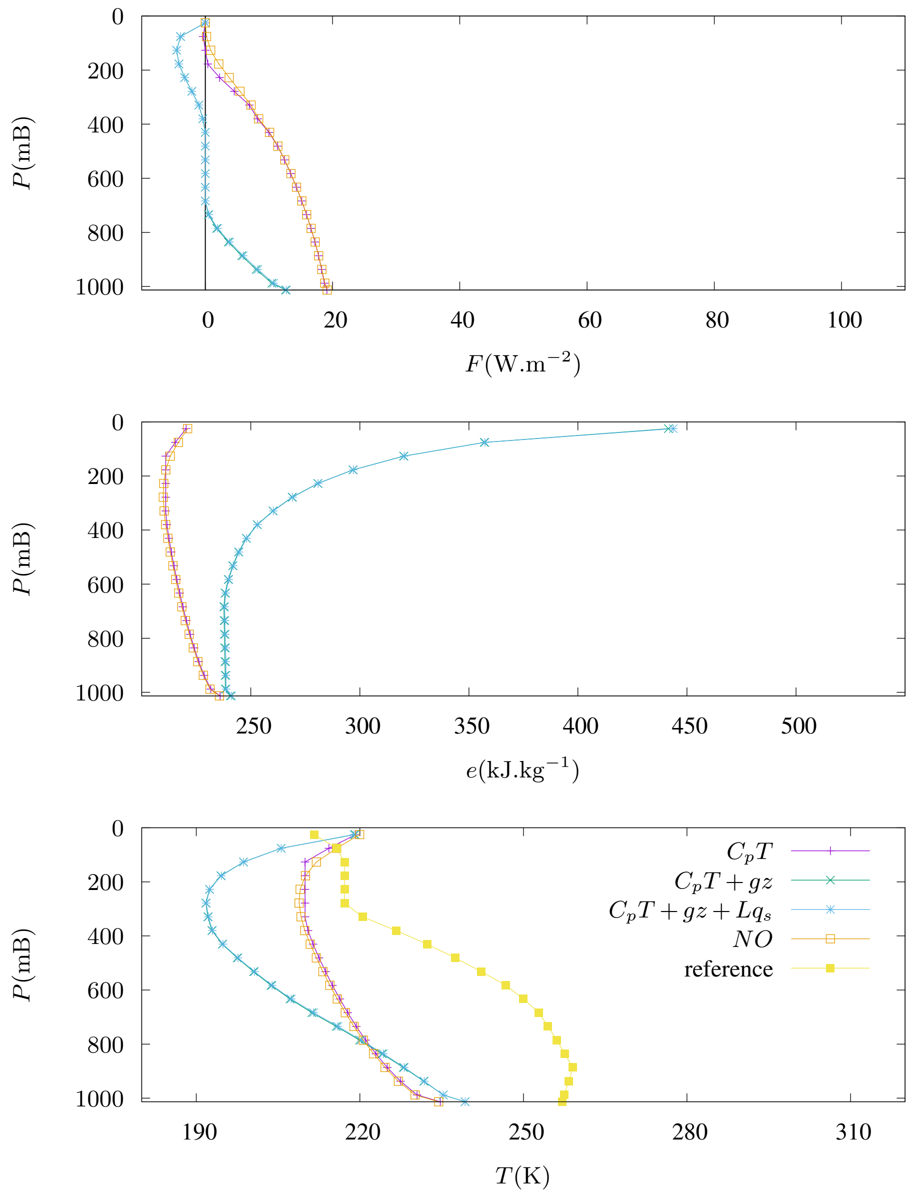

Figure 4. Energy per unit mass e, convective energy flux F and temperature T for sub-arctic winter conditions and different expressions for energy on the mechanical constraint ($e = C_pT$, $e = C_pT + gz$ and $e = C_pT + gz + Lq_s$). The elevation is given with pressure level P. Results for the unconstrained model (Herbert et al. , 2013), labelled as "NO", are shown. The reference temperature profile corresponding to meteorological observations (McClatchey , 1972) is also provided for qualitative comparison.

3. An inversion layer at the top of the atmosphere where the energy is increasing with altitude.

– A vanishing convective energy flux in the upper part of the profile (around $P \simeq 300$ mB for tropics and $P \simeq 700$ mB for sub-arctic winter). This correspond to the stratification except for a small negative downward flux that will be discussed.

We note the thermal gradient is divided roughly by a factor of 2 when we consider $e = C_pT + gz$ or $e = C_pT + gz + Lq_s$, which gives a more realistic temperature profile.

3.3 Comparison between profiles

3.3.1 Model outputs for different climatic conditions

The constrained model is obviously sensitive to the water content: $e = C_pT + gz$ or $e = C_pT + gz + Lq_s$ gives approximately the same results for sub-arctic winter conditions (since $q_s(T)$ is weak for low temperatures) while predictions differ for tropical conditions (where T and then $q_s(T)$ are more important). The influence of the surface albedo explains the large part of temperature's modification when we compare different climatic conditions.

3.3.2 Model output vs reference profile

Before discussing the differences between model results and observations, we note that this conceptual model does not take into account some important physical processes:

– Insolation is assumed to be constant, fixed at $1368/4$ W.m^{-2} for all conditions. But in reality, it varies with respect to seasons, diurnal cycle and latitude due to the Earth's obliquity. So, the model does not take into account the variation of radiative budget because of this geometrical factors;

– Horizontal energy flux are not considered in this 1D description;

– No thermal capacity is taken into account;

– The effect of clouds, that plays an important role for the absorption and emission of radiations (Dufresne and Bony , 2008), is not implemented in the radiative code.

Therefore, the aim of this study is not to give realistic values of temperature profiles nor vertical energy fluxes, but to give a qualitative evaluation of the model. However, we can make some remarks when we compare our results to reference temperature profiles. We observe that our model with $e = C_pT + gz + Lq_s$ provides better results for tropical conditions (figure 3) whereas the computed profiles are not so good for sub-arctic winter conditions (figure 4). Considering the previous remarks, one can explain the gap between our model and observations as follow. A constant insolation is valid for tropics, but it varies strongly with time for high latitudes. So, our model is not adapted to represent sub-arctic winter conditions. If we only take into account this effect, our model must predict warmer temperatures (because there is much less insolation in high latitudes).

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However the modification of albedo and over defects of the model (see below) explain why we obtain colder temperatures. Tropical regions are submitted to strong vertical motion due to radiative heating. Horizontal energy fluxes are less important and the 1D description may be more adapted to this case. In contrast, radiative heating is less important for arctic (specially in winter), so convection is weaker. Then horizontal energy fluxes are more important for the description of high latitudes since they play a major role for the heat transport from hot equatorial regions to cold poles. This probably explains why we underestimate temperature for high latitudes (figure 4).

3.4 Sensitivity to atmospheric composition

3.4.1 Ozone

When we do not take into account the influence of O_3 on the radiative budget, we observe small changes for energy content and large changes for temperature in the stratosphere (cf figure 5). Indeed, O_3 absorbs solar radiation at the top of the atmosphere which induces an heating of this region. It follows that the temperature in the high atmosphere is more important with O_3, and we even observe an inversion of the temperature gradient. It follows that less solar radiation heats the ground, resulting in a slightly smaller surface temperature. This effect is only due to the radiative model (Herbert et al. , 2013). In the constrained model with geo-potential including ozone influence, a downward convective energy flux at the top of the atmosphere appears.Ozone is therefore associated to heating from the top with an inversion of temperature gradient and downward energy fluxes in the high atmosphere. This last effect only appears when both geo-potential energy and O_3 are taken into account. When geo-potential is not considered, upward energy flux is so overestimated that the effect is undetectable (figures 3, 4).

3.4.2 Carbon dioxide

We also have performed the classic experiment of doubling CO_2 concentration (Randall et al. , 2007). The climate sensitivity, defined as the surface temperature differences between computations with $[co2] = 560$ ppm and $[co2] = 280$ ppm, is reported for the different conditions in table 1.
Conventional models usually represent various processes like water vapour, ice-albedo, lapse rate and clouds feedbacks. They play an important role in amplifying the climate sensitivity. While comparing our values with the literature, we must keep in mind that our model does not represent all those feedbacks. The lapse rate feedback is taken into account. Water vapour feedback is partially represented in a crude way by fixing relative humidity (changes in temperature have an impact on water content and change the radiative budget), but there is no explicit representation of the hydrological cycle. It is technically possible to include the ice-albedo feedback in a MEPM (Herbert et al. , 2011a), but it is not the case here. Clouds are not represented in the model. So we will focus on comparison between constrained and unconstrained model using the same radiative scheme. Nevertheless, typical values of climate sensitivity for multi models averages with only relevant feedbacks are given (Dufresne and Bony , 2008). Fixed absolute water profile outputs are compared to references models with only lapse rate feedback while the fixed relative humidity ones are compared to references models with both lapse rate and water vapour feedbacks.

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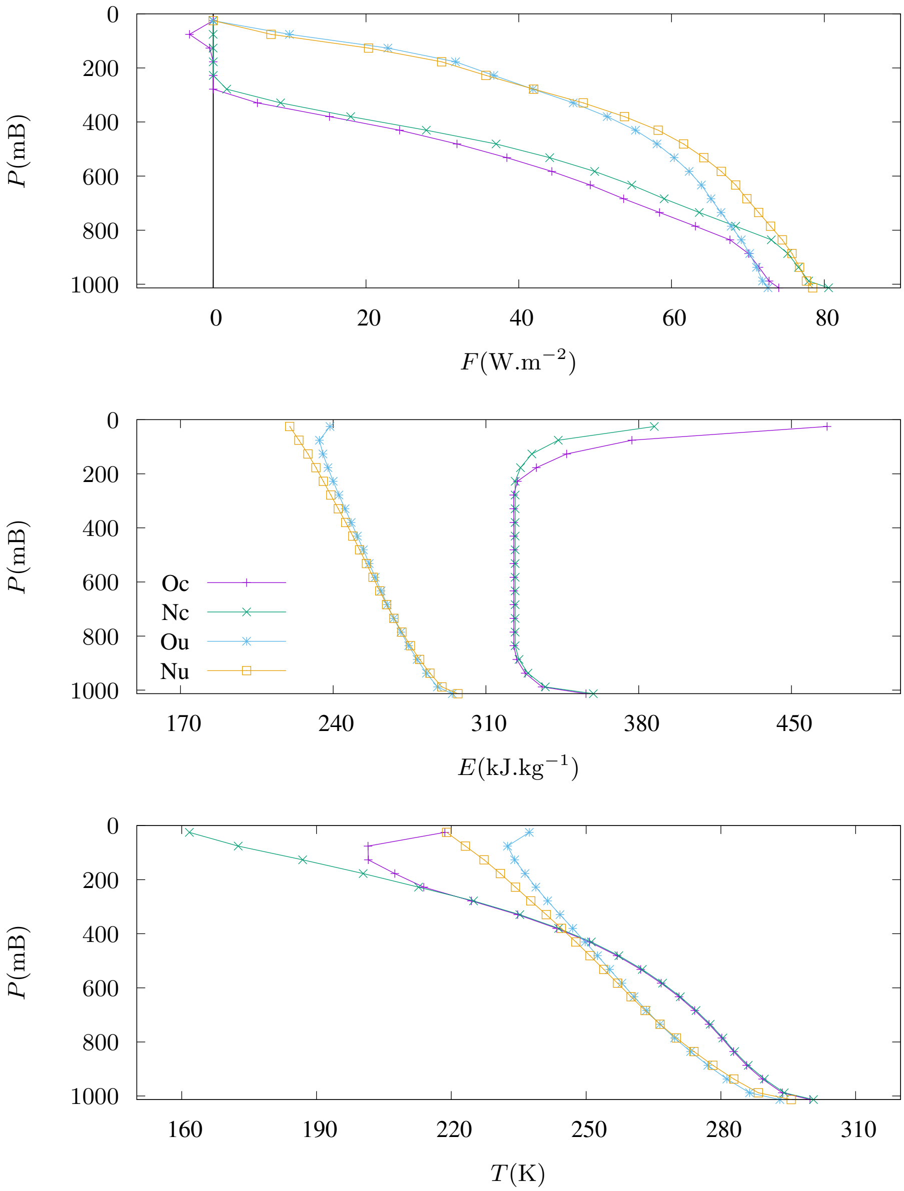

Figure 5. Temperature T, energy per unit mass e and convective energy flux F with ozone and without ozone are represented for constrained ($e = C_pT + gz + Lq_s$) and unconstrained cases in tropical conditions and $[co2] = 280$ ppm. Oc : with O_3, constrained. Nc : without O_3, constrained. Ou : with O_3, unconstrained. Nu : without O_3, unconstrained.

Table 1. Climate sentivity (warming of surface for $2\times CO_2$ compared to $1\times CO_2$) in K of the constrained model (with $e = C_pT + gz + Lq_s$), unconstrained model (Herbert et al. , 2013), and litterature (Dufresne and Bony , 2008). We give the values for different climatic conditions, and for fixed absolute or relative water vapor profiles.

Moisture	Atmospheric composition (McClatchey , 1972)	Surface albedo	Unconstrained model (K) (Herbert et al. , 2013)	Constrained model (K)	Literature (K) (Dufresne and Bony , 2008)
Absolute	Tropical	0.1	0.90	1.06	0.4 ± 0.3
	Mid-latitude summer	0.1	0.79	0.93	
	Mid-latitude winter	0.1	0.46	0.24	
	Sub-arctic summer	0.6	0.53	1.43	
	Sub-arctic winter	0.6	0.20	0.31	
Relative	Tropical	0.1	1.04	1.60	2.1 ± 0.2
	Mid-latitude summer	0.1	0.97	1.30	
	Mid-latitude winter	0.1	0.82	1.19	
	Sub-arctic summer	0.6	0.15	0.39	
	Sub-arctic winter	0.6	0.09	0.19	

Note that sensitivity values computed here for the unconstrained model differ from (Herbert et al. , 2013). We have checked that it is only a discretization effect ($N = 20$ atmospheric layers here instead of $N = 9$ in (Herbert et al. , 2013)). The climate sensitivity is higher for the constrained model than the unconstrained one (despite one exception with fix absolute moisture for Mid-latitude winter). This result may be interpreted as follow. If we start with a radiative forcing induced by a CO_2 doubling, it is the same for the two model since we fix the same atmospheric composition and surface albedo. However, upward energy fluxes are limited for the constrained case which induces a more important warming of the lower part of the atmosphere. The induced ground temperature elevation is therefore more important for the constrained case. This effect is observable (figure 6) when we look at the perturbation of flux, energy and temperature. The constrained model provides more realistic values of sensitivity for fixed absolute humidity, particularly for tropics. Indeed, the sensitivity 1.60 K computed in this case is closer to the literature values (Dufresne and Bony , 2008) 2.1 ± 0.2 K than the unconstrained model.

4 Discussion

MEPMs are different from the usual GCMs or EMICs. Generally, atmospheric models are based on:

1. Kinematics: equations describing how the fluid moves.
2. Dynamics: equations describing why the fluid moves. They are based on Navier-Stokes equations linking the fluid acceleration to the forces.
3. Thermodynamics: energy budget equation involving dissipation, radiative budget, phases changes, ...

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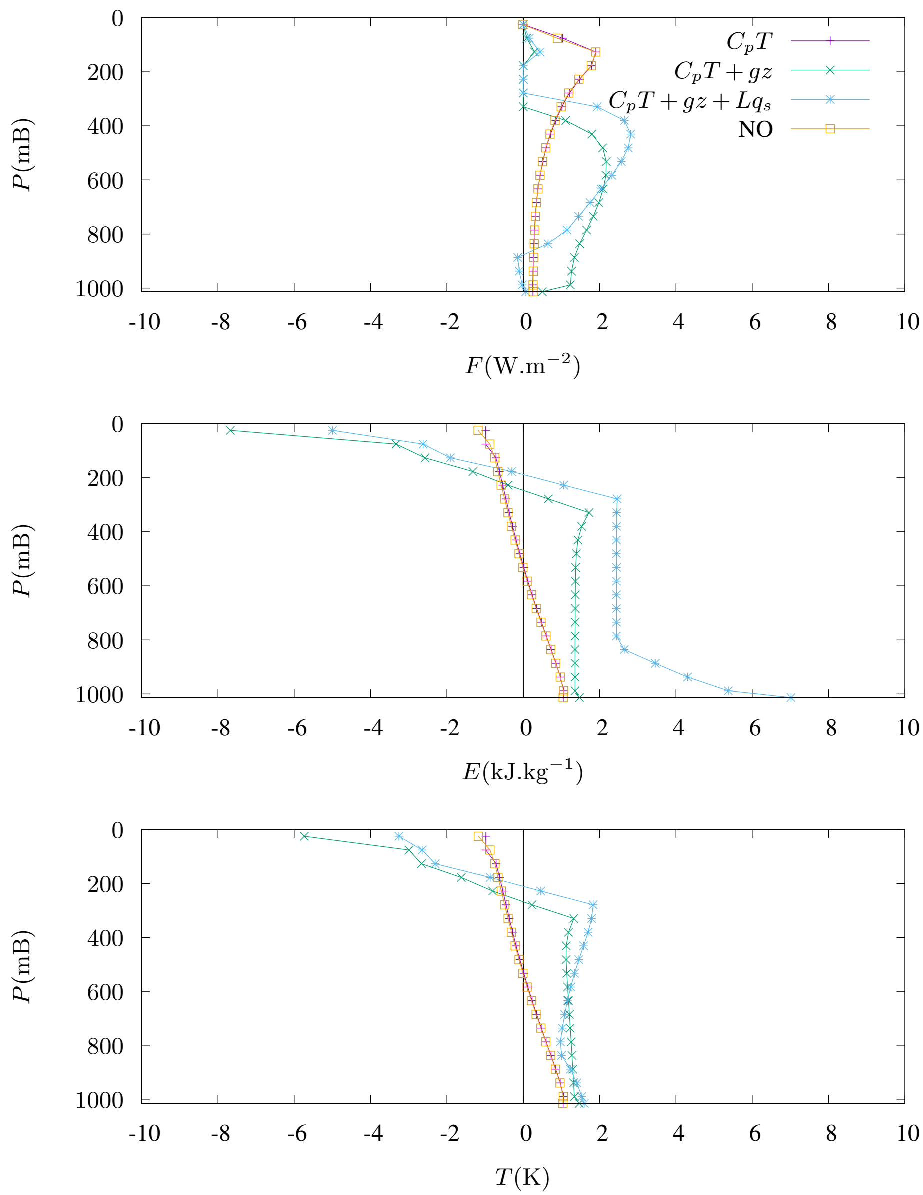

Figure 6. Differences of temperature T, energy per unit mass e and convective energy flux F between $[co2] = 560$ ppm and $[co2] = 280$ ppm in tropical conditions are represented for constrained ($e = C_pT$, $e = C_pT + gz$ and $e = C_pT + gz + Lq_s$) and "unconstrained" (labelled as "NO") models (Herbert et al. , 2013).

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4. A state equation like perfect gas relation, or approximations like hydrostatic to simplify the problem.

5. Closure hypothesis and/or parametrizations to represent sub-grid processes.

Then energy fluxes are then obtained after the computation of the velocity, and other fields.

Former MEPMs only use energy thermodynamics (i.e energy budget) and closure hypothesis to compute energy fluxes. Neither kinematics nor dynamics are taken into account. This paper provides a first attempt to introduce kinematics by fixing a simple conservative mass scheme transport. However, energy fluxes are still imposed by MEP. The later step is based on a really strong assumption and its validity must be investigated.

MEP may be viewed as a statistical inference (Jaynes , 1957; Dewar , 2003, 2009) (i.e draw a prediction with a partial knowledge which are here the radiative budget and the mechanics here). There are recent attempts to link MEP to others variational principles for dynamical systems/non equilibrium statistical physics like the maximum of Kolmogorov-Sinaï entropy (Mihelich , 2015). Similar methods like the maximization of dynamical entropy or "Maximum Caliber" (Monthus , 2010; Dixit et al. , 2018) may be also be relevent.

The gap between model and observations are easily understood, and explains why our 1D description with constant insolation is more adapted for tropics than arctic conditions. Further improvements are needed to solve these problems. Firstly, a more general 2 or 3 dimensional mass scheme transport is required. Secondly, it is formally possible to compute wind (Karkar and Paillard , 2015), moisture profiles, humidity fluxes using MEP. Finally, we have to include a time dependence in our model as a seasonal or diurnal cycles. The long term objective is to constructed an robust SCM, with as little adjustable parameters as possible.

5 Conclusions

We have investigated the possibility of computing the vertical energy fluxes and temperature in the atmosphere using the MEP closure hypothesis into a simple thermodynamical model. The energy fluxes are then computed in an implicit way, which avoids tuning parameters.

Former models (Ozawa and Ohmura , 1997; Pujol and Fort , 2002; Herbert et al. , 2013) failed to reproduce realistic energy fluxes and temperature on the vertical (Pascale et al. , 2012). This is due to the fact that some key ingredients was not considered. In our model, we assume that a mass scheme is responsible of energy transport. This can be seen a first attempt to introduce mechanics in MEP based models. We show that such hypothesis results in a better energetic description of convection. Different energy terms can be considered: sensible heat, geo-potential and latent heat for a saturated profile. The improved model allows to give more realistic temperature, energy content, and energy fluxes profiles, without any adjustable parameter. In particular, considering geo-potential leads to stratification in the upper atmosphere and allows us to reproduce a temperature gradient closer to the observed one.

We have investigated the sensitivity of the model when atmosphere's composition is modified. The results was compared to the

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literature. Our model is more sensitive to CO_2 concentration elevation because of geo-potential limits upward energy fluxes. We hope that the present model may be helpful to construct SCMs with a reduced number of adjustable parameters in order to study past climates other long time periods.

Appendix A: Resolution

In order to solve the optimization problem (8), we express it in Lagrangian formalism, assuming strong duality holds Boyd and Vandenberghe (2004). We therefore search the critical points of the Lagrangian associated to this problem

$$\mathcal{L} = \sigma_s - \sum_{i=1}^{N} \mu_i\, m_i \qquad \text{with} \qquad \begin{cases} m_i \geq 0 \\ \mu_i \geq 0 \end{cases} \qquad \text{and} \qquad \mu_i\, m_i = 0 \qquad i = 1, ..., N. \tag{A1}$$

Where $\mu_1, ..., \mu_N$ are Lagrange multipliers associated to the mechanical constraint (mass flux positivity). In order to express the problem in term of energy flux F, we must express the inverse temperature $X = 1/T$ and the mass flux m in term of F.

A1 Flux-temperature relation

The energy balance equation in stationary state can be written as follow

$$\mathcal{R}_i(X) + F_i - F_{i+1} = 0. \tag{A2}$$

We linearise the radiative budget $\mathcal{R}_i(X)$ around a reference temperature profile X^0 :

$$\mathcal{R}_i(X) \simeq \mathcal{R}_i(X^0) + \sum_{j=0}^{N} R_{ij}\, (X_j - X_j^0), \tag{A3}$$

where R is an invertible squared matrix of size N and $\mathcal{R}(X^0)$ is the radiative budget for the profile X^0. If we assume R to be invertible, we obtain a linear relation between the energy flux and the temperature profile

$$X_i = X_i^0 - \sum_{j=0}^{N} R_{ij}^{-1}\, \left(F_j - F_{j+1} + \mathcal{R}_j(X^0)\right). \tag{A4}$$

A2 Flux-mass relation

We first consider the dry static energy $e^d = C_p T + gz$. Considering the atmosphere is an ideal gas at hydrostatic equilibrium, and for prescribed pressure levels, layer volume depends only on temperature. Therefore, the elevation of a layer is a function of temperatures of layers below only and we can express the energy of a layer as (see Annexe B)

$$e_i^d = \sum_{k=0}^{N} E_{ik}^d\, T_k, \tag{A5}$$

where E^d is a squared triangular matrix of size N. If we linearise around X^0, one obtain

$$e_i^d \simeq \sum_{k=0}^{N} \frac{E_{ik}^d}{X_k^{0\,2}}(2X_k^0 - X_k). \tag{A6}$$

Using equation (A4), it gives the expression of energy in term of flux

$$e_i^d(F) \simeq \sum_{k=0}^{N} \frac{E_{ik}^d}{X_k^{0\,2}} \left(X_k^0 + \sum_{j=0}^{N} R_{kj}^{-1} \; (F_j - F_{j+1} + \mathcal{R}_j(X^0)) \right). \tag{A7}$$

We also can take into account the moist static heat of saturated atmosphere. The absolute ratio moisture at saturation q_s depends only of temperature T (in K) and pressure p (in Pa). It is given by the Bolton equation Bolton (1980):

$$q_s(T,p) = \frac{h_s(T)}{p - h_s(T)} \qquad \text{with} \qquad h_s(T) = 6.112 \exp\left(\frac{17.62(T - 273.15)}{T - 30.03} \right). \tag{A8}$$

where h_s is the mixture's saturation vapour pressure (in Pa). At fixed pressure, the moist static energy at saturation $e^s = C_pT + gz + Lq_s$ of layers is only function of temperatures. If we linearise around the profile X^0, we obtain

$$e_i^s = \sum_{k \in V} \frac{E_{ik}^s}{X_k^{0\,2}}(2X_k^0 - X_k) \qquad \text{with} \qquad E_{ik}^s = E_{ik}^d + L \left. \frac{\partial q_s}{\partial X} \right|_{X_k^0} \delta_{ik}, \tag{A9}$$

where δ_{ik} is the Kronecker symbol. Then, we can use the same reasoning as for the dry static heat and replace the matrix E^d by E^s if we want to consider the effect of latent energy for a saturated moisture profile. However, radiative budget is still computed with reference water vapour profiles. In the following, e can represent e^d or e^s

A3 Constrain

By multiplying both side of

$$F_i = -m_i \left(e_i - e_{i-1} \right) \tag{A10}$$

by $(e_i - e_{i-1})$, we obtain

$$F_i \left(e_i - e_{i-1} \right) = -m_i \left(e_i - e_{i-1} \right)^2 \tag{A11}$$

So the mechanical constrain $m_i \geq 0$ is equivalent to

$$\alpha_i(F) \equiv -F_i \; \left(e_i(F) - e_{i-1}(F) \right) \geq 0, \tag{A12}$$

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A4 Associated Lagrangien in flux space

Using the constrain (A12) and the linearised energy budget (A4), the problem (8) is supposed to be equivalent to research of critical points of the following Lagrangian

$$\begin{aligned}\mathcal{L}(F,\mu) &= \sigma_s(F) - \sum_{i=1}^{N} \mu_i\ \alpha_i(F)\\ &= \sum_{i=0}^{N} X_i\ (F_i - F_{i+1}) - \sum_{i=1}^{N} \mu_i\ \alpha_i \qquad \text{(A13)}\\ &\simeq \sum_{i=0}^{N} \left(X_i^0 - \sum_{j=0}^{N} R_{ij}^{-1}\ (F_j - F_{j+1} + \mathcal{R}_j(X^0)) \right) (F_i - F_{i+1}) - \sum_{i=1}^{N} \mu_i\ \alpha_i(F) \qquad \text{(A14)}\end{aligned}$$

while respecting the Karush-Kuhn-Tucker (KKT) conditions

$$\frac{\partial \mathcal{L}}{\partial F_i} = 0 \quad \text{with} \quad \begin{cases} \alpha_i(F) \geq 0 \\ \mu_i \geq 0 \end{cases} \quad \text{and} \quad \mu_i\ \alpha_i(F) = 0 \quad i = 1, ..., N. \qquad \text{(A15)}$$

The problem is solved by using an Interior point method Boyd and Vandenberghe (2004) with random initial temperature profile.

Appendix B: Computing geo-potential

In order to express the dry static energy $e_i^d = C_p T_i + g z_i$ in term of temperatures, we need to compute $z_i(T)$. We have

$$z_i = z_i - z_{i-\frac{1}{2}} + \sum_{j=1}^{i-1} \Delta z_j \qquad \text{(B1)}$$

where $\Delta z_j = z_{j+\frac{1}{2}} - z_{j-\frac{1}{2}}$ is the height of the layer j (cf figure 1). So, if ρ is the density of the air, R is the specific air constant and we assume the atmosphere is an ideal gas at hydrostatic equilibrium

$$g\Delta z_j = \int_{z_{j-\frac{1}{2}}}^{z_{j+\frac{1}{2}}} g\ \mathrm{d}z = -\int_{p_{j-\frac{1}{2}}}^{p_{j+\frac{1}{2}}} \frac{\mathrm{d}p}{\rho} = -\int_{p_{j-\frac{1}{2}}}^{p_{j+\frac{1}{2}}} RT\ \frac{\mathrm{d}p}{p}. \qquad \text{(B2)}$$

Then, we can compute the mean elevation of a layer with two possible prescriptions :

– Isothermal layers ($T = T_j$ in the integrand) :

$$g\Delta z_j = RT_j \ln\left(\frac{p_{j-\frac{1}{2}}}{p_{j+\frac{1}{2}}}\right). \qquad \text{(B3)}$$

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So the geopotential reads

$$gz_i = R\left[T_i \ln\left(\frac{p_{i-\frac{1}{2}}}{p_i}\right) + \sum_{j=1}^{i-1} T_j \ln\left(\frac{p_{j-\frac{1}{2}}}{p_{j+\frac{1}{2}}}\right)\right]. \tag{B4}$$

– Dry isentropic layers ($T = T_j\left(\frac{p}{p_j}\right)^{\frac{R}{C_p}}$ in the integrand):

$$g\Delta z_j = C_p T_j \left[\left(\frac{p_{j-\frac{1}{2}}}{p_j}\right)^{\frac{R}{C_p}} - \left(\frac{p_{j+\frac{1}{2}}}{p_j}\right)^{\frac{R}{C_p}}\right]. \tag{B5}$$

So the geo-potential reads

$$gz_i = C_p\left[T_i\left(\left(\frac{p_{i-\frac{1}{2}}}{p_i}\right)^{\frac{R}{C_p}} - 1\right) + \sum_{j=1}^{i-1} T_j\left(\left(\frac{p_{j-\frac{1}{2}}}{p_j}\right)^{\frac{R}{C_p}} - \left(\frac{p_{j+\frac{1}{2}}}{p_j}\right)^{\frac{R}{C_p}}\right)\right]. \tag{B6}$$

In both cases, for imposed pressure levels, we obtain

$$e_i^d \equiv \sum_{j=0}^{N} (C_p\delta_{ij} + G_{ij})T_j \equiv \sum_{j=0}^{N} E_{ij}^d T_j, \tag{B7}$$

where δ_{ij} is the Kronecker symbol, G and E^d are constant matrices.

Competing interests. Authors declare no conflict of interest

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