ESD Ideas: The stochastic climate model shows that underestimated Holocene trends and variability represent two sides of the same coin

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Abstract. Holocene sea surface temperature trends and variability are underestimated in models as compared to paleoclimate data. The idea is presented that the local trends and variability are related which is elaborated in a conceptual framework of the stochastic climate model. The relation is a consequence of the fluctuation-dissipation theorem, connecting the linear response of a system to its statistical fluctuations. Consequently, the spectrum can be used to estimate the timescale-dependent climate ~~sensitivity~~response. The non-normality in the propagation operator introduces enhanced long-term variability related to non-equilibrium and/or Earth system sensitivity. The simple model can guide us to analyze comprehensive models' behaviour.

Climate and Earth system models are widely used to evaluate the impact of anthropogenic emissions on future climate. The validation of these models by simulating different climate scenarios is essential to understand the sensitivity of the climate system to external forcing. The models are clearly unrivalled in their ability to simulate a broad range of large-scale phenomena on seasonal to decadal time scales (Flato et al., 2013). However, the reliability of models to simulate climate variability on multidecadal and longer time scales requires additional evaluation. Climate records derived from paleo-environmental proxy-parameters facilitate the testing of models across these time scales.

Interglacial periods provide the means for evaluating the performance of general circulation models in representing sea surface temperature (SST) anomalies and trends (e.g. Lohmann et al., 2013). One key finding is that the models do not capture the magnitude of the derived SSTs from marine proxy records in all climate simulations of the Holocene where the simulated SST trends systematically underestimate the local marine proxy-based temperature (Alkenone) trends. It is suggested that a part of such discrepancies can be caused either by too simplistic interpretation of the proxy data and/or by underestimated regional responses in climate models. Fig. 1a shows the scatter plot of simulated and reconstructed SST trends for the mid-to-late Holocene, based on results obtained within the Paleoclimate Modelling Intercomparison Project PMIP2/3 (Braconnot et al., 2007, 2012). Note that the orbital forcing has different signs at high and low latitudes (Berger, 1978). The slopes in Fig. 1a indicate that the response in the models is underestimated by an order of magnitude as compared to the SST reconstructions.

By using long-term multi-millenial climate model runs and paleoclimate data, a discrepancy is detected also with respect to variability (Fig. 1b)(see, Laepple and Huybers, 2014a,b). While most state-of-the-art climate models realistically simulate inter-annual variability (in this particular model the interannual variability is overeststimated), they underestimate variability on multidecadal to millennial time scales. This was revealed by a systematic comparison of climate model simulations, instrumental records and paleo-observations.

In order to reconcile both local sensitivity and variability, a model is presented which takes into account the mean as well as the variability, based on Hasselmann (1976). Imagine that the temperature of the ocean is governed by

$$\frac{dT}{dt}\frac{\mathrm{dT}}{\mathrm{dt}} = -\lambda \mathrm{T} + \mathrm{Q_{net}} + \mathrm{f(t)} = -\lambda T + Q_{net} + f(t), \tag{1}$$

where the air-sea fluxes due to weather systems are represented by a white-noise process Q_{net} with zero average ~~$< Q_{net} >= 0$~~ $< \mathrm{Q}_{net} >= 0$ and δ-correlated in time

$$< Q_{net}(t)Q_{net}(t+\tau) >= \delta(\tau)\mathrm{Q_{net}(t)Q_{net}(t+\tau)} >= \delta(\tau) \quad . \tag{2}$$

The brackets $< \cdots >$ denote the ensemble mean. The function ~~$f(t)$~~ $\mathrm{f(t)}$ is a time dependent deterministic forcing. We assume furthermore that ~~$f(t) = c \cdot u(t)$ with $u(t)$~~ $\mathrm{f(t) = c \cdot u(t)}$ with $\mathrm{u(t)}$ as unit step or the so-called Heaviside step function. Because ~~$< Q_{net} >= 0$~~$< \mathrm{Q}_{net} >= 0$, the ensemble mean solution is

$$< T(t) >= T(0) \cdot \exp(-\lambda t) + \frac{c}{\lambda} 1 - \exp(-\lambda t) < \mathrm{T(t)} >= \mathrm{T(0)} \cdot \exp(-\lambda \mathrm{t}) + \frac{\mathrm{c}}{\lambda}(1 - \exp(-\lambda \mathrm{t})) \tag{3}$$

where we have ~~$< T(0) >= T(0)$~~$< \mathrm{T(0)} >= \mathrm{T(0)}$. As equilibrium response we have

$$\Delta T \Delta \mathrm{T} = \lim_{\mathrm{t}\to\infty} < \mathrm{T(t)} > = \frac{\mathrm{c}}{\lambda} = \lim_{t\to\infty} < T(t) > = \frac{c}{\lambda}. \tag{4}$$

~~which is also called equilibrium climate sensitivity.~~

The fluctuations can be characterized by the spectral Fourier component ~~$\hat{T}_\omega = \frac{\hat{Q}_\omega}{(\lambda+i\omega)}$~~ $\hat{\mathrm{T}}_\omega = \frac{\hat{\mathrm{Q}}_\omega}{(\lambda+\mathrm{i}\omega)}$ with the spectrum

$$S(\omega)\mathrm{S}(\omega) = < \hat{\mathrm{T}}\hat{\mathrm{T}}^* > = \frac{< \hat{\mathrm{Q}}\hat{\mathrm{Q}}^* >}{\lambda^2 + \omega^2} = \frac{1}{\lambda^2 + \omega^2} = <^* > = \frac{< \hat{Q}\hat{Q}^* >}{\lambda^2 + \omega^2} = \frac{1}{\lambda^2 + \omega^2} \tag{5}$$

showing that a too high value in ~~λ~~ λ is related to a too low low-frequency variance. The model-data differences on long time scales suggest that feedback mechanisms and internal variability may not well represented in current climate models. The relation of (4) and (5) is related to the fluctuation-dissipation theorem connecting the linear response to the statistical fluctuations (Nyquist, 1928), and the relation of a too low sensitivity (Fig. 1a) and too low variability (Fig. 1b) is qualitatively detected. Recently, one focus of research was to identify feedback mechanisms in the Earth system enhancing the sensitivity (Stärz et al., 2016) or variability (Bakker et al., 2017).

If we include more components and feedbacks into the system, we can introduce higher values for the climate sensitivity, called Earth system sensitivity. In the spectral domain, we can consider a series of processes like (1) and the spectrum is the sum

$$\cancel{S_n(\omega) = \sum_i \frac{1}{\lambda_i^2 + \omega^2}} \mathrm{S_n}(\omega) = \sum_{\mathrm{i}} \frac{1}{\lambda_{\mathrm{i}}^2 + \omega^2} \quad . \tag{6}$$

over the components in (5). However, in the fluid dynamical context we have typically a non-normal matrix with non-orthogonal eigenvectors ~~extracting energy from the mean flow (~~with $\mathrm{S}(\omega) > \mathrm{S_n}(\omega)$. The plausible physical interpretation of the nature of the non-diagonal terms is related to the extraction of energy from a mean state/mean flow. In terms of the simple Stommel ocean model, this is the mean ocean circulation (Lohmann and Schneider, 1999). In fluid dynamical context, this has been discussed in terms of shear flow instabilities (e.g., Trefethen et al., 1993; Farrell and Ioannou, 1996~~; Palmer, 1999; Lohmann and Schneider, 1999)with $\mathbf{S}(\omega) > \mathbf{S_n}(\omega)$.~~).

Consider as an example a 2 dimensional system with a 2×2 matrix

$$\mathbf{A} = \begin{bmatrix} -1 & N \\ 0 & -5 \end{bmatrix} \tag{7}$$

replacing the scalar number $-\lambda$ in (1), and unit matrix in (2). The eigenfrequencies are independent on N (yellow and cyan vertical lines in Fig. 1c). However, ~~$\mathbf{S}(\omega)$~~ $\mathrm{S}(\omega)$ can be amplyfied relative to ~~$\mathbf{S_n}(\omega)$~~ $\mathrm{S_n}(\omega)$ (6) by orders of magnitude (Fig. 1c), affecting the low-frequency climate variability. The spectra in Fig. 1c are calculated analytically. For real problems, the estimation of A can be done via the POP method (Hasselmann, 1988) where the dynamical propagator has in general a non-normal structure. As a logical next step, the spectral properties can be used to estimate the time-scale dependent climate and Earth system sensitivity.

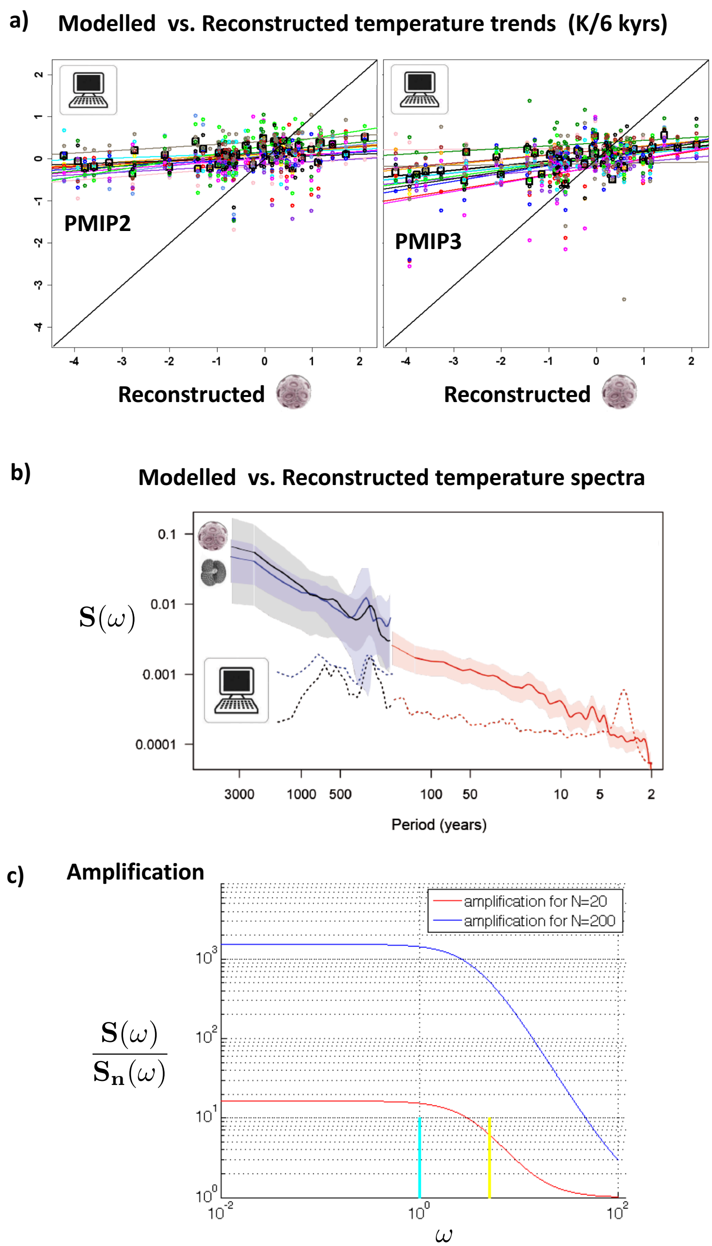

Figure 1. a) Global alkenone-based SST trends compared to simulated annual mean SST anomalies in the models listed in PMIP2 and PMIP3. The black squares represent the ensemble median mean and the colours correspond to a specific model (cf. Lohmann et al., 2013). Units are K per 6 kyrs. **b)** Spectrum of long-term multi-millenial climate model runs, instrumental and paleoclimate data (courtesy of T. Laepple, modified from Laepple and Huybers, 2014b). Inherent uncertainties of the paleo-observations are accounted (compare for instance the blue and black solid lines). **c)** Amplification of the spectrum ~~$\mathbf{S}(\omega)/\mathbf{S_n}(\omega)$~~ $S(\omega)/S_n(\omega)$ of the non-normal 2 dimensional dynamics. The amplification can be several orders of magnitudes depending on the degree of non-normality related here as the parameter N in (7).

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Answer to Referee 1

Thanks for you detailed comments on the manuscript *ESD Ideas: The stochastic climate model shows that underestimated Holocene trends and variability represent two sides of the same coin* . In the following I give answers to all the issues raised.

Answer to the General comments:

1. Comment: "The results show that the observational SST trends are poorly defined, varying from -4K to +2K over the 6 kyr period. The modelled trends are considerably smaller, being confined mainly to the range -1K to +1K over the same period (Figure 1(a))."

Answer: The analysis is a local one, i.e. the points at high latitudes have a general cooling trend whereas the low latitute points show a warming trend through the late Holocene. The general pattern of warming and cooling are consistent in the data and models (Figures 5a, 7a, 8a in Lohmann et al., 2013; see also Braconnot et al., 2012). In any of the analysis the local temperature trends based on proxy reconstructions and climate simulation are taken.

Action: In the revised manuscript, I will explicitly state that it is the local temperature trend as the response to latitude-varying orbital forcing. I wrote this in the ESD manuscript at line 20: " Note that the orbital forcing has different signs at high and low latitudes (Berger, 1978)."

2. Comment: "In view of the poor definition of the observational trends and the lack of knowledge regarding the partitioning of the observational variability, very strong caveats should be placed on any conclusions drawn from this observational/modelling comparison. In particular, since there is no global mean orbital forcing over the 6 kyr period studied, extreme care should be exercised in drawing any conclusions from the study as to the value of climate sensitivity to greenhouse gas increase."

Answer: As written above, we are considering the local temperature trends based on proxy reconstructions and climate simulations. Indeed the global forcing is weak. The pattern of climate response to orbital forcing is a combination of the system's response to precession and obliquity. On the basis of the observed insolation-temperature relationship, different temperature response regimes across the Earth can be identified. Linear relationships dominate extratropical land areas whereas in mid-latitude oceans, the seasonally varying mixed layer depth renders the temperature more sensitive to summer than to winter insolation (Laepple and Lohmann, 2009).

Action: In the revised manuscript, I will explicitly state that I am not analyzing the value of climate sensitivity to greenhouse gas increase.

Answer the Referee's Specific comments:

3. Comment: "In the theoretical part of the study, a zero-dimensional stochastic model represented by Equation (1) is used in an attempt to gain conceptual understanding of the observational and modelling results described above. The term f(t) is used to describe the deterministic forcing and this is assumed to be of the form f(t) = c u(t), where c is a constant and u(t) is a unit Heaviside step function. This means that a non-zero global average forcing is assumed, in contrast to the situation prevailing in the late Holocene period 6 kyrs to present, where the global average orbital forcing is zero. "

Answer: Indeed, the global average orbital forcing is almost zero. In the approach analyzing the climate sensitivity to external forcing such as orbital forcing, a local analysis is necessary.

Action: In the revised manuscript, I will explicitly emphasize that the global average orbital forcing is almost zero and therefore a regional analysis of the temperature trends and variabilty are analyzed.

4. Comment: "From this conceptual model, it is concluded that an underestimation of variability forced by a white noise stochastic forcing implies an underestimation of climate sensitivity to the c u(t) forcing. However, this form of conceptual model does not adequately describe the climate system as forced by the late Holocene orbital forcing. A conceptual model of minimum complexity to do this would be a three-box model such as used by Stap et al. (2018) to study paleoclimate sensitivity. I recommend that such a model instead of that represented by Equation (1) be used to gain theoretical insight into the observational and modelling results described above."

Answer: I know the approach of Stap et al. (2018). Here the attempt is made to define a minimal model for global climate sensitivity and response to greenhouse gases. Here I refrain from this approach and show that the underestimated local variability in the models can be reconciled with the underestimated local responses. Future work may follow a more explicitely resolved low and high-latitude climate model (Bates, 2016; Stap et al., 2018).

Action: In the revised manuscript, I will strongly emphasize the regional aspect of the stochastic climate model in the revised version.

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Answer to Referee 2 (M. Crucifix)

Thanks for your instructive comments on the manuscript *ESD Ideas: The stochastic climate model shows that underestimated Holocene trends and variability represent two sides of the same coin* . In the following I give answers to all the issues raised.

Answer to the comments:

1. Comment: "The key message of the article is that the underestimation, by models, of low-frequency variance could be caused by a mis-representation of non-normal modes. The article describes how non-normal modes in a forced Langevin equation amplify the low frequency variance. The conclusion of the article is that further constraints on the time-dependent climate sensitivity matrix could be obtained from the spectrum of climate fluctuations. The idea deserves to be formulated, although it would benefit some more critical examination of the frequency range over which it could be applied, because the underlying theory is linear and low-dimensional."

Answer: Indeed, the main idea behind the manuscript is to show that the underestimated local variability in the models can be reconciled with the underestimated local responses. The forced Langevin equation is the most simple dynamics to relate the spectrum with parameter λ to the local climate response to insolation forcing, again related to the damping λ. This is related to the more general fluctuation-dissipation theorem. The fluctuation-dissipation theorem relies on the assumption that the response of a system in thermodynamic equilibrium to a small applied force is the same as its response to fluctuations. Therefore, the theorem connects the linear response relaxation of a system from a prepared non-equilibrium state to its statistical fluctuation properties in equilibrium. As compared to the termination or the Early Holocene, the linear assumption for the mid-to-late Holocene trend is a valid assumption when analyzing the SST paleoclimate data (Lohmann et al., 2013). In my approach, the missing variance in the system appears naturally from the too high λ.

The second idea is that a higher dimentional system exhibits a higher variance if the underlying dynamics is non-normal. Indeed many fluid-mechanical systems extract energy out of the mean flow and show a transient amplification Trefethen et al., 1993; Farrell and Ioannou, 1996). Without changing the eigenvalues, the system can have enhanced variance in the spectrum which is due to transient growth. Therefore, the equilibrium climate sensitivity might be lower than the transient dynamics. The paragraph of this non-normal dynamics is admittedly sketchy, the two-dimensional dynamics is not explicitely worked out, but I found it instructive in this ESD ideas manuscript. In a simple 2-d system of the ocean thermohaline circulation (Stommel model), the systems response is far from normal introducing long-term fluctuations (Lohmann and Schneider, 2000).

You mention the frequency range over which the stochastic climate model can be applied. When looking at the spectra of the Holocene temperatures, the mid-to-late Holocene can be described very well by the linear model. Again, the termination, DO cycles or even the Early Holocene are not suitable and the variances may change over time (e.g., Wirtz et al., 2010; Wassenbrug et al., 2016).

Action: In the revised manuscript, I will explicitly state that it is the local temperature trend as the response to latitude-varying orbital forcing which is quasi-linear for the mid-to-late Holocene.

2. Comment: "Previous investigators have indeed shown why it is not straightforward to estimate climate sensitivity by application of the fluctuation-dissipation theorem. Kirk-Davidoff (2009) provided a critique of a previous attempt by Schartz (2007) to constrain climate sensitivity from interannual variability, and Fuchs et al. (2015) as well as Cooper and Haynes (2011) provided some further technical discussion about the scope of the fluctuation-dissipation theory in atmospheric sciences. Simply put, in a simple 1- potential well system forced by Brownian motion (the Langevin equation), there is only one relaxation force in the system, which acts in a similar way at all time scales. In other words, the physical forces causing the phenomenon of relaxation (e.g.: gravitational forces in a pendulum; spring tension in a mass attached to a spring) are the same as those which determine the sensitivity to a constant forcing. This ceases to be true in the atmospheric system. Processes of relaxation at the annual time scale (which involve geophysical fluid dynamics) involve different processes than the radiative relaxation which determine climate sensitivity."

Answer: Thanks a lot for these hints. Indeed, several articles are dealing with the FDT, but I have not seen a contribution to explore the Holocene trends and variability. One motivation of the stochastic climate model with introducing λ in the response as well as in the fluctuation is that it provides a framework for further GCM studies. Preliminary analyses of high-resolution climate models indicate a higher local SST variance as well as a more heterogenous, enhanced SST response to external forcing.

Action: In the revised manuscript, I will explicitly mention the goal of the FDTs. As you wrote, a comprehensive overview "would require a more systematic review, which, to be fair, is out of the scope of an “idea” paper". I will mention the benefit of simple models in guiding us to analyze comprehensive models’ sensitivity and variability.

3. Comment: "Despite these reservations, it is plausible that the linear assumption expressed by the equation (1) of the article under review be indeed valid over a range of time scales greater than the interannual time scale, and hence, that the idea suggested by this author has some scope for application. However, in order to determine which range of time scales it could be, it seems necessary to provide some plausible physical interpretation of the nature of the non-diagonal terms. Indeed, the author mentions the ‘fluid dynamical context’, but what does it mean ?"

Answer: Correct. As mentioned in comments 1. and 2., the range of timescales is given by the mid-to-late Holocene. The plausible physical interpretation of the nature of the non-diagonal terms is related to the extraction of energy from a mean state / mean flow. In terms of the simple Stommel model, this is the mean ocean circulation. In fluid dynamical context, this has been discussed in terms of shear flow instabilities (e.g., Trefethen et al., 1993).

Action: In the revised manuscript, I will explicitly mention the physics the non-normal dynamics, but will try to reduce the number of references in this direction.

4. Comment: "There is also some concern about the mathematical notations. Equation (1) is originally presented with T as vector (if bold notation is indeed supposed to indicate a vector), with λ, a scalar. What would be the components of T? If they are different climatic components (ocean, and atmosphere), then we need different relaxation time scales. Let us suppose that the original interpretation of equation (1) assumes T as a scalar, and that T becomes a vector only at the point of introducing equation (7). Then, we can legitimately consider that the different components of T correspond to different components of the climate system, in which case we would expect some non-diagonal (linear, symmetric) coupling terms. There are no such terms in matrix A. So the reader needs to infer that the system was rotated in order to get rid of the coupling terms. What is in vector Q then? The second component of Q needs to be strictly positive, in order to excite the second component of T, and finally generate the extra variance produced by the factor N. This leaves a bit too much guess work to the reader."

Answer: Sorry. The notation in (1) was meant to be for a scalar. Indeed, the vector is only introduced with (7). The vector Q is related to the variances of the individual components. In the ESD ideas mansucript, I have not specified it explicitely and normalized it to one.

Action: In the revised manuscript, I will explicitly mention Q to avoid guess work to the reader. Furthermore, it will be clearly stated that (1) is a scalar stochastic differential equation.

5. Comment: "Assuming these questions can be answered, there is, finally, some concern about the quality and performance of spectral estimators that would be needed to do the job of estimating A. Does the power spectrum contain enough information to constrain the non-diagonal elements of the transfer matrix? If it does, would it plausibly work given the palaeoclimate data available ?"

Answer: In the paper, the spectra in Fig. 1c are calculated analytically. For real problems, the estimation of A can be done via the POP method (Hasselmann, 1988). Then the dynamical propagator has in general a non-normal structure. The POPs can be calculated from the paleoclimate time series, which would be a logical next step. For recent climates, there exists very nice examples in the framework of (linearized) stochastically forced dynamics (e.g., Whitaker and Sardeshmukh, 1998; Kwasniok, 2004).

Action: I will try to give a short outlook in this direction.

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