Earth Syst. Dynam. Discuss.,
doi:10.5194/esd-2017-8-RC2, 2017

Earth System Dynamics Discussions

Interactive comment on "Inverse stochastic-dynamic models for high-resolution Greenland ice-core records" *by* Niklas Boers et al.

T. Mitsui (Referee)

takahito321@gmail.com

Received and published: 5 May 2017

1 General comments

This work is a contribution to inverse stochastic-dynamic modeling of abrupt climate changes, so-called Dansgaard-Oeschger (DO) events, using high-resolution Greenland ice core records. While many of the previous stochastic-dynamic models of DO events have been introduced in intuitive, ad-hoc ways (for example, assuming a double-well potential or an additive white noise), their stochastic delay differential equation is grounded in the Mori-Zwanzig (MZ) formalism of statistical physics. It is interesting to consider the coupling between different proxies δ^{18}O and log(dust) though its physical mechanism is not mentioned in this paper. The authors show that the coupling and the

Printer-friendly version

Discussion paper

memory terms due to MZ formalism are important to capture the asymmetric sawtooth oscillations and, in part, they are contributing to the bimodal character of the probability density functions (PDFs). These points are particularly valuable in inverse modeling of DO events. On the other hand, as mentioned below, some remarks are based on visual assessments, and thus sound less convincing. There are also confusions in the use of information criteria. Therefore, I recommend the publication of this article if the following points are revised.

2 Specific comments

(1) The successful reproduction of the time-reversal asymmetry of the sawtooth oscillations is a key result of this article. However, the quantitative assessment of the time-reversal asymmetry has not been done. The visual comparison between Fig. 2 and Fig. 4A has to be complemented by a more quantitative assessment. For instance, one may claim that simulated events during 56–58 ka and 29–30 ka in Fig. 2 look time-symmetric. I suggest some quantitative assessment of the time-reversal asymmetry (for example, by using the third-order moment $x(t)x^2(t+\tau) - x^2(t)x(t+\tau)$ as in Kwasniok, 2013).

Also, the authors write *"In particular, the variations between quiet and burst episodes observed in the dust series are not reproduced in this case (Fig. A6)"*, but I recognize quiet episodes during 34–38 ka, 42–45 ka, and 47–50 ka and burst episodes during the others in Fig. A6.

(2) There are confusions in the use of information criteria BIC and AICc[1]. If n is the number of time points of observation vectors, they should be BIC$= p\log(n) - 2\log(L^*)$ and AICc$= 2pn/(n-p-1) - 2\log(L^*)$, where p is the number of parameters and L^*

[1] I suggest to mention that the sample-size-corrected AIC, AICc, is used in this paper instead of the standard AIC.

is the maximum likelihood. The authors seem to follow the notation in Krumscheid et al., 2015, but in their paper, the number of observations is $n+1$. Also the sign in front of $2\log(L^*)$ should be minus. The number n in BIC is the number of time points of observation vectors, and not the total number of the elements in the observation vectors (e.g. Raftery 1995; see Yamaguchi and Higuchi 2006 for the case of state-space model). Thus, the authors should not multiply the number of time points 7529 by 2 to obtain the value of n used for BIC.

(3) The number of memory terms $d = 2$ and the delay $\tau = 75$ a are chosen such as to have the average PDFs of the simulated time series as close as possible to those of the observed ones. However, given that the purely Markovian form of the model approximates the PDFs of the observed time series relatively well (Fig. A5), I'm not sure that matching PDFs is an appropriate way to choose the values of d and τ. Does BIC or AICc select similar values?

(4) A comment about the conclusion: *"Our results demonstrate that the statistical characteristics of the roughly 40-ka–long, high-resolution NGRIP time series of δ^{18}O and dust considered here can be reproduced by a nonlinear inverse model without taking into account exogenous forcing, whether astronomical, solar or volcanic. There is thus no reason to assume that the temporal evolution of the δ^{18}O ratios and dust concentrations – and hence that of the climatic variabilities they represent, in particular the transitions between stadials and interstadials – are externally forced."*

I agree that abrupt DO warmings and coolings themselves are not governed by external forcings, but, for example, Mitsui and Crucifix (2017) show the influence of external forcings on the DO events. In that paper, the slow decay of the sample autocorrelation function of the NGRIP $\log_{10}$Ca^{2+} record is simulated only in the presence of ice volume forcing. The BIC evidence of forced models against unforced model is also reported for some model classes. Thus, there are some reasons to assume that the temporal evolution of the δ^{18}O ratios and dust concentrations have a non-autonomous character.

3 Technical corrections and minor comments

Page 1, Line 16: Is "consation" right?

Page 2, Line 10: F should be bold as in Eq. (1)?

Page 6, Line 7: twodimensional $\rightarrow$ two-dimensional

Page 6, Line 12: If n and s are both non-dimensional indices, it is strange that τ (=75 a) is dimensional in Eqs. (4)–(7).

Page 7, Line 20: A square root $\sqrt{\cdot}$ is missing for $(2\pi)^2|\Sigma|$ in Eq. (7)?

Page 8, Lines 5 and 31: (5) $\rightarrow$ Eq. (5) is desirable?

Page 9, Line 5: what is the unit for $\delta t = 10^{-5}$?

Page 11, Line 21: Fig. A4"A" is referred to in the text, but "A" is not labeled on the corresponding panel.

Page 11, Line 10: Although the authors mention that *"non-Markovian contributions* *in an SDE model have, to the best of our knowledge, not been considered so far in the paleoclimate literature"*, it is considered by Pelletier (2003) for glacial-interglacial cycles (cf. non-Markovian 'deterministic' models are considered by Rial (2004) for DO oscillations and by Berger (1999) for glacial-interglacial cycles).

The number of observation vectors is 7529 according to Page 12, Line 3, but it is $N = 7528$ according to Page 8, Line 19.

The caption of Figure 3C refers to "interpolated", "smoothed", and "simulated", but its legend refers to "interpolated", "preprocessed", and "simulated".

Figures 3C and 3D: What are the number of tapers and the time-bandwidth parameter used to estimate these spectra? The spectra seem strongly smoothed.

A reference (Mitsui and Crucifix, 2016) is updated (please see below).

Some of the previous models of DO events exhibit self-sustained oscillations. How much noises are important to generate DO transitions in this model? Do oscillations disappear if the noise covariance matrix Q is set to zero?

References

Kwasniok, F.: Analysis and modelling of glacial climate transitions using simple dynamical systems, Philosophical Transactions of the Royal Society of London A: Mathematical, Physical and Engineering Sciences, **371**, 20110472, 2013.

Krumscheid, S., Pradas, M., Pavliotis, G.A., and Kalliadasis, S.: Data-driven coarse graining in action: Modelling and prediction of complex systems, Physical Review E, **92**, 042139, 2015.

Raftery, A.E.: Bayesian Model Selection in Social Research, Sociological Methodology, **25**, 111–163, 1995.
Yamaguchi, R. and Higuchi, T.: State-space approach with the maximum likelihood principle to identify the system generating time-course gene expression data of yeast, Int. J. Data Mining and Bioinformatics, **1**, 77–87, 2006.
Pelletier, J.D.: Coherence resonance and ice ages, Journal of Geophysical Research, **108**, 4645, 2003.
Rial, J.A.: Abrupt climate change: chaos and order at orbital and millennial scales, Global and Planetary Change, **41**, 95–109, 2004.
Berger, W.H., The 100-kyr ice-age cycle: Internal oscillation or inclinational forcing?, Journal Earth Sciences, **88**, 305–316, 1999.
Mitsui, T. and Crucifix, M.: Influence of external forcings on abrupt millennial-scale climate changes: a statistical modelling study, Climate Dynamics, **48**, 2729–2749, 2017.

Interactive comment on Earth Syst. Dynam. Discuss., doi:10.5194/esd-2017-8, 2017.