Response to Anonymous Referee RC 1

We thank the referee for the careful reading and the useful comments and will adapt the manuscript accordingly. Below is a point by point reply with the referee's comments in bold font, our reply in italic font and the changes in manuscript in normal font.

Note: The reply to comment 6 of Referee RC 2 led to the removal of Figure 3, and various textual changes led to changes in line numbers. Therefore we consistently refer here to figure and line numbers in the previous version, not the revised one.

1. Comment from referee:
   The authors assess temperature rises to 2100 only. For some scenarios the global mean temperature will continue to increase well after this date. Those cases should be acknowledged.

   Author's response:
   Such possibility can indeed occur and is implicitly acknowledged in Figures 1, 2 and 4. We do not treat these cases in much detail because a) the response function model is based on 140-year long simulations so extrapolations far into the future are more uncertain and b) such scenarios exceed our temperature targets and are therefore of limited interest in this study.

   Changes in Manuscript:
   In line 40, after "usually taken as the year 2100." we will add the following sentence: "The choice of a particular year is necessarily arbitrary and neglects the possibility of additional future warming."

2. Comment from referee:
   The authors note that 2K warming is commonly seen as a "safe threshold". It may be seen that way, but that is a value judgment subject to considerable uncertainty, and this should be acknowledged.

   Author's response:
   We agree with the referee.

   Changes in Manuscript:
   In line 32, after "the 2 K warming threshold commonly seen" we will add " – while gauging considerable uncertainties – ".

3. Comment from referee:
   The assessment of delta T depends on the baseline period chosen. This point is addressed later in the report and is said to introduce a sensitivity to the PNR of up to 10 years. The new IPCC special report on warming of 1.5C and 2C indicates potentially large differences in delta T for different baseline choices. It would be nice to see the authors address this issue more explicitly to have confidence that their PNR sensitivity is as low as reported.

   Author's response:
   Within the scope of our model the effect of the baseline on the PNR is such that a lower baseline increases the currently realized warming. Therefore, a given temperature threshold is crossed at an earlier point in time.

   Changes in Manuscript:
   To clarify this point, the paragraph referring to the temperature baseline (from line 293, "This also") will be replaced by the following: "This also illustrates the importance of the temperature baseline relative to which ΔT is defined, as has been found previously (Schurer et al., 2017). Switching to a (lower) 18th century baseline increases current levels of warming by 0.13 K (Schurer et al., 2017) and thereby brings forward the PNR. For example, for a maximum temperature threshold of 1.5K the PNR moves from 2022 to 2016 in the MM scenario and from 2038 to 2033 for the EM scenario."

4. Comment from referee:
   The authors use the concept of "negative emissions" in their simulations, but don't say much about the feasibility of negative emissions. Some elaboration would be helpful for the reader.

   Author's response:
   It is not within the scope of this article to provide a detailed discussion of the question of feasibility of negative emissions, which is a research area in its own right. Scenarios such as the ones presented here and taken from

Rogelj et al., (2016a) are usually based on cost-minimization in Integrated Assessment Models (IAMs), and are feasible within the constraints and choices enforced there.

Changes to Manuscript:
In line 223, at the end of the paragraph, we will add the sentence "For details on the scenarios refer to Rogelj et al., (2016a). With carbon budgets rapidly running out and the PNR approaching fast, negative emissions may have to become an essential part of the policy mix. Such policies are cheap but may only be a temporary fix and lead to undesirable spillover effects on neighboring countries (e.g., Wagner and Weitzman, 2015). We abstract from these discussions here, since this is beyond the scope of the present paper".

We will add: "Wagner, G. and M.L. Weitzman (2015). *Climate Shock. The Economic Consequences of a Hotter Planet*, Princeton University Press, Princeton, New Jersey" to reference list.

5. Comment from referee:
The trajectory of warming from the present point to exceeding the specified temperature threshold will not be smooth as it will include multidecadal scale internal variability. That implies that the threshold will not be exceeded at a single point in time, but only in some average sense. The degree to which this is an issue depends on how well the CMIP5 runs represent multidecadal internal variability and how one treats temporal variability and overshoot in relation to the threshold. The authors could provide some discussion of this issue in relation to their analysis.

Author's response:
It is indeed the case that, due to internal variability, crossing the threshold takes place in some average sense. Commonly this is done by temporal averaging. In our case, averaging is done across the ensemble of simulations. Therefore, it is indeed possible to pinpoint the crossing of the threshold (at a chosen probability level) to a given year, as the large ensemble smooths out the variability (Figure 6). The model is not capable of accurately displaying modes of internal variability, nor is it designed to predict (esp. in a one time-series sense) the crossing of the threshold.

Changes to Manuscript:
Before the final paragraph starting in line 345 ("We have shown the constraints..."), we will add the following paragraph:
"In this work a large ensemble of simulations was used in order to average over stochastic internal variability. This allows to pinpoint the point in time where a threshold is crossed at a chosen probability level. Such an ensemble is not possible for more realistic models, nor do GCMs agree on details of internal variability. Therefore, in practice, the crossing of a threshold will likely be determined with hindsight and using 30-year temporal means. This fact should lead us to be more cautious in choosing mitigation pathways."

<u>Response to Anonymous Referee RC 2</u>

We thank the referee for the careful reading and the useful comments and suggestions and will adapt the manuscript accordingly. Below is a point by point reply with the referee's comments in bold font, our reply in italic font and the changes in manuscript in normal font.

General remarks to the referee:

The referee remarks that giving precise years for the Point of No Return (PNR) may be misleading due to many uncertainties associated with such approaches. This is certainly true and in fact the primary motivation to conduct this study in a probabilistic fashion, with the aim to capture climate system uncertainties in the model itself.

We see the presentation of a stochastic model as a major novelty of this paper, building upon and extending previous work such as Stocker (2013). The aim was to a) include uncertainties as captured by the CMIP5 ensemble and b) get a handle on risk tolerance, allowing us to choose with which probability a certain warming target should not be exceeded. Clearly, tighter constraints (i.e. an earlier Point of No Return (PNR)) are intuitively expected for a smaller risk tolerance but the model allows us to quantify this.

The stochastic state space model is described in section 2.2 and summarized in Table 2 (where also the noise terms are detailed), as stated in line 161. Noise is included in several of the carbon and temperature boxes, where W_t denotes the Wiener process. These boxes are added to form the total CO2 concentration and temperature anomaly (eqs 10a, 10b). The introduction of additive and multiplicative noise is central to this paper, and turns the temperature evolution DT(t) into the evolution of a probability density p(DT,t) (Figure 4), capturing the spread of the CMIP5 ensemble.

Reponses to the referee's specific comments:

1. Comment from referee:

   The new approach is essentially twofold: first a very simple deterministic model is developed that reproduces global characteristics of CMIP5, and second, emission pathways are given as an exponential increase at rate g (information not found in the paper: g=??) multiplied by a linearly decreasing factor (mitigation effect). In addition, negative emissions due to carbon capture and storage can be considered in this model framework. It would be useful to quantify the difference of the considered paths (11c) to an even more basic choice of just a simple exponential decrease of emissions at a constant rate from t_s onwards, as used by Stocker (2013). Obviously, the discontinuity of emission rates at t_s (increasing exponentially before, and then decreasing) are avoided here, but how would that matter for the PNR? Incidentally, for a given mitigation rate PNR can be read off Fig. 2A of Stocker (2013): it is the required starting time of emission reductions. Therefore, much of the information, which is the focus of the present paper, has been available already from an even simpler framework. This should be mentioned in the introduction.

   Author's response:

   - *The referee is right to point out that the original response function model (eqs 8) is deterministic. However, as pointed out in the introductory paragraph, this deterministic model is turned into a stochastic one through the introduction of stochastic noise terms (see section 2.2, Table 2, Figure 4), and used throughout the paper.*
   - *We thank the referee for noticing the omitted definition of the emissions growth rate. It will be corrected.*

- *In the final paragraph of the introduction (lines 66-76) we refer to Stocker (2013) and point out how our approach differs from his, in particular by using a stochastic model that is capable of capturing climate uncertainties and risk tolerance. We agree with the referee that a comparison of our mitigation pathways (11) with exponential pathways (Stocker) is interesting. We have performed such an analysis and show the results here, in the manner of Fig. 2A of Stocker (2013). From Figure RC1 one can see that the notable novelty of this work is the introduction of probabilities (top right and bottom panels). Comparing Fig. 2A of Stocker with the top left panel we find that our results are more optimistic than Stocker's, allowing for smaller reduction rates to reach the same target. Our model is more complex than Stocker's, and considering the good reconstruction of relevant RCP scenarios (Figure 4), we have confidence in our results. Under exponential mitigation, the PNR is substantially earlier (Table RC1) when using a value for the exponential reduction rate r that is equal to m1. A problem with exponential pathways is that emissions never reach exactly zero and can still be non-negligible by 2100, e.g. when starting reduction in 2038 at r=0.05 emissions in 2100 still reach 0.56 GtC/yr and 0.26 GtC/yr when starting in 2025. This is difficult to bring into agreement with the "net zero emissions" target of the Paris Agreement. We therefore choose to continue to use the mitigation pathways as defined in the paper.*

Changes in Manuscript:
- No changes
- In section 2.3, line 179, we will replace "rate g due" by "rate g = 0.01 due".
- No changes

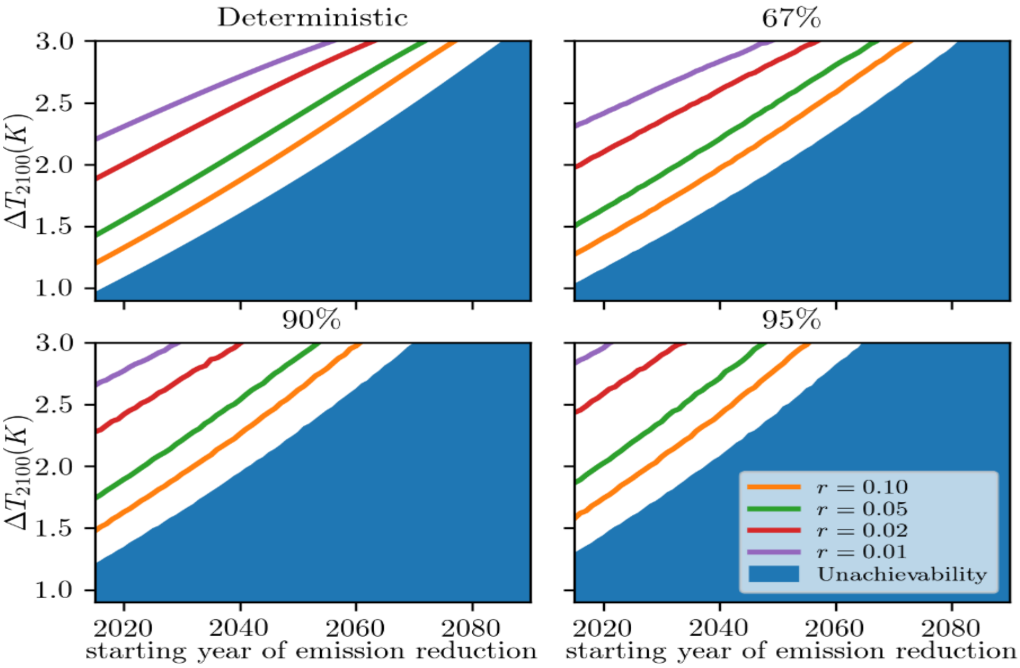

Figure RC1: Reconstruction of Fig. 2A from Stocker (2013) (top left) and panels for different probability threshold. E.g. the top right panel gives the year (x-axis) where exponential emission reduction at different rates (lines) needs to be initiated to limit warming below a given threshold (y-axis) with a probability of 67%. Increasing the required probability tightens the constraint.

	β	**0.5**	**0.67**	**0.9**	**0.95**	**noise-free**
Scenario	Threshold					
r = 0.1	1.5 K	2028	2024	2016	–	2027
	2.0 K	2046	2042	2033	2028	2045
r = 0.05	1.5 K	2019	–	–	–	2018
	2.0 K	2038	2033	2024	2020	2037
r = 0.02	1.5 K	–	–	–	–	–
	2.0 K	2022	2017	–	–	2020

Table RC: PNR with exponential mitigation at different rates r

2. Comment from referee:
Uncertainty is only substantively addressed in the text of the appendix. As this is a short text, I suggest to incorporate the appendix into the main text and amplify it. Regarding uncertainty, a general caveat would be useful in the abstract and the conclusion. Otherwise, the stated years of PNR are somewhat misleading.

   Author's response:
   Uncertainty is an essential part of this work. We assume that the spread in the CMIP5 ensemble captures all kinds of uncertainties, including parameter uncertainties (for example, in climate sensitivity). To this distribution we fit our stochastic model, accounting for all variations between the climate models. Nevertheless, an additional sensitivity study is certainly useful and was therefore performed. We thank the referee for the suggestion and will move the appendix to the end of the results section.

   Changes in Manuscript:
   At the end of the abstract, we will add the following sentence: "Sensitivity studies show that the PNR is robust with uncertainties of at most a few years.". Table 8 will be adapted, the appendix modified appropriately and moved to the end of section 3 (Results).

3. Comment from referee:
A constant factor A in the forcing (8b) is used to optimize the agreement with CMIP5. The size of this factor is quite large (1.48). For alpha_CO2 (in 8b) the correct value is taken (see Tab 3 - however inconsistent parameter notation - only alpha there!). The authors justify the factor A with the existence of non-CO2 GHG drivers in the CMIP5 results (RCP scenarios), but the effects of these drivers have a time evolution and characteristic time scales that are very different from the primary driver CO2. So I don't quite understand how is it possible to achieve a better match with CMIP5 by using a simple scaling of (8b).

   Author's response:
   The factor A captures all processes that are not represented in our (simple) model. This includes non-CO2 drivers as well as non-fossil CO2 drivers. In addition, our carbon model and our temperature model come from different model ensembles that are here joined together, and A is a matching factor. Thirdly, as discussed in the paper, the used carbon model is pulse-size-independent, which is a simplification that underestimates concentrations at high emissions. The factor A scales up the forcing from the unrealistically low concentrations to still give the required high radiative forcing.

   Changes in Manuscript:
   In line 143, we will replace "and non-CO2 GHG emissions." by "and non-CO2 GHG emissions, and matches the carbon and temperature models estimated from different model ensembles) together. The constant A = 1.48 was found in order to optimize the agreement of \Delta T with CMIP5 RCPs. The resulting reconstruction of temperatures from RCP CO_2 concentrations overlaid with CMIP5 data (Figure 1b) gives a good agreement." The lines 119-124 ("In order to apply … gives a good agreement.") are deleted.

4. Comment from referee:
It is not clear, why in (11) both mitigation and abatement are used. Also, there is a conflict of parameters (a_0 in 6 and 11). Is 11b, i.e. a(t), really needed and relevant in this paper? I see no discussion in the text or the figures relating to the difference of m(t) and a(t) pathways. In fact, inspecting (11c) I can see no benefit why one would consider both mitigation and abatement. Both have the same linear time dependence, even the same rate. Therefore, the difference seems to lien in the quadratic (positive) contribution a(t)*m(t) to the emission factor, essentially (m1ˆ2)(t-t_s)ˆ2, presumably a rather small contribution. Therefore, for simplicity, I suggest that you would eliminate a(t) altogether, which would also remove the parameter conflict of a_0.

   Author's response:
   We thank the referee for noticing the parameter conflict which will be resolved by renaming the coefficients in (6) from a_0, a_1, a_2, a_3 to \mu_0, \mu_1, \mu_2, \mu_3.
   We think like many others that there are several important dimensions to climate policy. This includes the substitution of fossil fuel by renewable energies (mitigation) as well as directly reducing the CO2 output via sequestration mechanisms (abatement). We consider it important to include both these dimensions (as a third dimension one might point to negative emissions which we briefly cover as well). It is true that the abatement pathway is chosen very similar to mitigation, both because of simplicity and due to a lack of better estimates. Many now believe that some form of abatement will be necessary, for example to deal with the problem of "stranded assets". Neglecting abatement would clearly require much higher mitigation rates to reach the same targets. For these reasons we decided to include both abatement and mitigation into our modelling framework. Note also that the quadratic term is not necessarily small, for m1 = 0.02, m0 = 0.14 it reaches >40% of the linear term after 40 years, which slows the decay to zero.

Changes in Manuscript:
The coefficients in (6), a_0, a_i will be renamed \mu_0, \mu_i.

5. Comment from referee:
Further to the emission pathway described in 11c, I note that E_neg is included. However, it is not clear from the text, how Fig. 3 is constructed. From the rather short caption I surmise that this is taken from Rogelj et al., and then just prescribed here. This must be stated in section 2.3 more clearly.

Author's response:
The referee is correct that Fig. 3 is constructed from scenarios simply taken from Rogelj et al, as is discussed in the final paragraph of the Methods section.

Changes in Manuscript:
Considering the response to comment 6, resulting in the removal of Fig. 3, no changes will be performed.

6. Comment from referee:
You seem to consider only the strong negative emission of Fig. 3 for the calculation of PNR in Tab. 6. As this strong case appears nearly exponential in nature, I would suggest that you simply approximate the Rogelj negative emissions by an exponential and a starting time, and give it explicitly in eq 11 with its associated rate. This would eliminate Fig. 3, be more transparent for the reader and actually more consistent with the simple scenario approach that you chose in eq 11.

Author's response:
The referee is correct that only results for the strong pathway are presented, so for clarity only it will now be mentioned. We thank the referee for the excellent suggestion to approximate the negative emissions by an exponential. It turns out that the fit is very good.

Changes in Manuscript:
In the first paragraph of section 2.3 the sentence "in addition, negative … concentration." will be replaced by: "In addition, negative emission technologies may be employed. They cause a direct reduction in atmospheric CO2 concentration and are here modelled as an exponential E_neg(t) = E_{neg,\infty} * (1 – exp(r *time))." A footnote is added to "exponential" in this sentence: "For long timescales, these (after a transient) constant negative emissions may not be realistic. However, we are interested in timescales until 2100."

The final paragraph of section 2.4 ("Since it is now … (red) pathway.") will be removed.
Figure 3 will be removed (in this response we continue to the original Figure numbers).
As the final paragraph of section 2.3 the following will be added:
"From these scenarios we obtain a family of negative emission scenarios out of which we pick a pathway with strong negative emissions. It is very well approximated by setting E_{neg,\infty} = 4.21 and r=-0.0283."

7. Comment from referee:
In order to construct ensembles, the mitigation rate m_1 is drawn from a Beta distribution. It would be helpful for the reader to have an explanation why this distribution is chosen and what difference a simple uniform or normal distribution would make.

Author's response:
The Beta distribution is chosen for purely practical reasons to get a better coverage of emission scenarios. m0 is drawn from a uniform distribution [0,0.7], so when drawing m1 from e.g. a uniform distribution, many of the m0, m1 pairs would result in a very quick mitigation, resulting in an under-sampling of scenarios with high cumulative emissions. The Beta distribution has the advantage that it is both bounded and (with these parameter values) highly skewed towards small m1, so that the scenario sample is more uniform in terms of cumulative emissions. The choice of distribution has no consequences on the results.

Changes in Manuscript:
In line 245, we will add the following sentence after "latest in 2080." : "The Beta distribution is chosen for practical reasons to a sample of (m0, m1) pairs. As m0 is drawn from a uniform distribution, doing likewise for m1 would result in many pathways with very quick mitigation and low cumulative emissions. Choosing a Beta distribution for m1 makes draws of small m1 much more likely and leading to a better sampling of high cumulative emission scenarios. The choice of distribution has no consequences on the results."

8. Comment from referee:
Some noise is added to the model as stated on line 167ff. It seems of only minor relevance for the results (see Tab 5 and 6 - PNR changes only by about 1 year compared to the 50%-probability case). I wonder then why the addition of noise should be necessary at all. I cannot see any new insight from this. If you retain the noise, a more detailed description would be necessary. In particular, the noise should be evident in eqs 10a and 10b as additional terms.

Author's response:
We would like to point the referee to the opening paragraph of this response. Our stochastic state space model consists of four carbon and three temperature boxes, as shown in Table 2. The noise is in several of the carbon and temperature boxes, with W_t denoting the Wiener process. The boxes are simple added (eqs 10a, 10b) to obtain the total, so no additional noise terms are required in this summation. The introduction of additive and multiplicative noise is central to this paper, allowing to get probability distributions (Figure 4). The referee is right to point out the similar values for PNR (Table 5 and 6) for the "noise-free" and 50%-probability case, which is because the deterministic model (setting the noise terms to zero) is very similar to the 50th percentile of the distribution (as can be seen in Figure 5). However, the temperature distributions are in fact not symmetric (Figure 4), so (this being a crucial result) the PNR changes substantially when requiring higher safety probabilities \beta (Tables 5 and 6) – in practice, it is likely preferable to have a probability higher than 50% (IPCC works with 67%).

Changes in Manuscript:
We thank the referee for pointing this out and will do our best to clarify the introduction on this point.
The caption of Table 2 will be changed to the following: "Stochastic State Space Model. Carbon model on the left, temperature model on the right. Wt denotes the Wiener process".
In line 63, we will replace "stochastic model is then" by "stochastic model – representing all kinds of uncertainties in the climate model ensemble – is then".
In line 61f, we will replace "stochastic model" by "stochastic state-space model".

9. Comment from referee:
Table 5, 6, and 7 could be presented in a more effective way. Table 7 is trivial (just the difference Tab6 - Tab5) and could therefore be omitted. I further suggest to combine Tables 5 and 6 into one table. Each probability column should then contain two subcolumns, one without E_neg the other one with E_neg. The small difference caused by E_neg makes would then be directly visible.

Author's response:
These suggestions are very welcome and the tables will be formatted as suggested.

Changes in Manuscript:
Table 7 will be omitted. Table 6 will be combined into Table 5 by splitting the probability columns into sub-columns, for the case with/without negative emissions.

10. Comment from referee:
In the appendix and in Tab. 8 some parameters (\gamma_0, r_\gamma) are listed without explanation. Where do they come from? Are they needed in this paper?

Author's response:
These are parameters connected to related research not included in the final paper. They will be removed.

Changes in Manuscript:
Mentions and discussions of \gamma_0, r_\gamma will be removed from the appendix.

11. Comment from referee:
Line 374: please spell IPCC correctly. It is an edited document and that information is missing, as well as the total page number.

Author's response:
We thank the referee for this remark and will correct the formatting.

Changes in Manuscript: The reference will be formatted correctly.

12. Comment from referee:
Figure 2: Put the 10ˆ3 factor into the label unit (1000 ppm).

Author's response:
The formatting of Figure 2 will be adapted.

Changes in Manuscript:
The factor of 10^3 in Figure 2, top right panel will be included in the tick label (1000 and 2000 ppm), as an inclusion into the unit label did not fit well into the formatting.

13. Comment from referee:
Figure 2 and line 147. The discrepancy with the CMIP5 CO2 concentrations for RCP8.5 is quite worrying. This would imply that cumulative emissions will be way off, as well. The discrepancy for the forcing is removed by introducing the factor A, but what about CO2(t) and cumE(t)?? This must be addressed in a more convincing way.

Author's response:
Our model has indeed substantial discrepancies in CO2 concentrations for high-emission scenarios such as RCP8.5. The reason for this is the use of a pulse-size-independent carbon response function (essentially meaning that carbon sinks operate at the same efficiency independent of CO2 concentration, temperature, and reservoir sizes). This is introduced in section 2.1 (line 125-139) and discussed in section 4 (lines 310ff). This is indeed a problem for the CO2 concentration, but, as seen in Figure 2, not for radiative forcing or temperature due the factor A (see also comment 3). We are not focused on the intermediate variable CO2(t), and compute cumE(t) directly from the emissions, so this has no substantial effect on our results.

Changes in Manuscript:
In line 148, we will replace "natural sinks saturate." with "natural sinks saturate, which is a process the pulse-size-independent carbon response function cannot adequately capture."

14. Comment from referee:
Figure 6: Caption should be amplified by elaborating on the "different policies". You could add, e.g.: "... as described by m in eq 11, the rate of mitigation increase per year."

Author's response:
We thank the referee for his suggestion and will incorporate it.

Changes in Manuscript:
The caption of Figure 6 will be adapted. "different policies, without … negative emissions." is replaced by "different policies as described by in eq 11 with different choices for m1, the rate of mitigation increase per year. Top and bottom panels show the cases without and with strong negative emissions, respectively."

15. Comment from referee:
Figure 7: y-axis labels not complete.

Author's response:
We thank you for the remark and have corrected the labels.

Changes in Manuscript:
The y-axis labels of Figure 7 will be formatted correctly.

~~Risk and the~~ The Point of No Return for ~~Climate Action~~climate action: effects of climate uncertainty and risk tolerance

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Abstract. If the Paris targets are to be met, there may be very few years left for policy makers to start cutting emissions. Here, we ask by what year at the latest one has to take action to keep global warming below the 2 K target (relative to preindustrial levels) at the year 2100 with a 67% probability; we call this the Point of No Return (PNR). Using a novel, stochastic model of CO_2 concentration and global mean surface temperature derived from the CMIP5 ensemble simulations, we find that cumulative CO_2 emissions from 2015 onwards may not exceed 424 GtC and that the PNR is 2035 for the policy scenario where the share of renewable energy rises by 2% per year. Pushing this increase to 5% per year delays the PNR until 2045. For the 1.5 K target, the carbon budget is only 198 GtC and there is no time left before starting to increase the renewable share by 2% per year. If the risk tolerance is tightened to 5%, the PNR is brought forward to 2022 for the 2 K target and has been passed already for the 1.5 K target. Including substantial negative emissions towards the end of the century delays the PNR from 2035 to 2042 for the 2 K and to 2026 for the 1.5 K target, respectively. We thus show the impact on the PNR not only of the temperature target and the speed by which emissions are cut, but also of risk tolerance, climate uncertainties and the potential for negative emissions. Sensitivity studies show that the PNR is robust with uncertainties of at most a few years.

1 Introduction

The Earth System is currently in a state of rapid warming that is unprecedented even in geological records (Pachauri et al., 2014). This change is primarily driven by the rapid increase in atmospheric concentrations of greenhouse gases (GHG) due to anthropogenic emissions since the industrial revolution (Myhre et al., 2013). Changes in natural physical and biological systems are already being observed (Rosenzweig et al., 2008), and efforts are made to determine the 'anthropogenic impact' on particular (extreme weather) events (Haustein et al., 2016). Nowadays, the question is not so much if, but by how much and how quickly the climate will change as a result of human interference, whether this change will be smooth or bumpy (Lenton et al., 2008) and whether it will lead to dangerous anthropogenic interference with the climate (Mann, 2009).

The climate system is characterized by positive feedbacks causing instabilities, chaos and stochastic dynamics (Dijkstra, 2013) and many details of the processes determining the future behavior of the climate state are unknown. The debate on action on climate change is therefore focused on the question of *risk* and how the *probability* of dangerous climate change can be reduced. In scientific and political discussions, targets on 'allowable' warming (in terms of change in Global Mean Surface Temperature (GMST) relative to pre-industrial conditions[1]) have turned out to be salient~~, with the~~. The $2\,\mathrm{K}$ warming threshold ~~commonly seen~~ is commonly seen – while gauging considerable uncertainties – as a safe threshold to avoid the worst effects that might occur when positive feedbacks are unleashed (Pachauri et al., 2014). Indeed, in the Paris COP21 conference it was agreed to attempt to limit warming below $1.5\,\mathrm{K}$ (United Nations, 2015). It is, however, questionable whether the commitments made by countries (the so-called Nationally Determined Contributions (NDCs)) are sufficient to keep temperatures below the 1.5 K and possibly even the 2.0 K target (Rogelj et al., 2016a).

A range of studies has appeared to provide insight on the safe level of cumulative emissions to stay below either the $1.5\,\mathrm{K}$ or $2.0\,\mathrm{K}$ target at a certain time in the future with a specified probability, usually taken as the year 2100. The choice of a particular year is necessarily arbitrary and neglects the possibility of additional future warming. Early studies made use of Earth System Models of Intermeditate Complexity (EMICs) (Zickfeld et al., 2009; Huntingford et al., 2012; Steinacher et al., 2013) to obtain such estimates. Because it was found that peak warming depends on cumulative carbon emissions E_Σ but is independent of the emission pathway (Allen et al., 2009; Zickfeld et al., 2012), focus has been on the specification of a safe level of E_Σ values corresponding to a certain temperature target. In more recent papers, also emulators derived from either C4MIP models (Sanderson et al., 2016) or CMIP5 (Coupled Model Intercomparison Project 5) models (Millar et al., 2017b), with specified emission scenarios, were used for this purpose. Such methodology was recently used in (Millar et al., 2017a) to argue that a post-2015 value of $E_\Sigma \sim 200$ GtC would limit post-2015 warming to less than 0.6°C (so meeting the 1.5 K target) with a probability of 66%.

In this paper we pose the following question: assume one wants to limit warming to a specific threshold in the year 2100, while accepting a certain risk tolerance of exceeding it, then, when, at the latest, does one have to start to ambitiously reduce fossil fuel emissions? The point in time when it is 'too late' to act in order to stay below the prescribed threshold is called (van Zalinge et al., 2017) the Point of No Return (PNR). The value of the PNR will depend on a number of quantities, such as the climate sensitivity and the means available to reduce emissions. To determine estimates of the PNR, a model is required of global climate development that a) is accurate enough to give a realistic picture of the behavior of GMST under a wide range of climate change scenarios, b) is forced by fossil fuel emissions, c) is simple enough to be evaluated for a very large number of different emission and mitigation scenarios and d) provides information about risk, i.e., it cannot be purely deterministic.

The models used in van Zalinge et al. (2017) are clearly too idealized to determine adequate estimates of the PNR under different conditions. In this paper, we therefore construct a stochastic state space model from the CMIP5 results where many global climate models were subjected to the same forcing for a number of climate change scenarios (Taylor et al., 2012). This stochastic model – representing all kinds of uncertainties in the climate model ensemble – is then used together with a broad range of mitigation scenarios to determine estimates of the PNR under different risk tolerances.

~~If~~ Stocker (2013) showed that if the Paris temperature targets are to be met, only a few years are left for policy makers to take action by cutting emissions~~(Stocker, 2013)~~: with an emissions reduction rate of $5\,\%\,\mathrm{yr}^{-1}$, the $1.5\,\mathrm{K}$ target has become unachievable and the $2.0\,\mathrm{K}$ target becomes unachievable after 2017. The ~~Stocker (2013)~~ Stocker (2013) analysis highlights the

[1]We define pre-industrial temperature as the 1861-1880 mean temperature, in accordance with IPPC AR5.

crucial concept of the closing door or PNR of climate policy, but it is deterministic. It does not take account of the possibility that these targets are not met, and does not allow for negative emissions scenarios. We here show how the considerable climate uncertainties captured by our stochastic state-space model~~of the carbon dynamics and temperature inertia,~~, the degree to which policy makers are willing to take risk, and the potential of negative emissions affect the carbon budget and the date at which climate policy becomes unachievable (the PNR). The climate policy is here not defined as an exponential emission reduction as in Stocker (2013) but as a steady increase in the share of renewable energy in total energy generation.

2 Methods

We let ΔT be the annual-mean area-weighted Global Mean Surface Temperature (GMST) deviation from pre-industrial conditions of which the 1861-1880 mean is considered to be representative (Pachauri et al., 2014; Schurer et al., 2017). From the CMIP5 scenarios we use the simulations of the pre-industrial control, abrupt quadrupling of atmospheric CO_2, smooth increase of 1% CO_2 per year, and the RCP (Representative Concentration Pathways) scenarios 2.6, 4.5, 6.0 and 8.5 (Taylor et al., 2012). The data is obtained from the German Climate Computing Center (DKRZ), the ESGF Node at DKRZ, and KNMI's Climate Explorer. The CO_2 forcings (concentrations (Meinshausen et al., 2011) and emissions (van Vuuren et al., 2007; Clarke et al., 2007; Fujino et al., 2006; Riahi et al., 2007)) are obtained from the RCP Database (available at http://tntcat.iiasa.ac.at/RcpDb).

As all CMIP5 models are designed to represent similar (physical) processes but use different formulations, parametrizations, resolutions and implementations, the results from different models offer a glimpse into the (statistical) properties of future climate change, including various forms of uncertainty. We perceive each model simulation as one possible, equally likely, realization of climate change. Applying ideas and methods from statistical physics (Ragone et al., 2016), in particular Linear Response Theory (LRT), a stochastic model is constructed that represents the CMIP5 ensemble statistics of ~~the~~ GMST.

2.1 Linear Response Theory

We use only those ensemble members from CMIP5 for which the control run and at least one perturbation run are available, leading to 34 members for the abrupt (CO_2 quadrupling) and 39 for the smooth-forcing experiment. Considering those members from the RCP runs also available in the abrupt forcing run, we have 25 members for RCP2.6, 30 for RCP4.5, 19 for RCP6.0 and 29 for RCP8.5.

The CO_2 concentration as a function of time for the abrupt quadrupling and smooth CO_2 increase is prescribed as

$$C_{CO2,abrupt}(t) = C_0(3\theta(t)+1) \tag{1}$$

$$C_{CO2,smooth}(t) = \begin{cases} C_0 & , \quad t \leq 0 \\ C_0 1.01^t & , \quad t > 0 \end{cases} \tag{2}$$

with time in years from the start of the forcing, pre-industrial CO_2 concentration C_0 and Heaviside function $\theta(t)$. The radiative forcing ΔF due to CO_2 relative to pre-industrial conditions is given as

$$\Delta F = \alpha_{CO2} \ln\left(\frac{C_{CO2}(t)}{C_0}\right) \tag{3}$$

with $\alpha_{CO2} = 5.35\,\mathrm{W\,m^{-2}}$ (Myhre et al., 2013). With LRT, the Green's function for the temperature response is computed from the abrupt forcing case as the time derivative of the mean response (Ragone et al., 2016)

$$G_T(t) = \frac{1}{\Delta F_{abrupt}} \frac{d}{dt} \Delta T_{abrupt} \tag{4}$$

where $\Delta F_{abrupt}(t) = \ln(4C_0/C_0) = \ln(4)$. The temperature deviation from the pre-industrial state for any forcing ΔF_{any} in then obtained, via the convolution of the Green's function, as

$$\Delta T_{any}(t) = \int_0^t G_T(t')\Delta F_{any}(t-t')\,dt' \tag{5}$$

Because equation (4) is exact we expect that (5) with $\Delta F_{any} = \Delta F_{abrupt}$ will exactly reproduce the abrupt CMIP5 response. In addition, for the LRT to be a useful approximation, the response has to reasonably reproduce the smooth $1\,\%\,\mathrm{yr}^{-1}$ CMIP5 response with $\Delta F_{any} = \Delta F_{smooth}$. Figure 1a shows that LRT applied to the abrupt perturbation recovers perfectly – as required – the abrupt response and is well able to recover the response to a smooth forcing. The correspondence is very good for the mean response and also the variance is captured quite well. ~~In order to apply LRT to the RCP scenarios, the radiative forcing has to be scaled up by a constant factor A as these - unlike the idealized abrupt and smooth scenarios - include non-fossil CO_2 emissions and non-CO_2 GHG emissions. The constant $A = 1.48$ was found in order to optimize the agreement of ΔT with CMIP5. The resulting reconstruction of temperatures from RCP CO_2 concentrations overlaid with CMIP5 data (Figure 1b), also gives a good agreement.~~

Beyond finding the temperature change as a result of CO_2 variations, eventually emissions E_{CO2} cause these CO_2 changes and have to be addressed explicitly. A multi-model study of many carbon models of varying complexity under different background states and forcing scenarios was recently presented (Joos et al., 2013). A fit of a three-timescale exponential with constant offset was proposed for the ensemble mean of responses to a $100\,\mathrm{GtC}$ emission pulse to a present-day climate of the form

$$G_{CO2}(t) = a\mu_0 + \sum_{i=1}^{3} a\mu_i e^{-\frac{t}{\tau_i}} \tag{6}$$

Coefficients ~~$a_i, i = 0\ldots3$~~ $\mu_i, i = 0,\ldots,3$ and timescales $\tau_i, i = 1\ldots3$ are determined using least-square fits on the multi-model mean. The CO_2 concentration then follows from

$$C_{CO2}(t) = \int_0^t G_{CO2}(t')\,E_{CO2}(t-t')\,dt' \tag{7}$$

In doing so, we use a response function that is independent of the size of the impulse, i.e. the carbon cycle reacts in the same way to pulses of all sizes other than $100\,\mathrm{GtC}$. This is of course a simplification, especially as very large pulses might unleash positive feedbacks to do with the saturation of natural sinks such as the oceans (Millar et al., 2017b), but works reasonably well in the range of emissions we are primarily interested in.

The full (temperature and carbon) LRT model is summarized as

$$C_{CO2}(t) = C_{CO2,0} + \int_0^t G_{CO2}(t')\, E_{CO2}(t-t')\, dt' \tag{8a}$$

$$\Delta F_{CO2}(t) = A\, \alpha_{CO2} \ln(C_{CO2}(t)/C_0) \tag{8b}$$

$$\Delta T(t) = \Delta T_0 + \int_0^t G_T(t') \Delta F_{CO2}(t-t') dt' \tag{8c}$$

and relates fossil CO_2 emissions E_{CO2} to mean GMST perturbation ΔT with initial conditions $C_{CO2,0}$ for CO_2 and ΔT_0 for GMST perturbation. This is quite a simple model with few 'knobs to turn'. The only really free parameter is the constant A that scales up CO_2-radiative forcing to take into account non-fossil CO_2 and non-CO_2 GHG emissions ~~.~~ (not present in the idealized scenarios), and matches the carbon and temperature models (estimated from different model ensembles) together. The constant $A = 1.48$ was found in order to optimize the agreement of ΔT with CMIP5 RCPs. The resulting reconstruction of temperatures from RCP CO_2 concentrations overlaid with CMIP5 data (Figure 1b) gives a good agreement. Internally, emissions need to be converted from $\mathrm{GtC\,yr^{-1}}$ to $\mathrm{ppm\,yr^{-1}}$ using the respective molar masses and the mass of the Earth's atmosphere as $E_{CO_2}[\mathrm{ppm\,yr^{-1}}] = \gamma E_{CO_2}[\mathrm{GtC\,yr^{-1}}]$ with $\gamma = 0.46969\,\mathrm{ppm\,GtC^{-1}}$. ~~In Table ?? we summarize our~~ Our estimates of the model's ten parameters are found in Table 2.

In Figure 2 we show the results obtained for RCP emissions. For very high emission scenarios we underestimate CO_2 concentrations because for such emissions natural sinks saturate, which is a process the pulse-size independent carbon response function cannot adequately capture. However, the up-scaling of radiative forcing is quite successful, yielding a good temperature reconstruction.

2.2 Stochastic State Space Model

The model outlined above still contains a data-based temperature response function and it informs only about the *mean* CMIP5 response. However, our main motivation is to obtain new insights on the possible evolution to a 'safe' carbon-free ~~,~~ state and such paths necessarily depend strongly on the variance of the climate and on the risk one is willing to take. This variance in temperature is quite substantial, as is evident from Figure 1b and 1c. Therefore we translate our response function model to a state-space model and incorporate the variance via suitable stochastic terms.

The response function G_T from the 140-year abrupt quadrupling ensemble is well approximated by

$$G_T(t) = \sum_{i=0}^{2} b_i e^{-\frac{t}{\tau_{bi}}} \tag{9}$$

Although $\tau_{b0} \to \infty$, we require a finite τ_{b0} for temperatures to stabilize at some level. Hence, we choose a long time scale $\tau_{b0} = 400\,\mathrm{yr}$ that cannot really be determined from the $140\,\mathrm{yr}$ abrupt forcing (CMIP5) runs. By writing

$$C = C_P + \sum_{i=1}^{3} C_i \tag{10a}$$

$$\Delta T = \sum_{i=0}^{2} \Delta T_i \tag{10b}$$

the LRT model can be transformed into the 7-dimensional Stochastic State Space Model (SSSM) shown in Table 1 with parameters in Table 2. Initial conditions are obtained by running the noise-free model forward from pre-industrial conditions ($C_P = C_0$ and $C_i = \Delta T_i = 0, i = 1,2,3$) to present-day, driven by historical emissions [2]. As these temperatures are now given relative to the start of emissions, i.e. 1765, we add the 1961-1990 model mean to the HadCRUT4 dataset to get observed temperature deviation relative to 1765, and compute ΔT relative to 1861-1880 by adding the 1861-1880 mean of this deviation time series.

The major benefit of this formulation is that we can include stochasticity. We introduce additive noise to the carbon model such that the standard deviation of the model response to an emission pulse as reported by (Joos et al., 2013) is recovered. For the temperature model we introduce (small) additive noise to recover the (small) CMIP5 control run standard deviation. In the CMIP5 RCP runs the ensemble variance increases with rising ensemble mean. This calls for the introduction of (substantial) multiplicative noise, which we introduce in ΔT_2, letting these random fluctuations decay over an 8-year timescale. The magnitude of these fluctuations is (especially at high temperatures) likely to be unrealistic when looking at individual time series. However, the focus here is on ensemble statistics.

2.3 Transition Pathways

The SSSM described in the previous section is forced with fossil CO_2 emissions. We assume that, in the absence of any mitigation actions, emissions increase from their initial value E_0 at an exponential rate ~~g~~ $g = 0.01\,\mathrm{yr}^{-1}$ due to economic and population growth. Political decisions cause emissions to decrease from starting year t_s onward as fossil energy generation is replaced by non-GHG producing forms such as wind, solar and water (mitigation m) and by an increasing share of fossil energy sources the emissions of which are not released but captured and stored away by Carbon Capture and Storage (abatement m). In addition, negative emission technologies ~~E_{neg}~~ may be employed~~that lead to a net~~. They cause a direct reduction in atmospheric CO_2 concentration and are here modelled as an exponential [3] $E_{neg}(t) = E_{neg,\infty}(1 - \exp(-rt))$. We model this

[2] ~~these~~ These are the fossil fuel and cement production emissions from (Le Quéré et al., 2016), accessed 28th March, ~~2017~~2017.

[3] For long time scales, these (after a transient) constant negative emissions may not be realistic. However, we are interested in the period until the year 2100.

in a very simple way by letting both mitigation and abatement increase linearly until emissions are brought to zero:

$$m(t) = \begin{cases} m_0 & t \le t_s \\ \min(m_0 + m_1(t - t_s), 1) & t > t_s \end{cases} \tag{11a}$$

$$a(t) = \begin{cases} a_0 & t \le t_s \\ \min(a_0 + m_1(t - t_s), 1) & t > t_s \end{cases} \tag{11b}$$

$$E(t) = E_0 e^{gt}(1 - a(t))(1 - m(t)) - E_{neg}(t) \tag{11c}$$

with constants m_0, a_0 giving the mitigation and abatement rates at the start of the scenario and m_1 the incremental year-to-year increase. The simplified model (11) is very well able (not shown) to reproduce the IAM pathways from that fulfil the NDCs until 2030 and afterwards reach the 2 K target with a 50-66% probability (Rogelj et al., 2016a). These pathways are exemplary for those that continue on the low-commitment path for a while, followed by strong and decisive action. From them we obtain a family of negative emission scenarios out of which we pick a pathway with strong negative emissions. It is very well approximated by setting $E_{neg,\infty} = 4.21\,\mathrm{GtC}$ and $r = 0.0283\,\mathrm{yr}^{-1}$.

2.4 Point of No Return

With the emission scenarios and the SSSM - returning CO_2 concentrations and GMST for any such scenario - one can now address the issue of transitioning from the present-day (year 2015) to a carbon-free era such as to avoid catastrophic climate change. We need to take into account both the *target* threshold and the *risk* one is willing to take to exceed it. The maximum amount of cumulative CO_2 emissions that allows reaching the 1.5 and 2 K targets, as a function of the risk tolerance, is called the Safe Carbon Budget (SCB). It is well established in the literature (Meinshausen et al., 2009; Zickfeld et al., 2009) but does not contain information on how these emissions are spread in time. This is where the Point of No Return (PNR) comes in: The PNR is the point in time where starting mitigating action is insufficient to stay below a specified target with a chosen risk tolerance.

Concretely, let the temperature target ΔT_{max} be the maximum allowable warming and denote the parameter β as the probability of staying below a given target (a measure of the risk tolerance). For example the case $\Delta T_{max} = 2\,\mathrm{K}$ and $\beta = 0.9$ corresponds to a 90% probability of staying below $2\,\mathrm{K}$ warming, i.e. 90 of 100 realizations of the SSSM, started in 2015 and integrated until 2100, do not exceed $2\,\mathrm{K}$ in the year 2100.

Then, in the context of (11), the PNR is the earliest t_s that does not result in reaching the defined 'Safe State' (van Zalinge et al., 2017) in terms of ΔT_{max} and β. It is determined from the probability distribution $p(\Delta T_{2100})$ of GMST in 2100. Both SCB and PNR depend on temperature target, climate uncertainties and risk tolerance, but the PNR also depends on the aggressiveness of the climate action considered feasible (here given by the value of m_1). This makes the PNR such an interesting quantity, since the SCB does not depend on the time path of emission reductions. Clearly there is a close connection between the PNR and the SCB. Indeed, one could define a PNR also in terms of the ability to reach the SCB. The one-to-one relation between cumulative emissions and warming gives the PNR in 'carbon space'. Its location in time, however, depends crucially on how fast a transition to a carbon-neutral economy is feasible.

~~Since it is now recognized that negative emissions may be essential in meeting temperature targets, we include this possibility into the PNR computation. From the IAM scenariosthat Rogelj et al. (2016a) found to fulfill NDCs until 2030 and stay below 2 K with 50-66% probability, we obtain a family of negative emission pathways (Figure ??) out of which we pick a 'moderate' (orange) and a 'strong' (red)pathway.~~ For details on the scenarios, we refer to Rogelj et al. (2016a). With carbon budgets rapidly running out and the PNR approaching fast, negative emissions may have to become an essential part of the policy mix. Such policies are cheap but may only be a temporary fix and lead to undesirable spillover effects on neighboring countries (e.g. Wagner and Weitzman (2015)). We abstract from these discussions here since this is beyond the scope of the present paper.

3 Results

To demonstrate the quality of the SSSM we initialise it at pre-industrial conditions, run it forward and compare the results with those of CMIP5 models. The SSSM is well able to reproduce the CMIP5 model behavior under the different RCP scenarios (Figure 3, shown for RCP2.6 and 4.5). As these scenarios are very different in terms of rate of change and total cumulative emissions this is not a trivial finding. It is actually remarkable that the SSSM, which is based on a limited amount of CMIP5 model ensemble members, performs so well. As an example, the RCP2.6 scenario contains substantial negative emissions, responsible for the downward trend in GMST, which our SSSM correctly reproduces. The mean response for RCP8.5 is slightly underestimated (not shown) because the uncertainty in the carbon cycle plays a rather minor role compared to that in the temperature model. In addition, for such large emission reductions positive feedback loops set in from which our SSSM abstracts. The temperature perturbation ΔT is very closely log-normally distributed while for weak forcing scenarios (e.g., RCP2.6 and RCP4.5) the distribution is approximately Gaussian. The CO_2 concentration is found to be Gaussian distributed for all RCP scenarios. These findings (log-normal temperature and Gaussian CO_2 concentration) result from the multiplicative and additive noise in temperature and carbon components of the SSSM, respectively.

To determine the SCB, 6000 emission reduction strategies (with $E_{neg}(t) = 0$) were generated and, using the SSSM, an 8000-member ensemble for each of these emission scenarios starting in 2015 was integrated. Emission scenarios are generated from (11) by letting $a(t) = 0$, a uniform $m_0 \in [0, 0.7]$ and m_1 drawn from a beta distribution (with distribution function $p(m) = \frac{1}{B(\alpha,\delta)} m^\alpha (1-m)^{(\delta-1)}$, where $B(\alpha, \delta)$ is the beta function; parameters are chosen as $\alpha = 1.2, \delta = 3$), with the [0,1] interval scaled such that $m = 1$ latest in 2080. The Beta distribution is chosen for practical reasons to sample (m_0, m_1) pairs. As m_0 is drawn from a uniform distribution, doing likewise for m_1 would result in many pathways with very quick mitigation and low cumulative emissions. Choosing a Beta distribution for m_1 makes draws of small m_1 much more likely and leads to a better sampling of high cumulative emission scenarios. The choice of distribution has no consequences on the results.

The temperature anomaly in 2100 (ΔT_{2100}) as a function of cumulative CO_2 emissions E_Σ is shown in Figure 4. The same calculation is also shown for the deterministic case without climate uncertainty (no noise in the SSSM). In Figure 4, the SCB is given by the point on the E_Σ-axis where the (colored) line corresponding to a chosen risk tolerance crosses the (horizontal) line corresponding to a chosen temperature threshold ΔT_{max}. The curves $\Delta T_{2100} = f(E_\Sigma)$ (Figure 4) are very well described by expressions of the type

$$f(E_\Sigma) = a \ln\left(\frac{E_\Sigma}{b} + 1\right) + c \tag{12}$$

with suitable coefficients a, b and c, each depending on the tolerance β. For the range of emissions considered here, a linear fit would be reasonable (Allen et al., 2009). However, our expression also works for cumulative emissions in the range of business as usual (when fitting parameters on suitable emission trajectories). From Figure 4 we easily find the SCB for any combination of ΔT_{max} and β, as shown in Table 3.

Allowable emissions are drastically reduced when enforcing the target with a higher probability (following the horizontal lines from right to left in Figure 4). These results show in particular the challenges posed by the $1.5\,\mathrm{K}$ compared to the $2\,\mathrm{K}$ target. ~~The sensitivity of the SCB to the relevant model parameters is shown in the Appendix and the values are robust.~~ From IPCC-AR5 (IPCC, 2013) we find cumulative emissions post-2015 of $377\,\mathrm{GtC}$ to $517\,\mathrm{GtC}$ in order to 'likely' stay below $2\,\mathrm{K}$ while we find an SCB of $424\,\mathrm{GtC}$ for $\Delta T_{max} = 2\,\mathrm{K}, \beta = 0.67$ which lies in the same range. Like Millar et al. (2017a) we find approximately $200\,\mathrm{GtC}$ to stay below $2\,\mathrm{K}$ with $\beta = 0.67$.

To determine the PNR, we resort to three illustrative choices to model the abatement and mitigation rates with $E_{neg}(t) = 0$. Following (11) we construct Fast Mitigation (FM) and Moderate Mitigation (MM) scenarios with $m_1 = 0.05$ and 0.02, respectively. In addition, in an Extreme Mitigation (EM) scenario $m = 1$ can be reached instantaneously. This corresponds to the most extreme physically possible scenario and serves as an upper bound. When varying t_s to find the PNR for the three scenarios, we always keep $m_0 = 0.14$ and $a_0 = 0$ at 2015 values (World Energy Council, 2016).

As an example, $t_s = 2025$ leads to total cumulative emissions from 2015 onward of 109, 183 and 335 GtC for the mitigation scenarios EM, FM and MM, respectively. ~~Note that while~~ MM is the most modest scenario, but it is actually quite ambitious, considering that with $m = 0.1355$ in 2005 and $m = 0.14$ in 2015 (World Energy Council, 2016) the current year-to-year increases in the share of renewable energies are very small.

Figure 5 shows the probabilities for staying below the 1.5 and $2.0\,\mathrm{K}$ thresholds in 2100 as function of t_s for different policies, including FM ($m_1 = 0.05$) and MM ($m_1 = 0.02$), while the EM policy bounds the unachievable region. It is clear that this region is larger for the 1.5 than for the 2.0 degree target, and shrinks when including negative emissions. From the plot we can directly see the consequences of delaying action until a given year. For example, if policy makers should choose to implement the MM strategy only in 2040, the chances of reaching the 1.5 (2.0) degree target are only 2% (47%). We conclude that the remaining 'window of action' may be small, but a window still exists for both targets. For example, the $2\,\mathrm{K}$ target is reached with a probability of 67% even when starting MM is delayed until 2035. However, reaching the $1.5\,\mathrm{K}$ target appears unlikely as MM would be required to start in 2018 for a probability of 67%. When requiring a high (≥ 0.9) probability, it is impossible to reach with the MM scenario. The PNR for the different targets and probabilities is ~~given in Table ??. The robustness of these PNR values is shown in the Appendix.~~ shown in Table 4 and Figure 5.

~~We also see from Figure 5and Table ?? that the inclusion of~~ Including strong negative emissions delays the PNR by 6-10 years~~(see Table ??)~~, which may be very valuable especially for ambitious targets. For example, ~~when including 'strong'~~

~~negative emissions one can~~ one can then reach 1.5 K with a probability of up to 66% in the MM scenario when acting before 2026, 8 years later than without. The PNR varies substantially for slightly different temperature targets. This also illustrates the importance of the temperature baseline relative to which ΔT is defined~~. This~~, as has been found previously (Schurer et al., 2017)~~, and we find (not shown) that switching to an~~. Switching to a (lower) 18^{th} century baseline ~~can move the PNRearlier by up to 10 years~~increases current levels of warming by 0.13 K (Schurer et al., 2017) and thereby brings forward the PNR. For example, for a maximum temperature threshold of 1.5 K the PNR moves from 2022 to 2016 in the MM scenario and from 2038 to 2033 for the EM scenario.

It is clear that an energy transition more ambitious than RCP2.6 is required to stay below 1.5 K with some acceptable probability, and whether that is feasible is doubtful. For all other RCP scenarios, exceeding 2 K is very likely in this century (Figure 6).

The parameter sensitivities of SCB and PNR were determined by varying each parameter by ±5%. Table 5 shows the results for selected parameters for a small ($T_{max} = 1.5\,\mathrm{K}, \beta = 0.95$), intermediate ($T_{max} = 1.5\,\mathrm{K}, \beta = 0.5$), and large ($T_{max} = 2.0\,\mathrm{K}, \beta = 0.5$) SCB, corresponding to a close, intermediate and far PNR.

The biggest sensitivities are found for the radiative forcing parameter A. The parameters of the carbon model (μ_i, τ_i) do not have big impacts on the found SCB, on the order of $0 - 17\,\mathrm{GtC}$, with larger numbers found for larger absolute values of SCB. The temperature-model parameters are more important, changing the SCB by up to around 10% for large and 50% for small values. The model is particularly sensitive to changes in the intermediate timescale (b_2, τ_{b2}). The PNR sensitivities are generally small. We find the most relevant, yet small, sensitivities in the temperature model parameters. For example, a 10% error in τ_{b2} can move the PNR by 3-4 years.

The sensitivity of SCB and PNR to the noise amplitudes is small, with largest values found for the multiplicative noise amplitude σ_{T2} that is responsible for most of the spread of the temperature distribution. Increasing noise amplitudes decreases the SCB, in accordance with the expectation that larger climate uncertainty leads to tighter constraints. It is useful to remember that the stochastic formulation of our model is designed with the explicit purpose to incorporate parameter uncertainty in a natural way via the noise term, without having to make specific assumptions on the uncertainties of individual parameters.

4 Summary, Discussion and Conclusions

We have developed a novel stochastic state space model (SSSM) to accurately capture the basic statistical properties (mean and variance) of the CMIP5 RCP ensemble, allowing us to study warming probabilities as function of emissions. It represents an alternative to the approach that contains stochasticity in the parameters rather than the state. Although the model is highly idealized, it captures simulations of both temperature and carbon responses to RCP emission scenarios quite well.

A weakness of the SSSM is the simulation of temperature trajectories beyond 2100 and for high emission scenarios. The large multiplicative noise factor leads – especially at high mean warmings – to immensely volatile trajectories that in all likelihood are not physical (on the individual level, the distribution is still well-behaved). It might be ~~a worthy endeavour~~ worthwhile to investigate how this could be improved. Another weakness in the carbon component part of the SSSM is that the real carbon cycle is not ~~pulse-independent~~pulse size-independent. Hence, using a single constant response function has inherent problems, in particular when running very high-emission scenarios. This is because the efficiency of the natural carbon sinks to the ocean and land reservoirs is a function both of temperature and the reservoir sizes. The SSSM has therefore slight problems reproducing CO_2 concentration pathways (Figure 2), a price we accept to pay as we focus on the CMIP5 temperature reproduction. Taking account of non-CO_2 emissions more fully beyond our simple scaling and also avoiding temporary overshoots of the temperature caps would reduce the carbon budgets (Rogelj et al., 2016b) and thus lead to earlier PNRs than given here. Therefore the values might be a little too optimistic.

In Millar et al. (2017b), the authors draw a different conclusion from studying a similar problem. They introduce in their FAIR model response functions that dynamically adjust parameters based on warming to represent sink saturation. Consequently, their model gives much better results in terms of CO_2 concentrations. It would be an interesting lead for future research to conduct our analysis ~~here~~ (in terms of SCB and PNR) with other simple models (such as FAIR or MAGICC) to discover similarities and differences. However, only rather low-emission scenarios are consistent with the 1.5 or 2 K targets, so we do not expect ~~this~~ such nonlinearities to play a major role, and indeed our carbon budgets are very similar to Millar et al. (2017a).

The concept of a Point of No Return introduces a novel perspective into the discussion of carbon budgets that is often centered on the question of when the remaining budget will have ‘run out’ at current emissions. In contrast, the PNR concept recognizes the fact that emissions will not stay constant and can decay faster or slower depending on political decisions. With these caveats in mind, we conclude that, first, the PNR is still relatively far away for the 2.0 K target: with the MM scenario and $\beta = 67\%$ we have 17 years left to start. When allowing to set all emissions to zero instantaneously, the PNR is even delayed to the 2050s. Considering the slow speed of large-scale political and economic transformations, decisive action is still warranted, as the MM scenario is a large change compared to current rates. Second, the PNR is very close or passed for the 1.5 K target. Here more radical action is required – 9 years remain to start the FM policy to avoid 1.5 K with a 67% chance, and strong negative emissions ~~gives~~ give us 8 years under the MM policy.

Third, we can clearly show the effects of changing $\Delta T_{max}, \beta$ and the mitigation scenario. Switching from 1.5 to 2 K buys an additional ≈ 16 years. Allowing a one-third, instead of one-tenth exceedance risk, buys an additional 7-9 years. Allowing for the more aggressive FM policy instead of MM buys an additional 10 years. This allows to assess trade-offs, for example between tolerating higher exceedance risks and implementing more radical policies. Fourth, negative emissions can offer a brief respite but only delay the PNR by a few years, not taking into account the possible decrease in effectiveness of these measures in the long term (Tokarska and Zickfeld, 2015).

In this work a large ensemble of simulations was used in order to average over stochastic internal variability. This allows to determine the point in time where a threshold is crossed at a chosen probability level. Such an ensemble is not possible for more realistic models, nor do GCMs agree on details of internal variability. Therefore, in practice, the crossing of a threshold will likely be determined with hindsight and using long temporal means. This fact should lead us to be more cautious in choosing mitigation pathways.

We have shown the constraints put on future emissions by restricting GMST increase below 1.5 and 2 K, respectively, and the crucial importance of the safety probability. Further (scientific and political) debate is essential on what are the right values for both temperature threshold and probability. Our findings are sobering in light of the bold ambition in the Paris agreement, and add to the sense of urgency to act quickly before the PNR has been crossed.

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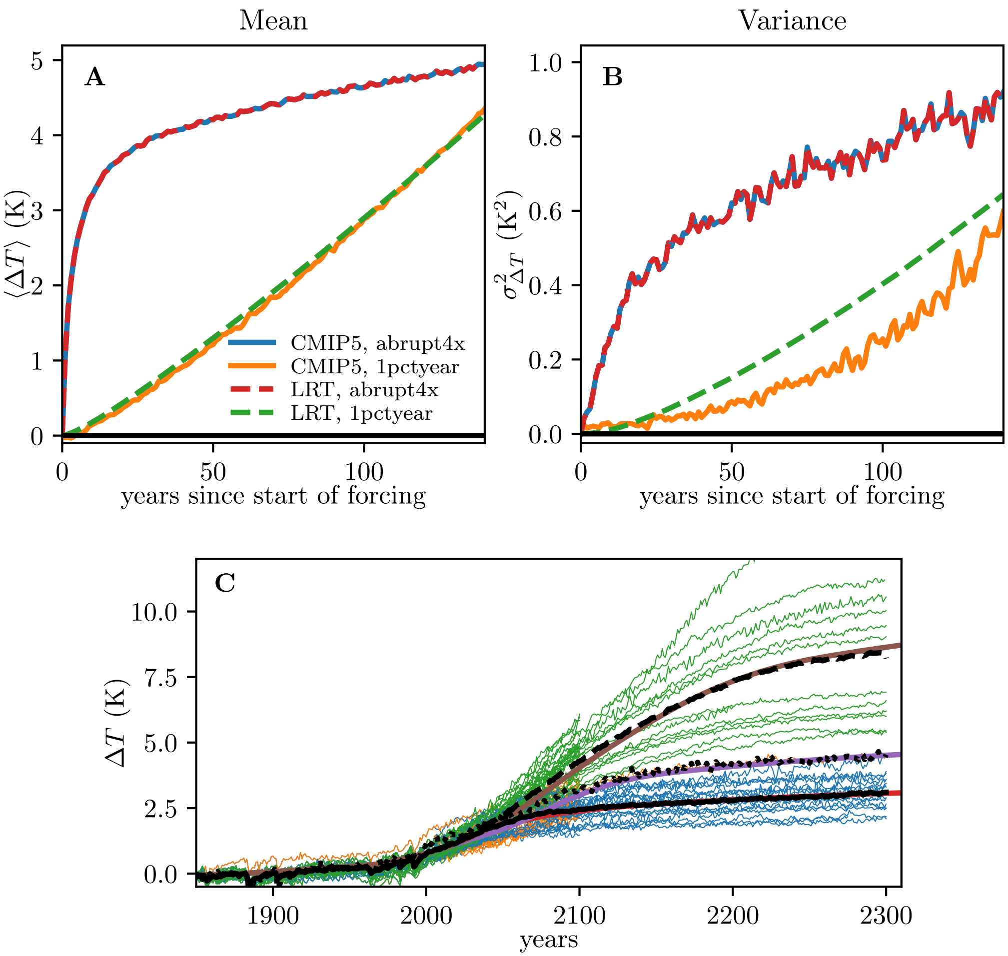

Figure 1. Ensemble mean (**A**) and variance (**B**) of temperature response from CMIP5 (solid) and LRT reproduction (dashed). Year 0 gives the start of the perturbation. (**C**) Reconstruction of RCP temperature evolution from concentration pathways using CO_2 only. Blue, orange and green lines gives CMIP5 data for RCP4.5, RCP6.0 and RCP8.5, respectively, with the ensemble mean given in black solid (RCP4.5), dotted (RCP6.0) and dashed (RCP8.5) black. Reconstruction using CO_2 radiative forcing in red (RCP4.5), purple (RCP6.0) and brown (RCP8.5).

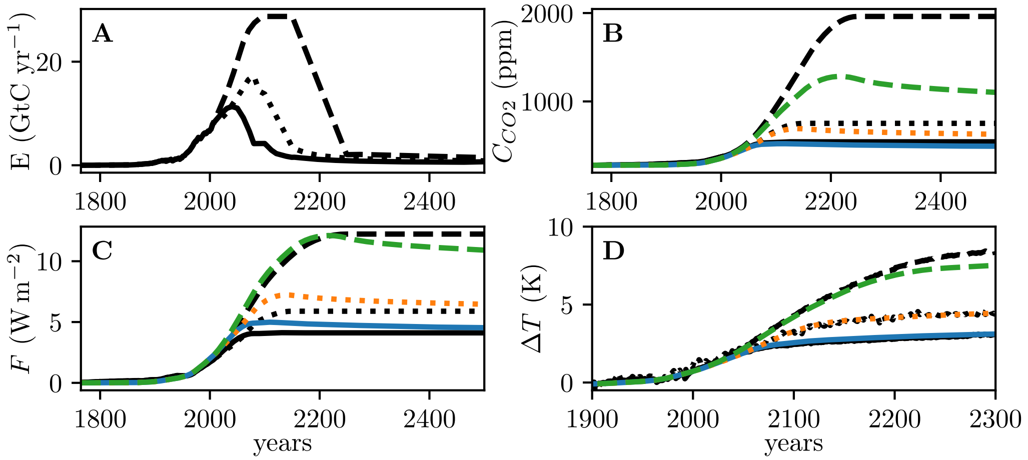

Figure 2. Reconstruction of RCP results using the Response Function Model. In all panels, solid lines refer to RCP4.5, dotted to RCP6.0 and dashed lines to RCP8.5. Black lines show RCP data while colors (blue: RCP4.5, orange: RCP6.0, green: RCP8.5) give our reconstruction. **(A):** Fossil CO_2 emissions. **(B):** CO_2 concentrations from RCP and reconstructed using G_{CO2}. **(C):** Total anthropogenic radiative forcing (black) and radiative forcing from CO_2 only (red) (both from RCP) and reconstructed forcing using the relations above. **(D):** Temperature perturbation from CMIP5 RCP (ensemble mean) and the our reconstruction.

~~Negative Emissions from IAM scenarios (Rogelj et al., 2016a), with two sample pathways marked.~~

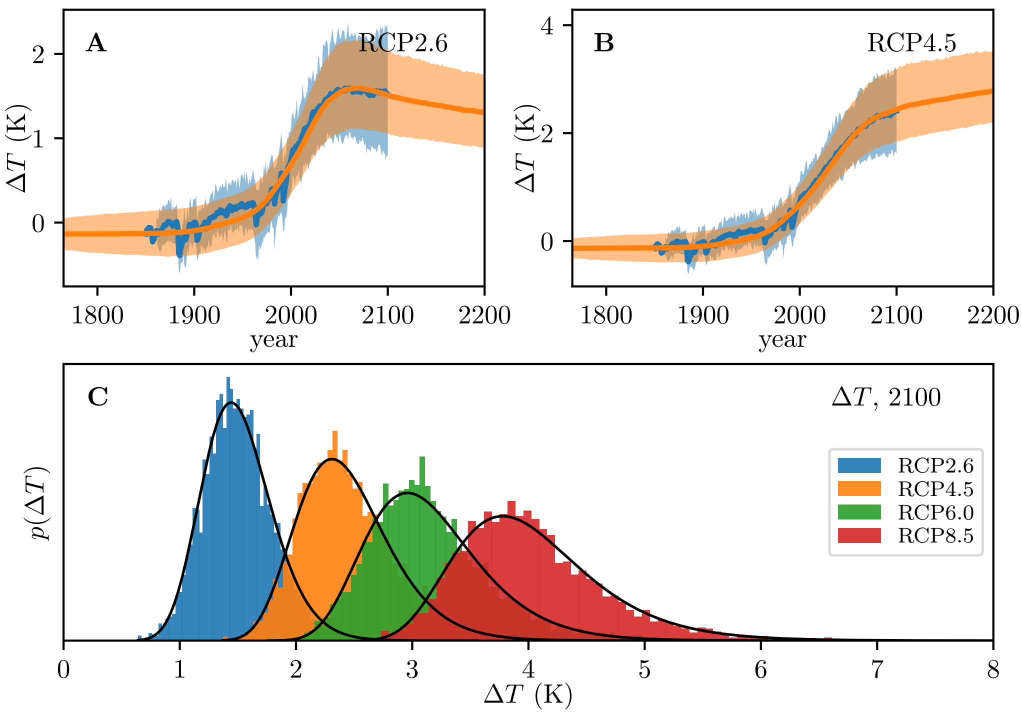

Figure 3. Stochastic State Space Model applied to RCP scenarios. **(A,B):** Ensemble mean and 5^{th}, 95^{th} percentile envelopes of CMIP5 RCPs (blue) and stochastic model (orange). **(C):** Probability density functions for ΔT in 2100 based on 5000 ensemble members, and driven by forcing from RCP2.6 (blue), RCP4.5 (orange), RCP6.0 (green) and RCP8.5 (red). In black are fitted lognormal distributions.

~~C_0 (ppm) a_0 a_1 a_2 a_3 278 0.2173 0.2240 0.2824 0.2763 A α ($\mathrm{W\,m^{-2}}$) τ_1 τ_2 τ_3 1.48 5.35 394.4 36.54 4.304 Response Function Model Parameters. All timescales τ_i are in years and the carbon model amplitudes a_i are dimensionless for E in $\mathrm{ppm\,yr^{-1}}$.~~

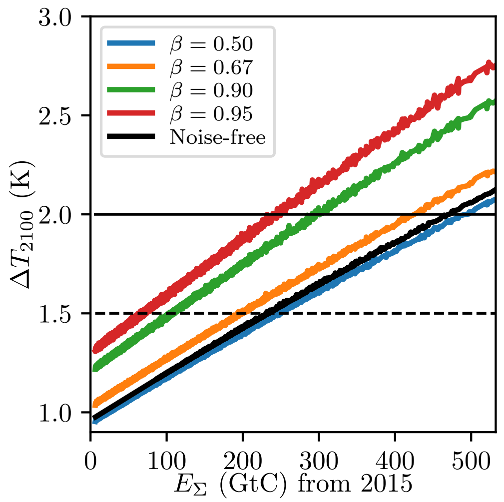

Figure 4. The Safe Carbon Budget. ΔT_{max} in 2100 such that $p(\Delta T_{2100} \leq \Delta T_{max}) = \beta$ as a function of cumulative emissions for different β. The black curve gives the deterministic results with noise terms in the stochastic model set to zero.

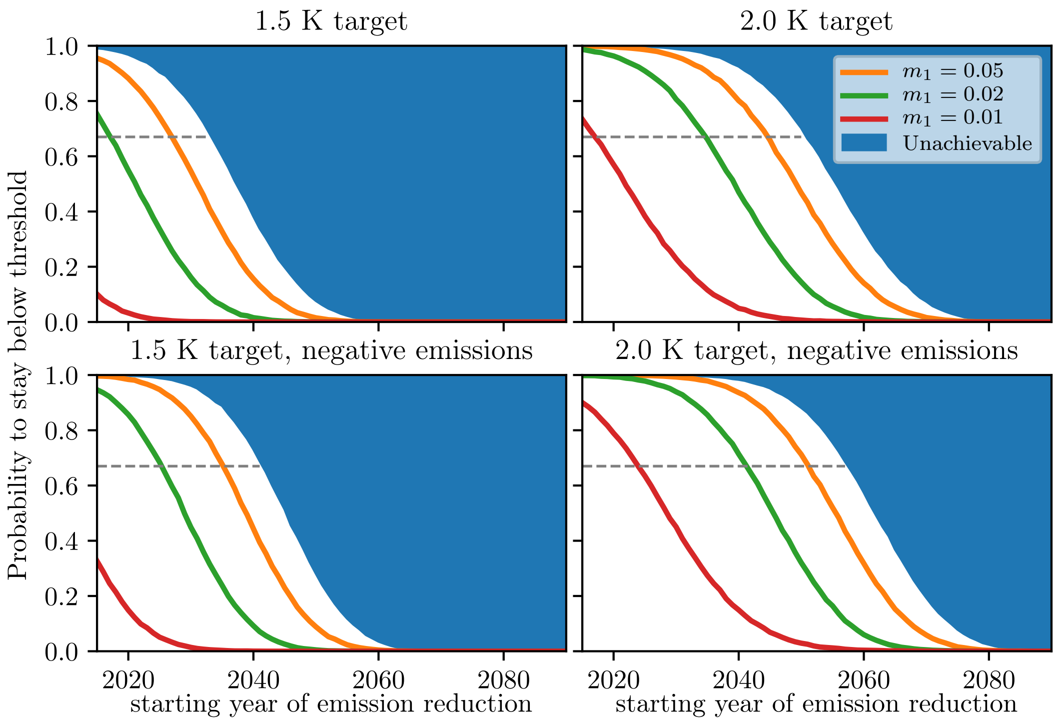

Figure 5. The Point of No Return. Probability of staying below the 1.5 K (left) or 2.0 K (right) threshold when starting emission reductions in a given year, for different policies as described by in equation 11 with different choices for m_1, the rate of mitigation increase per year. Top and bottom panels show the cases without ~~(top)~~ and with ~~(bottom)~~ strong negative emissions, respectively. The Point of No Return for a given policy is given by the point in time where the probability drops below a chosen threshold. The default threshold of two-thirds is dashed. The unachievable region is bounded by the extreme mitigation scenario.

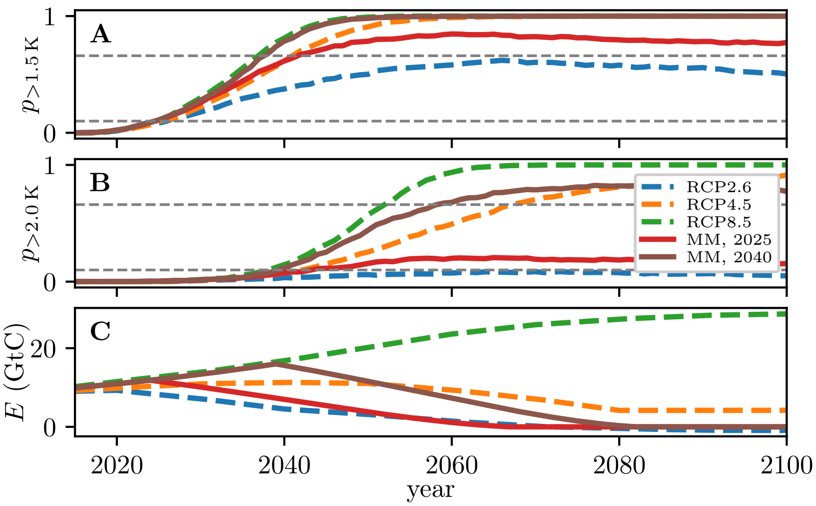

Figure 6. **(A,B):**Instantaneous probability to exceed 1.5 K **(A)** and 2.0 K **(B)** for different emission scenarios. RCP scenarios are shown as dashed lines while solid lines give MM scenario results starting in 2025 (red) and 2040 (brown). Dashed horizontal lines give $p = 0.1$ and 0.67, respectively. **(C)**: Fossil fuel emissions in GtC for the same scenarios.

$$
\begin{aligned}
dC_P &= \mu_0 E dt \\
dC_1 &= (\mu_1 E - \frac{1}{\tau_1} C_1) dt \\
dC_2 &= (\mu_2 E - \frac{1}{\tau_2} C_2) dt + \sigma_{C2}\, dW_t \\
dC_3 &= (\mu_3 E - \frac{1}{\tau_3} C_3) dt \\
C &= C_P + \sum_{i=1}^{3} C_i
\end{aligned}
$$

$$
\begin{aligned}
\Delta F &= A\, \alpha \ln(C/C_0) \\
d\Delta T_0 &= (b_0 \Delta F - \frac{1}{\tau_{b0}} \Delta T_0) dt + \sigma_{T0}\, dW_t \\
d\Delta T_1 &= (b_1 \Delta F - \frac{1}{\tau_{b1}} \Delta T_1) dt \\
d\Delta T_2 &= (b_2 \Delta F - \frac{1}{\tau_{b2}} \Delta T_2) dt \quad + \sigma_{T2} \Delta T_2\, dW_t \\
\Delta T &= \sum_{i=0}^{2} \Delta T_i
\end{aligned}
$$

Table 1. Stochastic State Space Model. Carbon model on left, temperature model on the right. W_t denotes the Wiener process.

$\cancel{a_0}\,\mu_0$	$\cancel{a_1}\,\mu_1$	$\cancel{a_2}\,\mu_2$	$\cancel{a_3}\,\mu_3$	τ_1 (yr)	τ_2 (yr)	τ_3 (yr)
0.2173	0.2240	0.2824	0.2763	394.4	36.54	4.304
C_0 (ppm)	b_0 ($\mathrm{K\,yr^{-1}\,W^{-1}\,m^2}$)	b_1 ($\mathrm{K\,yr^{-1}\,W^{-1}\,m^2}$)	b_2 ($\mathrm{K\,yr^{-1}\,W^{-1}\,m^2}$)	τ_{b0} (yr)	τ_{b1} (yr)	
278	0.00115176	0.10967972	0.03361102	400	1.42706247	
A	α ($\mathrm{W\,m^{-2}}$)	σ_{C2} ($\mathrm{ppm/yr^{1/2}}$)	σ_{T0} ($\mathrm{K/yr^{1/2}}$)	σ_{T2} ($\mathrm{yr^{-1/2}}$)	τ_{b2} (yr)	
1.48	5.35	0.65	0.015	0.13	8.02118539	

Table 2. Stochastic State Space Model Parameters. All timescales are in years, the carbon model amplitudes $\cancel{a_i}\,\mu_i$ are dimensionless for E in $\mathrm{ppm\,yr^{-1}}$, the temperature model amplitudes b_i are in $\mathrm{K\,W^{-1}\,m^2\,yr^{-1}}$.

β	0.5	0.67	0.9	0.95	Noise-free
$T_{max} = 1.5\,\mathrm{K}$	247	198	107	69	233
$T_{max} = 2.0\,\mathrm{K}$	492	424	298	245	469

Table 3. Safe Carbon Budget (in GtC since 2015) as function of threshold and safety probability β.

	β	0.5		0.67		0.9		0.95		noise-free	
	E_{neg}	none	strong	none	strong	none	strong	none	strong	none	strong
EM	$T_{max} = 1.5\,\mathrm{K}$	2038	2046	2034	2042	2026	2035	2022	2032	2037	2045
	$T_{max} = 2.0\,\mathrm{K}$	2056	2062	2051	2058	2042	2049	2038	2046	2055	2061
FM	$T_{max} = 1.5\,\mathrm{K}$	2032	2039	2027	2036	2020	2028	2016	2025	2030	2038
	$T_{max} = 2.0\,\mathrm{K}$	2050	2056	2045	2052	2036	2043	2032	2039	2048	2055
MM	$T_{max} = 1.5\,\mathrm{K}$	2022	2029	2018	2026	–	2019	–	–	2021	2029
	$T_{max} = 2.0\,\mathrm{K}$	2040	2046	2035	2042	2026	2033	2022	2030	2038	2045

Table 4. Point of No Return as function of threshold and safety probability β without and with strong negative emissions.

~~Appendix: SCB and PNR Parameter Sensitivity~~

~~SCB and PNR sensitivities were determined by varying each parameter by $\pm 10\%$ and running the calculation to see how the obtained value changes. Sensitivities were determined for all discussed values of T_{max}, β, and the EM, FM and MM scenarios in case of PNR. We show (Table 5) sample values for a small ($T_{max} = 1.5\,\mathrm{K}, \beta = 0.95$), intermediate ($T_{max} = 1.5\,\mathrm{K}, \beta = 0.5$), and large ($T_{max} = 2.0\,\mathrm{K}, \beta = 0.5$) SCB, corresponding to a close, intermediate and far PNR.~~

~~The biggest effects on the SCB are found for the initial condition of the large carbon reservoirs and the radiative forcing parameters A, α and C_0 that are essentially fixed constants. The parameters of the carbon model (a_i, τ_i) do not have big impacts on the found SCB, on the order of $0 - 17\,\mathrm{GtC}$, with the larger numbers found for larger absolute values of SCB. Varying the temperature-model parameters can have quite noticeable effects, up to 10% for large and up to 50% for small values of SCB. The model is particularly sensitive to changes in the intermediate timescale (b_2, τ_{b2}). Likely, possible variations in the (model) parameters are not independent, potentially canceling each other. The sensitivity of SCB and PNR to the noise amplitudes is small, with largest values found for the multiplicative noise amplitude that is responsible for much of the spread of the temperature distribution (so increasing σ_{T2} decreases the SCB).~~

~~The PNR sensitivities are generally small and in no way change our message qualitatively. The effect of initial conditions and carbon model parameters is small, often even unnoticeable (with the exception of the permanent carbon reservoir, due to its large size). We find the most relevant, yet small, sensitivities in the temperature model parameters. For example, a 10% error in τ_{b2} can move the PNR by 2-3 years. An interesting effect is the case of r_γ, the energy-saving progress (reduction in energy-intensity of a unit of economic output and in effect equivalent to a decrease in the emission growth rate) which is taken~~

	SCB		
T_{max}, β	$1.5K, 0.95$	~~$1.5K, 0.5$~~ $1.5K, 0.67$	~~$2.0K, 0.5$~~ 2.0
undisturbed	~~68.63~~ 69	~~247.02~~ 198	~~492.09~~ 4
~~C_1~~ μ_1	~~15.06, -15.01~~ 3, -3	~~14.75, -14.48~~ 8, -8	~~14.65, -13.41~~
~~ΔT_1~~ μ_2	~~-0.12, -0.05 0.09, -0.0 0.52, 0.04 0, 0 0, 0 0, 0~~ ~~ΔT_2~~ ~~-0.04, -0.03 0.05, 0.1 -0.04, -0.49~~ 1, ~~0~~ -1	~~0, 0~~ 2, -3	~~0, 0~~ ~~a_1~~ 5
τ_1	~~3.64, -3.02~~ 4, -3	~~4.73, -3.75~~ 4, -4	~~5.92, -4.43~~
τ_2	~~4.58, -4.48~~ 4, -4	~~7.6, -7.1~~ 6, -6	~~12.44, -11.08~~
A	~~55.59, -44.99~~ 56, -45	~~80.98, -64.43~~ 73, -59	~~118.57, -93.23~~
~~C_0~~ b_1	~~–, 169.67 -188.37, 182.48 -205.7, 199.12 –, 13 -15, 11~~ 12, -12 ~~, 10~~ ~~b_1~~	~~12.17, -11.57~~ 19, -19	~~22.74, -21.04~~
b_2	~~32.08, -28.29~~ 32, -28	~~38.94, -34.41~~ 37, -33	~~57.89, -50.29~~
τ_{b1}	~~12.31, -11.83~~ 12, -12	~~23.02, -21.17~~ 19, -18	~~34.51, -30.64~~
τ_{b2}	~~37.84, -33.21~~ 38, -33	~~38.13, -33.51~~ 38, -34	~~56.77, -49.36~~
~~γ_0~~ σ_{T2}	~~∼, ∼~~ 10, -10	~~∼, ∼~~ 0, 0	~~∼, ∼~~ 1, -1

Table 5. Sensitivity of Safe Carbon Budget and Point of No Return to selected parameter variations. Values as difference in GtC (SCB) and number of years (PNR) ~~from~~ relative to the undisturbed value (~~first~~ top row). The PNR values all refer to the EM scenario. First and second numbers give 10% parameter decrease and increase, respectively.~~Exception is r_γ (in orange) which is zero by default and where first and second numbers give $r_\gamma = 0.01$ and $r_\gamma = 0.02$, respectively. No sensitivities are calculated for the SCB for the economic parameters γ_0 and r_γ and replaced by (∼), whereas (–) implies no positive SCB/PNR could be calculated. The fields corresponding to the radiative forcing parameters A, α, C_0 are colored in cyan, while the most sensitive climate model parameters b_2, τ_{b2} are given in orange.~~

~~zero by default. Increasing it to 1% or 2% has little effect on *close* PONR (e.g. 2020) but is capable of delaying *late* PNR by up to 15 years, and the effect is more substantial for the less ambitious scenarios. This is an interesting finding, showing that in the long run increasing energy efficiency can play a role in avoiding the PNR.~~