

Interactive comment on “Inverse stochastic-dynamic models for high-resolution Greenland ice-core records” by Niklas Boers et al.

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Responses to Takahito Mitsui:

We thank you for this thorough evaluation of our manuscript. Your detailed comments and suggestions have been very helpful in our revision.

Regarding your Specific Comments:

1. Following your suggestion, we have computed the third-order moment,

$$M(\theta) = \langle x(t)x^2(t+\theta) - x^2(t)x(t+\theta) \rangle_t,$$

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as done by Kwasniok et al. (2013) to quantify the time-reversal asymmetry of simulated time series. Fig. 1B shows that, for delays θ up to roughly 1000 years, $M(\theta)$ computed from dust simulations does exhibit a somewhat similar behavior as the $M(\theta)$ computed from the observed dust. Quantitatively, the similarity between $M(\theta)$ computed for the observed and simulated dust, respectively, is also supported by Kendall's τ (red curve in Fig. 2D). The temporal asymmetry is, however, not reproduced by $\delta^{18}\text{O}$ simulations (Fig. 1A), as also confirmed by the blue curve in Fig. 2D.

Given the very strong correlations r^P between the two variables, with $r^P \approx 0.9$ for both observations and simulations, the discrepancies between the resulting $M(\theta)$ given $\delta^{18}\text{O}$ or dust data are rather surprising. These discrepancies suggest that $M(\theta)$ might not be the best measure for quantifying whether sawtooth-shaped oscillations are present or not.

Still, in terms of the M -score, the reproduction of the time-reversal asymmetry is not as successful as we inferred visually from comparing Figs. 2 and 4A in the original manuscript. We do acknowledge that the behavior of single simulated time series can be misleading, and will modify our conclusions accordingly, to include a paragraph on the quantitative results obtained on the basis of the third-order moment $M(\theta)$.

The main contribution of including memory terms into the model is to improve the average simulated waiting times between subsequent transitions from stadials to interstadials, cf. Fig. 2E, for $55 \text{ a} \lesssim \tau \lesssim 60 \text{ a}$. See also our response in item 3. below. The memory terms also help improve the probability density functions (PDFs) of simulated time series, as shown by a comparison of Figs. 3A and 3B to Fig. A5 in our original manuscript. The improvement is particularly noteworthy in reproducing the bimodality of the PDF of the $\delta^{18}\text{O}$ time series. Furthermore, the AICc and BIC criteria support, in general, the models that do include memory terms.

2. You are right, there were several mistakes in the stated formulae for both AICc and BIC, and we do appreciate your pointing them out. Apart from typos, we indeed took n to be the total number of data points, although it should be the number of (two-dimensional) observation vectors. Correcting for this mistake leads to slightly smaller values for AICc and BIC than stated in our manuscript, but the relative order of AICc and BIC values, corresponding to the different model candidates, remains unchanged. Therefore, the conclusions drawn from both model selection criteria are unaltered.
3. From a theoretical perspective, based on the Mori-Zwanzig formalism, there is no need to further increase the number of memory terms d because, for $d = 2$, the residuals are already white in space and time. This point is mentioned in the manuscript, but we will emphasize it further in the revision.

The appropriate length of the delay τ could, of course, be directly determined based on BIC or AICc criteria. For any τ such that $0 < \tau \leq 1000$ time steps (of 5 years), both selection criteria consistently favor the non-Markovian model, with the memory terms, over the Markovian one, without them. The AICc and BIC criteria, however, exhibit lowest values for $\tau = 1$ time step (5 years), and monotonically increase for longer τ , whilst staying below the respective values obtained for $\tau = 0$; see Fig. 2A.

Choosing $\tau = 5$ years (i.e. one time step) would, however, lead to less accurate approximations of the statistical properties of the observed NGRIP time series than either no memory or longer memory; see Figs. 2B–2E. The reproduction of the statistical characteristics of the NGRIP time series is thus not optimal for the value of τ that is suggested by AICc and BIC, but for considerably longer memory. Values of τ between 55 and 80 years (i.e., between 11 and 16 time steps) yield comparably good approximations when all statistical characteristics are taken into account.

In particular, a rather narrow range of possible memory step sizes, namely

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$\tau = 55$ a or $\tau = 60$ a, yields an accurate approximation of the average waiting times between subsequent transitions from stadials to interstadials. In the revised version, we will include these observations to explain and motivate the right choice of the memory parameter, for which a value of $\tau = 60$ a seems to be optimal when taking into account the AICc, together with the statistical characteristics shown in Figs. 2B–2E.

4. We acknowledge that including ice-volume forcing improves the slow decay of the sample autocorrelation function of the NGRIP $\log(\text{Ca}^{2+})$ record in your recent paper, and that the BIC supports external forcing for some of the studied models. We also agree that the relevance of external forcing cannot be ruled out by our results. In the conclusion, we merely wanted to make the point that a relatively simple model can approximate the dynamics of the NGRIP time series quite well without external forcing, as long as memory and couplings between oxygen isotope and dust are considered. We believe that the extent to which external forcing might contribute to the longer-term evolution of DO cycles remains open and subject to further inquiry. We will rephrase the corresponding sentences in the conclusions accordingly.

Regarding your Technical corrections and minor comments: In the revised version, we will address all of them following your suggestions:

- P1, L16: This should have been “consideration”.
- P2, L10: We agree!
- P6, L7: Correct, thank you!
- P6, L12: The memory-window length τ is 15 time steps and thus indeed non-dimensional. In the text, we translated this to 75 a because each time step corresponds to 5 a. We will clarify this in the revised manuscript.

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- P7, L20: Agreed!
- P9, L5 and L31: Yes, we agree.
- P9, L5: The step size for the numerical integration of our models (using the Euler-Maruyama scheme) is $\delta t = 10^{-5}$, which is thus non-dimensional. Compared to the observational data, this step size would correspond to 5 a. Essentially, we thus rescaled time in order to guarantee a stable numerical integration. In the revised version of our manuscript, this will be pointed out more explicitly.
- P11, L21: We will add the labels in the revised manuscript.
- P11, L10: Thank you for pointing us to these interesting references. We were not aware of them and will cite them in the revised manuscript.
- The total number of data points is $N = 7529$, but this leaves 7528 difference quotients to be fitted. We'll add a note on this in the revision.
- Yes, we will correct this in the revised manuscript.
- For the multitaper estimate of the PSD, we set the time-halfbandwidth parameter (NW) to a value of 4, corresponding to $2 \cdot NW - 1 = 7$ tapers. The PSD shown for the simulations is an average over 500 simulated time series, and is therefore strongly smoothed.
- We will update the reference to your paper in Climate Dynamics in the revised version.
- The drift term in our model corresponds to two stable states (stadial and interstadial conditions), and the parameters are kept fixed. Transitions between the two stable states are thus solely triggered by fluctuations. By setting the entries of the noise covariance matrix Q to zero, the system would thus stay at either one of the fixed points without any further dynamics.

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We will revise our manuscript in accordance with these responses once the editor agrees.

Figure captions:

Fig. 1. Third-order statistical moment $M(\theta) = \langle x(t)x^2(t+\theta) - x^2(t)x(t+\theta) \rangle_t$ for the observed NGRIP time series (solid blue), the full model including memory terms with step size $\tau = 60$ a (solid red), and the model without memory terms (dashed red). Note that for increasing delays θ the values of $M(\theta)$ are affected more and more by the non-stationarity of the data, and should therefore be interpreted with care. The upper panel (A) shows $M(\theta)$ for the $\delta^{18}\text{O}$ time series, while the lower one (B) shows $M(\theta)$ for the dust time series.

Fig. 2. A. Log-likelihood and AICc for different values of the memory step size τ . The AICc is computed as $\text{AICc} = 2pn/(n-p-1) - 2\log \mathcal{L}^*$. B. Difference between observed and simulated standard deviations. C. L^2 - and L^∞ -distances between observed and simulated probability density functions. D. Kendall's τ between the third-order moments $M(\theta)$, computed for observed and simulated time series, respectively. E. Difference between observations and simulations in terms of average waiting times between subsequent transitions from stadials to interstadials. In B–E, statistics for simulated time series are obtained as averages over 400 simulations using the full model.

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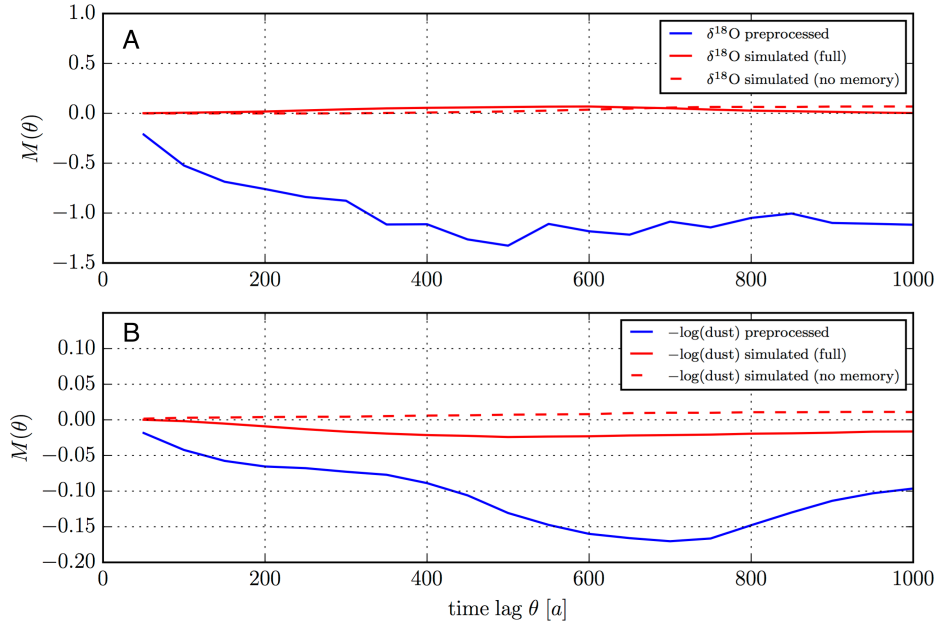


Fig. 1. see text for caption

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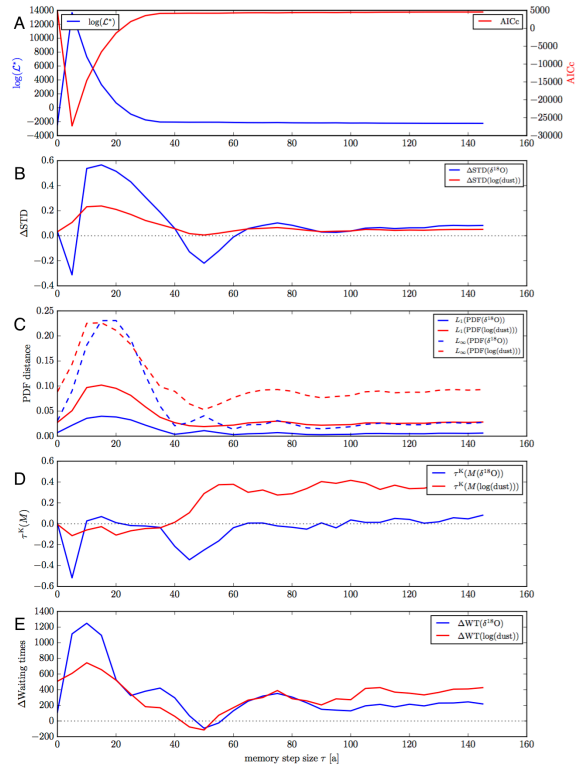


Fig. 2. see text for caption

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