

# ***Interactive comment on “ESD Ideas: The stochastic climate model shows that underestimated Holocene trends and variability represent two sides of the same coin” by Gerrit Lohmann***

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## **1 Main appreciation**

The key message of the article is that the underestimation, by models, of low-frequency variance could be caused by a mis-representation of non-normal modes. The article describes how non-normal modes in a forced Langevin equation amplify the low-frequency variance. The conclusion of the article is that further constraints on the

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time-dependent climate sensitivity matrix could be obtained from the spectrum of climate fluctuations.

The idea deserves to be formulated, although it would benefit some more critical examination of the frequency range over which it could be applied, because the underlying theory is linear and low-dimensional.

Previous investigators have indeed shown why it is not straightforward to estimate climate sensitivity by application of the fluctuation-dissipation theorem. Kirk-Davidoff (2009) provided a critique of a previous attempt by Scharz (2007) to constrain climate sensitivity from interannual variability, and Fuchs et al. (2015) as well as Cooper and Haynes (2011) provided some further technical discussion about the scope of the fluctuation-dissipation theory in atmospheric sciences. Simply put, in a simple 1-potential well system forced by Brownian motion (the Langevin equation), there is only one relaxation force in the system, which acts in a similar way at all time scales. In other words, the physical forces causing the phenomenon of relaxation (e.g.: gravitational forces in a pendulum; spring tension in a mass attached to a spring) are the same as those which determine the sensitivity to a constant forcing. This ceases to be true in the atmospheric system. Processes of relaxation at the annual time scale (which involve geophysical fluid dynamics) involve different processes than the radiative relaxation which determine climate sensitivity.

Further work on the fluctuation-dissipation theorem has since been published or shown in conferences, and would require a more systematic review, which, to be fair, is out of the scope of an “idea” paper.

Despite these reservations, it is plausible that the linear assumption expressed by the equation (1) of the article under review be indeed valid over a range of time scales greater than the interannual time scale, and hence, that the idea suggested by this author has some scope for application. However, in order to determine which range of time scales it could be, it seems necessary to provide some plausible physical in-

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interpretation of the nature of the non-diagonal terms. Indeed, the author mentions the ‘fluid dynamical context’, but what does it mean ?

There is also some concern about the mathematical notations. Equation (1) is originally presented with  $\mathbf{T}$  as vector (if bold notation is indeed supposed to indicate a vector), with  $\lambda$ , a scalar. What would be the components of  $\mathbf{T}$ ? If they are different climatic components (ocean, and atmosphere), then we need different relaxation time scales. Let us suppose that the original interpretation of equation (1) assumes  $\mathbf{T}$  as a scalar, and that  $\mathbf{T}$  becomes a vector only at the point of introducing equation (7). Then, we can legitimately consider that the different components of  $\mathbf{T}$  correspond to different components of the climate system, in which case we would expect some non-diagonal (linear, symmetric) coupling terms. There are no such terms in matrix  $\mathbf{A}$ . So the reader needs to infer that the system was rotated in order to get rid of the coupling terms. What is in vector  $\mathbf{Q}$  then? The second component of  $\mathbf{Q}$  needs to be strictly positive, in order to excite the second component of  $\mathbf{T}$ , and finally generate the extra variance produced by the factor  $N$ . This leaves a bit too much guess work to the reader.

Assuming these questions can be answered, there is, finally, some concern about the quality and performance of spectral estimators that would be needed to do the job of estimating  $\mathbf{A}$ . Does the power spectrum contain enough information to constrain the non-diagonal elements of the transfer matrix? If it does, would it plausibly work given the palaeoclimate data available ?

### 1.1 minor typo comments:

use  $\langle \rangle$  and  $\langle \rangle$  for brackets in  $\text{\LaTeX}$ .

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## 1.2 references:

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- Kirk-Davidoff D. B. (2009), On the diagnosis of climate sensitivity using observations of fluctuations, *Atmospheric Chemistry and Physics*, (9) 813–822 doi:10.5194/acp-9-813-2009
- Cooper F. C. and P. H. Haynes (2011), Climate Sensitivity via a Non-parametric Fluctuation–Dissipation Theorem, *J. Atmos. Sci.*, (68) 937–953 doi:10.1175/2010jas3633.1
- Fuchs D., S. Sherwood and D. Hernandez (2015), An Exploration of Multivariate Fluctuation Dissipation Operators and Their Response to Sea Surface Temperature Perturbations, *J. Atmos. Sci.*, (72) 472–486 doi:10.1175/jas-d-14-0077.1

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