

Response to comment by J. Heitzig

I like this model a lot. You may improve its exposition marginally and make it less time-dependent by introducing a fifth state variable, cd = deep ocean carbon stock, and rewrite eq. (8) as two ODEs, one for cd (containing the two integrals) and one for ca . This way the model's remaining time dependency is only on the two "control" variables $E(t)$ and $LUC(t)$ and it may thus be analysed more easily using tools from bifurcation analysis or topology of sustainable management (see this special issue).

We thank the commenter for the constructive proposal to improve the readability of the model and its potential for analysis. In the revised version of the manuscript, we will implement a slightly modified version of the commenter's proposal. We would prefer not to introduce a state variable that corresponds to a quantity (deep ocean carbon) that is outside the boundaries of our system of analysis (which is upper ocean, atmosphere and marine carbon). Instead, we will introduce a new state variable that counts the total amount of carbon over our three carbon stocks. The rate of increase of this quantity will be a differential equation given by the rate of carbon emissions minus the rates of the solubility and biological pumps (Eq. 9 in the revised manuscript). Conservation of carbon within the three internal stocks will then give a simple algebraic equation (Eq. 8 in the revised manuscript) to replace the former Eq. 8.

Response to review by C. D. Jones

We thank the reviewer for their considered and constructive comments. Please find below our responses to the reviewer's comments. We also attach our proposed revised manuscript, with changes marked.

This is a nicely designed study, and well presented manuscript which attempts to develop and document a simple ("stylized") model of the global climate-carbon cycle system, but in a way which enables analytical analysis of its behaviour and feedback mechanisms. The intended aim is to facilitate improved understanding of the system dynamics and more readily quantify which processes contribute to certain feedbacks and long-term responses.

Overall, I very much like the approach and the intention – such modelling studies can develop improved insight and can strip back confounding and complicating issues of model complexity to reveal more fundamental underlying behaviour. I have a few comments below and a few queries about the extent to which the intentions have been realised – I think these can be readily addressed with revised text and some more context/explanation.

We thank the reviewer for his support.

Overarching comments:

1. There are numerous simple/stylized carbon cycle models in the literature. You cite Raupach (2013) which I like very much. There is also one I developed and have published with several times, including as recently as last year (Jones et al., Tellus, 2003; Jones et al., Tellus, 2006; Jones et al., ERL, 2016). Then there is the MAGICC model often used in IAMs, the Joos IRF, and also the Oscar model which some of the current authors know very well. So I wonder if a bit more explanation is needed for why a new model is required – could you start from one of the existing ones and achieve the same thing? I guess your main driver is the ability to analytically derive the feedback functions – but it's not clear to me the same is not possible from these previous models (I haven't tried it with the Jones et al simple model, but may do!) – Mike Raupach derived eigenmodes, so I would imagine feedback metrics could follow also, but again I haven't tried.

A key motivation for the proposed model is that it contains mechanistic representations (albeit highly aggregated and stylised) of key climate-carbon processes. In contrast, few, if any, of the models cited above explicitly include a solubility or biological ocean pump. In comparison to many of the cited models, we also substantially simplify the representation of the terrestrial carbon cycle, in order to simplify the analysis. We suspect that a similar analytical feedback analysis could in theory be applied to most of the simple models that the reviewer cites, but the partitioned terrestrial carbon stocks in most of these models would complicate the analysis, and parametric fits to the ocean carbon cycle would make the results less meaningful. We will make clearer the added value of the model in the revised version (see list beginning bottom page 3).

2. On a similar line – I was looking forward to seeing the analytical derivation of feedback factors, but then realised this was not as "tractable" as your title suggests, and you have to

make a lot of assumptions in your sections 4.2 and 4.3. This seems a shame – if this is the case, have you not lost your unique and attractive feature? The resulting expressions are still useful, but not as analytical as you suggest. I also wondered, on seeing the expressions in table 3, if you had compared these to the expressions of Ric Williams et al – who have done a similar based assessment of terms controlling ocean heat/carbon uptake and TCRE. (See e.g., Richard G Williams et al 2016 Environ. Res. Lett. 11 015003)

We agree with the reviewer that while our feedback results are analytical (in the sense of closed-form mathematical expressions) they are not exact. It would be an interesting challenge to develop a model for which exact results could be achieved, but we suspect this would be at the cost of a mechanistic representation. In the revised version of the manuscript, we will clarify our use of the term of ‘analytical’.

Williams et al. (2016) split apart three key factors influencing TCRE: (1) influence of CO₂ emissions on radiative forcing from CO₂; (2) influence of radiative forcing from CO₂ on total radiative forcing; and (3) influence of total radiative forcing on temperature. They then use time series output from ESMs to drive each of these factors and calculate TCRE over time. For example, land and ocean uptake (which influence factor 1) are based directly on ESM output. In contrast, we formulate mechanistic models for land and ocean uptake. Our treatment of factor 3 is similarly highly stylised and we do not explicitly model ocean heat uptake. Regarding factor 2, we only consider CO₂ forcing.

In one subsection, Williams et al. analytically calculate an equilibrium TCRE. They use a similar formulation for ocean chemistry based on the Revelle (buffer) factor as ours. However as theirs is an equilibrium calculation, they unlike us do not consider time scales introduced by mechanisms such as the solubility and biological pumps. They also neglect land carbon uptake on this long time scale.

3. You discuss (page 18, line 16) that you might want to develop this model further to include other mechanisms and forcings. Modellers have attempted a two-box approach for the ocean – by splitting upper and deep ocean before. But I would suggest maybe a two-box approach for the land also – not splitting veg vs soil, but maybe by tropics and extra-tropics as these can have very different responses (even by sign) to changes in climate. IPCC AR5 fig 6.22 shows the latitudinal distribution of “gamma” – there is a clear change of sign towards high latitudes, and so a tropics/high-latitude 2-box approach might be a nice (and novel) development

To compartmentalise land carbon by tropics and extra-tropics is an interesting suggestion which could be followed in future studies. We will raise this idea in the revised version of the manuscript, see section 6. In doing so, one would define carbon pools not by residence times but by climate sensitivities. This is really interesting but requires some more thinking.

In addition, to our understanding the results in IPCC AR5 fig 6.22 are based on ESM runs that do not contain any representation of permafrost carbon, hence this strong difference between arctic and extra-arctic beta values seems to more reflect the vegetation response to climate change. It would be interesting to see such sensitivity study using a fully coupled

ESM run including permafrost carbon. For now, we therefore refrain from including these sensitivity gradients in our model.

4. In a few places, you discuss non-linearities – this is good, and important to bring out. It's not necessarily true that land dominates the non-linearity, but this does show up in your rapid-forcing transient runs. If you ran longer, or with slower changing forcing, the ocean would have more chance to exhibit these too – see, e.g. Schwinger et al (J. Climate, 2014). Hence both land and ocean can have pronounced non-linearities and for this reason, C4MIP took the decision to move back towards the Friedlingstein definition of feedbacks as the difference between COUPLED and BGC runs (see Jones et al., GMD, 2016 – documentation of C4MIP). Hence this differs from the Arora et al definition of using the RAD forced run.

We agree with the reviewer: since we chose to investigate effects on a 100-year policy-relevant time scale, many ocean effects are rendered insignificant. We will discuss in the revised version of the manuscript that the dominance of the land in our nonlinearity results is likely due to the time scale simulated (see section 5.3).

We thank the reviewer for drawing attention to the need for clarity in feedback definitions. Most of the the previous studies to which we compare our numerical feedback results use the Arora definition of feedbacks (Arora 2013 and Zickfeld 2011), the exception is Friedlingstein 2006. We have used the Arora definition to be consistent with the majority of cited previous studies, and will clarify which definition we use in the revised manuscript. We are prepared to also calculate the climate-carbon feedbacks under the Friedlingstein definition if the reviewer wishes, however we feel this would further complicate an already large table.

5. My final request would be to ask if you can more directly or relevantly bring this back to complex models – how does this approach help us develop/evaluate/constrain them further? For example you claim in the discussion that the carbon-climate feedbacks are “less sensitive” than the carbon-concentration ones. And that this is due to “the shape of $K(Ca,DT)$ ”. So how does that help with my ESM? What controls the shape of this in ESMs? And can we measure and constrain it from obs? If so, then your analysis brings a way in which we might narrow the spread in model projections, or at least evaluate a very relevant aspect of the models. If not, then all it does is leave us with a better feel of why the models continue to diverge – if you have any ideas how to make this jump that would be great to see.

We thank the reviewer for this relevant comment. Of course, the divergence amongst ESMs could well be due to diverging parameterisations, as well as different functional forms. As the reviewer suggests, an interesting area for future work would be to study what effects different forms for key functions such as K have on feedback strengths. Other steps to aid development of ESMs could include analysing the effective shape of functional forms such as K in ESMs or how to constrain these functional forms from data. These are beyond the

scope of the present work but in the revised manuscript we will point to these possible future directions in section 6.

Minor comments:

1. Page 4. Your NPP function of CO₂ claims to include the effects of climate change – but surely these also depend on the climate sensitivity. For models with high/low climate sensitivity, there is a different trade-off of the effects of CO₂ and climate. So I don't follow how the impact on NPP can be made without reference back to the temperature as well as the CO₂

We agree with the reviewer that an accurate treatment of NPP would separately parameterise the effects of CO₂, temperature, rainfall, nutrient availability, and so on. We fold all these effects into a CO₂ dependence through Keeling's formula. The references in section 2.1 that we cite for Keeling's formula support this simplification. A key assumption to support this folding, as the reviewer implies, is that we fix climate sensitivity to a constant value -- in the revised manuscript we will state this assumption in section 2.1.

2. Page 7. I couldn't quite see if you had a link between ocean heat and ocean carbon uptake – I don't think so. Should these be related? There might be a false extra degree of freedom in your model – I would expect for example rapidly mixed oceans to have high rates of both heat and carbon uptake – and vice versa for poorly mixing oceans. But if you have independent mechanisms of carbon uptake and transient response to climate do you miss this link?

The reviewer is correct that in our model ocean heat uptake (as represented by climate sensitivity) and ocean carbon uptake are parameterised independently. We also agree with the reviewer that higher ocean mixing rates ought to speed up both carbon and heat uptake. We have chosen to focus our model development and analysis on the carbon cycle; future work could involve incorporating mechanisms related to ocean heat uptake such as ocean circulation, and then specifying common drivers on ocean heat and carbon uptake could be worthwhile. We discuss in Section 6 that energy balance is a potential route for further model development.

3. Table 1: please be careful to stress “climate sensitivity” as “transient climate response” – you do say so, but using the wrong name makes it look like a very low value (1.8K)

We thank for the reviewer for the cautionary note. We will stress that the climate sensitivity in our model is transient climate response in the revised version (see section 3).

4. Page 14, line 23 – just to check here you mean “1% increase up to double CO₂” and not a step-change to double CO₂.

We thank the reviewer for prompting us to clarify this matter. In fact this simulation has a 1% increase up to *quadrupling* CO₂. We will clarify this matter in the revised manuscript (see section 4.3).

Response to review by M. Heimann

We thank the reviewer for their considered and constructive comments. Please find below our responses to the reviewer's comments. We also attach our proposed revised manuscript, with changes marked.

General comments

The authors introduce a new variant of a simple analytical, highly parameterised global carbon cycle - climate model, which is used to formally analyse the four major feedback loops in the system, i.e. the land and ocean concentration carbon feedbacks and the land and ocean climate carbon feedbacks. The simplicity of the approach allows the authors to derive analytical approximations to the definitions of various feedbacks metrics at play in the global carbon cycle - climate system.

Simple analytical global carbon cycle models and simple climate models have been used many times in the past. Also the literature contains several simple coupled carbon cycle - climate models (e.g. Gregory et al., 2009 or Meinshausen et al., 2011). It is not clear however, what this particular new variant adds to our understanding of the global carbon cycle - climate system. The motivation outlined in the introduction is not very convincing.

We thank the reviewer for prompting us to make explicit what our work “adds to our understanding of the global carbon cycle - climate system”. As we state in the revised version of the abstract, three specific results of our work are:

- that different feedback formalisms measure fundamentally the same climate-carbon cycle processes;
- that temperature dependence of the solubility pump, biological pump, and CO₂ solubility all contribute approximately equally to the ocean climate-carbon feedback; and
- that concentration-carbon feedbacks may be more sensitive to future climate change than climate-carbon feedbacks.

These results would not have been possible without the simple, mechanistically-based model that we develop in this manuscript.

Regarding the previous models that the reviewer cites: There is no explicit representation of biophysical processes in the model of Gregory et al. (2009), which consists of fits to fluxes between ocean, atmosphere and land carbon stocks predicted by C4MIP models, or the ocean carbon cycle component of Meinshausen et al. (2011), which is a parametric fit to predicted impulse response functions. That our model is a mechanistically based representation of the carbon cycle, even if that representation is highly aggregated and simplified, allows us to deliver the insights above. We concede however that our motivation of the model in the Introduction was not particularly detailed on these points. In the revised manuscript we will expand upon the model's motivation in the second paragraph of the Introduction, as well as refining the more detailed description at the start of section 2.

The dynamic characteristics of the chosen “mechanistic” model formulation clearly is determined by the simple model structure and the chosen parametrisations of the exchange

fluxes. Also the stated “biophysical or biogeochemical interpretation of the model parameters”, given that these represent global averages is plausible but not very compelling. E.g. why should the global CO₂ fertilisation effect work in reality in a way as parameterized here with a simple β -factor formulation? Or global respiration with a simple Q₁₀ temperature response?

We use the term “mechanistic” to convey that we have in our model representations of real-world processes, such as photosynthesis, respiration, ocean-atmosphere diffusion and the solubility and biological pumps. Our model is not a precise, first-principles mechanistic description of these processes at the microscopic scale, but then again all models are simplifications of reality; we merely choose to perform the simplification at a more aggregated level than most Earth System Models. The β -factor (or ‘Keeling formula’) and Q₁₀ temperature responses are previously used parameterisations of the response to climate change of globally aggregated NPP and respiration, respectively. We will clarify our use of the term ‘mechanistic’ in the revised manuscript (see second paragraph of the introduction).

Perhaps the main value of the simple model is educational, as it can easily be programmed by students and one can show in this simple model system how the feedback metrics are computed. But as a tool for policymakers nor for generating new carbon cycle science, this model does not provide added value to the already existing simple models. A simple model “tuned” to emulate one or several of the more complex models would be more useful.

Gregory et al. (2009) and Meinshausen et al. (2011), as well as others, already provide simple models “tuned” to emulate one or several of the more comprehensive models. We believe there is scope for a model such as ours, in which we do not force our model to closely fit historical data (or future projections) but rather parameterise each process with the best available (globally aggregated) knowledge about that process. See our response to the next comment below for further information.

Perhaps a missed opportunity for demonstrating the validity of the model is a more careful calibration and evaluation. Clearly the “mechanistic” model parameter values are not based on first principles, but contain large uncertainties. E.g. the Q₁₀ value used here (1.72) is highly uncertain (see e.g. Mahecha et al., 2010). Why not tune the model parameters so that the current global carbon budget is properly matched? The model substantially underestimates the historical ocean carbon uptake (Table 2), and, when driven with the historical emissions from the Global Carbon Project (Le Quere et al., 2017), the numerical version of the model underestimates the current ocean uptake. In addition, a graph showing the model performance against the atmospheric CO₂ record from ice cores and direct observations could demonstrate that at least on multi-decadal time scales the model performs reasonably. Figure 2 clearly is not sufficient as it does not show any observations. Another useful model evaluation would be to follow the impulse response simulation protocol defined by Joos et al. (2013) and compare the dynamics of this model with the impulse response simulations of more comprehensive models as shown in that paper.

We thank the reviewer for these comments and suggestions on model calibration and evaluation. Following the reviewer's suggestion, in the revised version of the paper we will include historical carbon fluxes and temperature anomalies alongside model predictions (see revised Fig 2).

We have attempted to 'tune' several different combinations of parameters to match current carbon stocks (one example is $K_C = 0.25$, $Q_R = 2.5$ and $w_0 = 0.2$). However the tuned parameter sets lie well outside the best available independent estimates of those parameters (see references in Table 1). This is not surprising since we do not expect a mechanistically based model of this simplicity to precisely reproduce historical carbon stocks.

Rather than forcing the model to fit historical data, we choose to parameterise each process with the best available knowledge about that process. Gaps between our model and observations then point to what other processes should be included in a more complex model to improve accuracy. This is in line with our stated model purposes of understanding and learning, rather than emulation and prediction. In the revised manuscript, we will clarify our choices taken during the parameterisation of our model (see section 3).

Specific comments

1. As shown in Table 3, the results of the analytical approximations of the feedback metrics compared to the numerical simulations is pretty poor. Does this not invalidate the simplifications made in deriving the analytical approximations?

We concede that in the submitted version of the manuscript, while the land feedback metrics were accurate, the agreement between the numerical and analytical results for ocean feedback metrics was poor. Deriving approximate metrics for ocean feedbacks is challenging, as the deep ocean does not reach equilibrium on the time scale of our simulation. We have taken the opportunity to derive alternative approximations to the ocean feedback metrics (see description in section 4.2). The approximated ocean climate-carbon feedback is now more accurate. The ocean concentration-carbon feedback remains in poor agreement. As we will explain in the revised manuscript (see second paragraph of section 5.2), this is partly due to an approximation made in analytically estimating the deep ocean uptake, but partly also due to numerical concentration-carbon feedback calculations requiring climate-carbon feedbacks to be switched off.

2. The comparison of the feedback metrics with the results of Zickfeld et al. (2011) and Friedlingstein et al. (2006) in Table 3 shows that the simple model with the chosen parameter values responds substantially different - the discrepancies range up to a factor of 2. This is clearly at odds to what is claimed in section 5.1 and 5.2.

We thank the reviewer for prompting us to clarify what we judge as 'agreement' between the results of our simple model and the results of previous simulations. First, we note that the results of complex models display considerable spread (as also noted by the reviewer in the following point below). While some of our results differ by nearly a factor of 2 from the mean results of Friedlingstein et al, all our feedback metric results are within their reported spread (Table 3). Second, we consider it remarkable that such a simple model can reproduce the

results of highly complex models so closely, and would not consider a discrepancy of a factor of 2 an invalidation of the simple model. We will discuss these discrepancies more carefully in the revised manuscript (see first paragraph of section 5.2).

3. On the other hand, also the comprehensive models show a large spread in the feedback metrics. A more useful analysis/comparison would be possible if the model parameters were tuned to emulate the various comprehensive models.

This is an interesting idea, but beyond the scope of our study. As discussed above, our goal is not to emulate or evaluate ESMs, but rather to develop process-based understanding.

4. The statements in section 5.2 and 5.3 about the behaviour of the carbon cycle - climate system and the feedback metrics under increasing emissions clearly refer to this particular simple model. While plausible, the real world may behave differently.

We thank the reviewer for raising this concern. It is correct that our model can only anticipate changes in the carbon cycle arising from those processes that it has modelled -- and may therefore neglect other important future changes in the carbon cycle. We will acknowledge this caveat in the manuscript (see second-last paragraph of section 5.2).

5. The direct ocean concentration-carbon feedback given as exact in Table A1 and approximated in Table 3 (5th line from bottom) differ very much: Evaluated with the standard model parameters at a value of c_a corresponding to 800 ppm the exact formula gives 0.0152 PgC/(ppm yr) while the approximation gives 0.396 PgC/(ppm yr). (I assumed in the exact formula that the symbol w is actually w_0). Also the solid red curve showing BO in Figure A1a is missing. Obviously there is some error in the listed formulas or the chosen approximation is very poor.

We respectfully disagree with the reviewer's calculations. By our calculations, under the conditions the reviewer indicated the approximation gives 0.0398 PgC/(ppm yr). While this is not as severe as the 20-30 times the reviewer suggested, it is still a significant difference at 2 to 3 times the exact expression. We took the opportunity to derive a more precise approximation (see Table 3 and the last sentence of section 4.3) that gives a value 0.0240 PgC/(ppm yr).

We thank the reviewer for noticing the omitted curve in Figure A1; this will be rectified in the revised manuscript (see our proposed revision).

Technical corrections

Technically, the formulas in the manuscript contain a some inconsistencies and not correctly defined symbols.

- *p. 4, line 25: In the exponent of QR the symbol T should be replaced by ΔT .*

Thank you, in the revised manuscript we will correct this mistake.

• p.5, line 13: *The way the Revelle factor is used here is weird: Formally, using the notation here, it is defined as:*

$$R = \frac{\partial p(c_m, 0)}{\partial c_m} \frac{c_{m0}}{p(c_{m0}, 0)}$$

Inserting the definition $p(c_m, \Delta T)$ given here (eq (5)) this expression does not evaluate to the constant r as it should according to the text.

We respectfully disagree with the reviewer's general definition of Revelle factor. According to Sabine et al. (2004) [see citation in our manuscript] and the AR4 [see https://www.ipcc.ch/publications_and_data/ar4/wg1/en/ch7s7-3-4-2.html], the general definition of Revelle factor is in our notation

$$R = \frac{\partial p(c_m, 0)}{\partial c_m} \frac{c_m}{p(c_m, 0)}$$

that is, the mixed-layer ocean carbon stock in the right-hand quotient should not be fixed at pre-industrial CO₂ levels. Substituting our model's expression for partial pressure of CO₂ [equation 5 in our manuscript] gives $R = r$ for all $c_m > c_{m0}$ as expected.

• p. 6, line 25: *The atmosphere equation, written as an integral equation is weird. Why not write it similar to the biosphere and ocean mixed layer equation as normal first order differential equation?*

<equation omitted>

where $e(t)$ are the emissions (in PgC/yr); $E(t)$ in equation (8) are the integrated emissions (this is nowhere defined in the text, and wrongly described on p.5 line 7).

We agree that the form of the atmosphere equation is unusual! In line with the suggestion of the comments provided by Heitzig (see above), we will rewrite equation 8 as an algebraic equation for conservation of carbon amongst our stocks, alongside a new differential equation to account for aggregate carbon flows into or out of our three stocks. This formulation will remove all integral equations.

We thank the reviewer for identifying that $E(t)$ is incorrectly defined. We will correct this mistake in the revised manuscript (see definition preceding the new equation 9 and section 3).

• p. 7, eq 9: *For consistency with the text the symbol T in the differential quotient on the left should be replaced by ΔT .*

Thank you, in the revised manuscript we will make this change to improve the clarity of the manuscript (see equation 10 in the revised manuscript).

• Table 3, 4th and 3rd line from bottom: *The references to the Figures A1a and A1b are not correct.*

Thank you for noting this mistake. We will correct the figure numbering in the revised manuscript.

- *Table A1: What is the meaning of w (without subscript)? Presumably it should be w_0 ?*

We thank the reviewer for noting this mistake. We confirm the w in Table A1 should be w_0 . We will correct this mistake in the revised manuscript.

Authors' overall comment

We thank the reviewers for their considered and constructive comments that have improved the manuscript.

Reviewer Jones wrote “[t]his is a nicely designed study, and well presented manuscript” and “I very much like the approach and the intention”. In line with the reviewer’s suggestions, we now compare our model against additional existing models, clarify our use of the terms “mechanistic” and “analytical”, include additional speculations on future work, and responded to some specific requests for clarification.

Reviewer Heimann raised concerns about the novelty of our model compared to previous models and about how accurately our numerical and analytical results compare to previous ESM results as well as to each other. We believe there exists a gap in the literature for our mechanistically-based but highly stylized model, as we have sought to make clearer in the revised manuscript. For a model as extremely simple as ours, and given the wide spread in the results of ESMs, we believe our results are sufficiently accurate. In line with Heimann’s very perceptive technical corrections we have modified our feedback calculations to bring them into closer agreement than in the initially submitted version.

We also appreciate Heitzig’s comment to modify the mathematical presentation of the model. We have modified the model to achieve Heitzig’s overall goal of presenting the model solely in terms of differential, instead of integro-differential, equations, although not exactly by the route he suggested.

Please find attached our proposed revised manuscript, with changes tracked.

Analytically tractable climate-carbon cycle feedbacks under 21st century anthropogenic forcing

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Abstract. Changes to climate-carbon cycle feedbacks may significantly affect the Earth System's response to greenhouse gas emissions. These feedbacks are usually analysed from numerical output of complex and arguably opaque Earth System Models (ESMs). Here, we construct a stylized global climate-carbon cycle model, test its output against **complex comprehensive** ESMs, and investigate the strengths of its climate-carbon cycle feedbacks analytically. The analytical expressions we obtain aid understanding of carbon-cycle feedbacks and the operation of the carbon cycle. **We use our results to analytically study the relative strengths of different** **Specific results include that: different feedback formalisms measure fundamentally the same** climate-carbon cycle **feedbacks and how they may change in the future, as well as to compare different feedback formalisms** **processes; temperature dependence of the solubility pump, biological pump, and CO₂ solubility all contribute approximately equally to the ocean climate-carbon feedback; and concentration-carbon feedbacks may be more sensitive to future climate change than climate-carbon feedbacks.** Simple models such as that developed here also provide 'workbenches' for simple but mechanistically based explorations of Earth system processes, such as interactions and feedbacks between the Planetary Boundaries, that are currently too uncertain to be included in **complex comprehensive** ESMs.

1 Introduction

The exchanges of carbon between the atmosphere and other components of the Earth system, collectively known as the carbon cycle, currently constitute important negative (dampening) feedbacks on the effect of anthropogenic carbon emissions on climate change. Carbon sinks in the land and the ocean each currently take up about one quarter of anthropogenic carbon emissions each year (Le Quéré et al., 2016). These feedbacks are expected to weaken in the future, amplifying the effect of

anthropogenic carbon emissions on climate change (Ciais et al., 2013). The degree to which they will weaken, however, is highly uncertain, with Earth System Models predicting a wide range of land and ocean carbon uptakes even under identical atmospheric concentration or emission scenarios (Joos et al., 2013).

Here, we develop a stylised model of the global carbon cycle and its role in the climate system to explore the potential weakening of carbon cycle feedbacks on policy-relevant time scales (<100 years) up to the year 2100. Whereas **complex comprehensive** Earth System Models (ESMs) are generally used for projections of climate, models of the Earth System of low complexity are useful for improving mechanistic understanding of Earth system processes and for enabling learning (Randers et al., 2016; Raupach, 2013). Compared to **complex comprehensive** Earth System Models, our model has far fewer parameters, can be computed much more rapidly, can be more rapidly understood by both researchers and policy-makers, and is even sufficiently simple that analytical results about feedback strengths can be derived. **Compared to previous stylised models (Gregory et al., 2009; Joos et al., 1996; Meinshausen et al., 2011a, c; Gasser et al., 2017a), our model features simple mechanistic representations, as opposed to parametric fits to ESM output, of aggregated carbon uptake both on land and in the ocean.** Our stylised and mechanistically based climate-carbon cycle model also offers a workbench for investigating the influence of mechanisms that are at present too uncertain, poorly defined or computationally intensive to include in current Earth System Models. Such stylised models are valuable for exploring the uncertain, but potentially highly impactful Earth system dynamics such as interactions between climatic and social tipping elements (Lenton et al., 2008; Kriegler et al., 2009; Schellnhuber et al., 2016) and the planetary boundaries (Rockström et al., 2009; Steffen et al., 2015).

Analyses of climate-carbon cycle feedbacks conventionally distinguish four different feedbacks (Fig. 1) (Friedlingstein, 2015; Ciais et al., 2013). (i) In the land concentration-carbon feedback, higher atmospheric carbon concentration generally leads to increased carbon uptake due to the fertilisation effect, where increased CO₂ stimulates primary productivity. (ii) In the ocean concentration-carbon feedback, physical, chemical and biological processes interact to sink carbon. Atmospheric CO₂ dissolves and dissociates in the upper layer of the ocean, to be then transported deeper by physical and biological processes. The concentration-carbon feedbacks are generally negative, dampening the effects of anthropogenic emissions. (iii) In the land climate-carbon feedback, higher temperatures, along with other associated changes in climate, generally lead to decreased storage on land at the global scale, for example due to the increase in respiration rates with temperature. (iv) In the ocean climate-carbon feedback, higher temperatures generally lead to reduced carbon uptake by the ocean, for example due to decreasing solubility of CO₂. The climate-carbon feedbacks are generally positive, amplifying the effects of carbon emissions.

We begin by introducing our stylised carbon cycle model and testing its output against historical observations and future **predictions projections** of Earth System Models. Having thus established the model's performance, we introduce different formalisms used to quantify climate-carbon cycle feedbacks and describe how they can be computed both numerically and analytically from the model. We use our results to analytically study the relative strengths of different climate-carbon cycle feedbacks and how they may change in the future, as well as to compare different feedback formalisms. We conclude by speculating on how this stylised model could be used as a 'workbench' for studying a range of complex Earth system processes, especially those related to the biosphere.

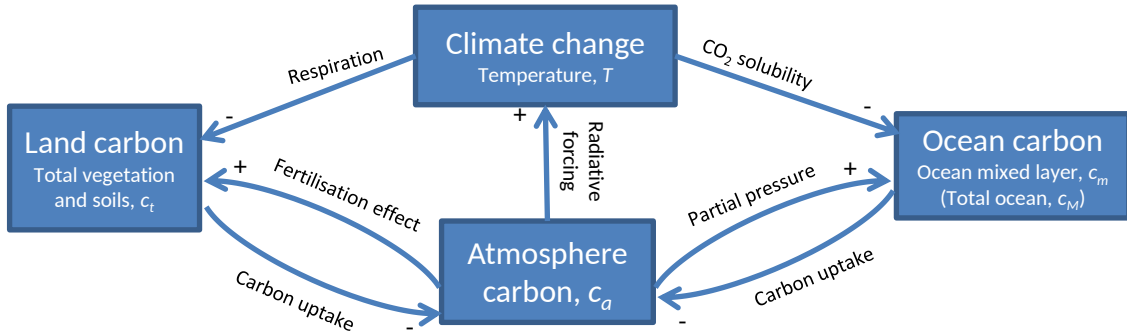


Figure 1. Climate-carbon cycle feedbacks and state variables as represented in the stylized model introduced in this paper. Carbon stored on land in vegetation and soils is aggregated into a single stock c_l . Ocean mixed layer carbon, c_m , is the only explicitly modelled ocean stock of carbon; though to estimate carbon-cycle feedbacks we also calculate total ocean carbon (Eq. (7)).

2 Model formulation

There is a well-developed literature on stylized models used for gaining a deeper understanding of Earth system dynamics and even for successfully emulating the outputs of **complex comprehensive** coupled atmosphere-ocean and carbon cycle models (Meinshausen et al., 2011a, c; Gasser et al., 2017a). Many such models are based on Budyko-Sellers (Budyko, 1969; Sellers, 1969) type energy balance models and come in two flavors: models of mathematical interest motivated by the Earth system dynamics, and models focused on capturing essential features of the Earth system to reproduce broad empirical patterns. The former tend to focus on characterizing stability (e.g. Cahalan and North, 1979), and the existence of multiple equilibria given particular feedbacks (ice cap albedo) (e.g. North, 1990; Diaz et al., 1997) or details of possible bifurcation structures Arcoya et al. (1998) in such models. Examples of the latter include studies of snowline stability (Mengel et al., 1988).

In the spirit of the energy-balance models described above, we constructed (Anderies et al., 2013; Gregory et al., 2009; Joos et al., 1996; Meinshausen et al., 2011a, c; Gasser et al., 2017a). We developed a combination of existing models and new formulations to construct a global climate-carbon cycle model with the following characteristics:

1. The model includes processes relevant to the carbon cycle and its interaction with climate on the policy-relevant time scale of the present to the year 2100. **Stylised carbon cycle models often do not, for example, include explicit representations of the solubility or biological pumps.**
2. The model produces quantitatively plausible output for carbon stocks and temperature changes.
3. All parameters have a direct biophysical or biogeochemical interpretation, although these parameters may be at an aggregated scale (for example, a parameter for the net global fertilisation effect, rather than leaf physiological parameters). We

avoid [models or model components constructed by purely parametric fits to Earth System Models](#), such as impulse response functions ([Kamiuto, 1994](#); [Gasser et al., 2017b](#); [Joos et al., 1996](#); [Harman et al., 2011](#)), to [historical data or projections of Earth System Models](#) ([Kamiuto, 1994](#); [Gasser et al., 2017b](#); [Joos et al., 1996](#); [Harman et al., 2011](#); [Gregory et al., 2009](#); [Meinshausen et al., 2011a](#)).

- 5 4. The model is sufficiently simple that calculation of the model’s feedback strengths is readily analytically tractable. This tractability may come at the expense of complexity, for example multiple terrestrial carbon compartments, or accuracy at millennial or longer time scales ([Lenton, 2000](#); [Randers et al., 2016](#)).

Building on the work of [Anderies et al. \(2013\)](#), we constructed a simple model with globally aggregated stocks of: atmospheric carbon in the form of carbon dioxide, c_a ; terrestrial carbon, including vegetation and soil carbon, c_t ; and dissolved inorganic carbon (DIC) in the ocean mixed layer, c_m . The model’s fourth state variable is global mean surface temperature relative to pre-industrial, $\Delta T = T - T_0$. Compared to [Anderies et al. \(2013\)](#), our model includes more realistic representation of terrestrial and ocean processes but without increase in model complexity, as well as time lags for climate response to CO_2 .

We now describe the dynamics of the land carbon stock, the ocean carbon stock, and atmospheric carbon and temperature in our model.

15 2.1 Land

Net primary production (NPP) is the net uptake of carbon from the atmosphere by plants through photosynthesis. NPP is expected to increase with concentration of atmospheric carbon dioxide c_a . A simple parameterisation of this so-called fertilisation effect is ‘Keeling’s formula’ for global NPP ([Bacastow et al., 1973](#); [Alexandrov et al., 2003](#)):

$$\text{NPP}(c_a) = \text{NPP}_0 \left(1 + K_C \log \frac{c_a}{c_{a0}} \right) \quad (1)$$

- 20 Throughout this article, the subscript ‘0’ denotes the value of the quantity at a pre-industrial equilibrium, and ‘log’ denotes natural logarithm. Keeling’s formula incorporates all climate change-related effects on global NPP occurring simultaneously with carbon dioxide changes, for example, precipitation and temperature effects, in addition to fertilisation effects. The curvature of the log function represents limitations to NPP such as changing carbon-use efficiency ([Körner, 2003](#)) or nutrient limitations ([Zaehle et al., 2010](#)). [Constant climate sensitivity is also a key assumption, otherwise the relative weight of climate and](#)
- 25 [CO₂ effects on NPP would change.](#)

At the same time, carbon loss from the world’s soils through respiration, R , is expected to increase at higher global mean surface temperature, ΔT . We approximate the net temperature response of global soil respiration using the Q10 formalism $R(\Delta T) = R_0 Q_R^{\Delta T/10} c_t/c_{t0}$ ([Xu and Shang, 2016](#)), where Q_R is the proportional increase in respiration for a 10 K temperature increase. We assume that pre-industrial soil respiration is balanced by pre-industrial net primary productivity, $R_0 = \text{NPP}_0$. To

30 avoid introducing multiple pools of carbon into the model, we also have to assume that global soil respiration is proportional to total land carbon (rather than soil carbon). Respiration in our model implicitly also includes other carbon-emitting processes such as wildfires or insect disturbances.

It follows that the change in global terrestrial carbon storage is

$$\frac{dc_t}{dt} = \text{NPP}_0 \left(1 + K_C \log \frac{c_a}{c_{a0}} \right) - \frac{\text{NPP}_0}{c_{t0}} Q_R^{T/10 \Delta T/10} c_t - \text{LUC}(t).$$

In this expression we have also included loss of terrestrial carbon due to land use emissions $\text{LUC}(t)$. We rearrange this expression to give

$$5 \quad \frac{dc_t}{dt} = \frac{\text{NPP}_0}{c_{t0}} Q_R^{\Delta T/10} [K(c_a, \Delta T) - c_t] - \text{LUC}(t) \quad (2)$$

where the terrestrial carbon carrying capacity is

$$K(c_a, \Delta T) = \frac{1 + K_C \log \frac{c_a}{c_{a0}}}{Q_R^{\Delta T/10}} c_{t0}. \quad (3)$$

For model simplicity, we do not explicitly model factors affecting terrestrial carbon uptake such as seasonality, species interactions, species functionality, migration, and regional variability.

10 2.2 Ocean

In the upper ocean mixed layer, mixing processes allow exchange of carbon dioxide with the atmosphere. The solubility and biological pumps then transport carbon from the mixed layer into the deep ocean. Since the residence time of deep ocean carbon is several centuries, we explicitly only model the dynamics of upper ocean carbon while the deep ocean is treated merely as an extremely large carbon reservoir. We include the effects of ocean carbon chemistry, the solubility and biological pumps, and
 15 ocean-atmosphere diffusion on upper ocean mixed layer carbon.

Ocean uptake of carbon dioxide from the atmosphere is chemically buffered by other species of dissolved inorganic carbon such as HCO_3^- and CO_3^{2-} , which are produced when dissolved CO_2 reacts with water. The reaction of CO_2 with water, producing these other species, reduces the partial pressure of CO_2 in water allowing for more ocean CO_2 uptake before equilibrium with the atmosphere is achieved. The Revelle factor, r , is defined as the the ratio of the proportional change in
 20 carbon dioxide content to the proportional change in total dissolved inorganic carbon (Sabine et al., 2004; Goodwin et al., 2007). For simplicity, we assume a constant Revelle factor, except for the temperature dependence, D_T , of the solubility of CO_2 in sea water. Therefore CO_2 diffuses between the atmosphere and ocean mixed layer at a rate proportional to

$$c_a - p(c_m, \Delta T), \quad (4)$$

where

$$25 \quad p(c_m, \Delta T) = c_{a0} \left(\frac{c_m}{c_{m0}} \right)^r \frac{1}{1 - D_T \Delta T}, \quad (5)$$

since at pre-industrial equilibrium $p(c_{m0}, 0) = c_{a0}$.

There are two main mechanisms by which carbon is transported out of the upper ocean mixed layer into the deep ocean stocks: the solubility and biological pumps. In the solubility pump, overturning circulations exchange mixed layer and deep

ocean water. We assume that the large size of the deep ocean means its carbon concentrations are negligibly changed over the 100-year time scales relevant for the model. The net transport of carbon to the lower ocean by the solubility pump can therefore be represented by

$$w_0(1 - w_T \Delta T)(c_m - c_{m0}),$$

- 5 where w_0 is the (proportional) rate at which mixed layer ocean water is exchanged with the deep ocean and w_T parameterises weakening of the overturning circulation that is expected to occur with future climate change (Collins et al., 2013).

The biological pump refers to the sinking of biomass and organic carbon produced in the upper ocean to deeper ocean layers (Volk and Hoffert, 1985). In the models on which the IPCC reports are based, a weakening of the biological pump is predicted under climate change, mostly due to a decrease in primary production, in turn due to increases in thermal stratification of ocean
10 waters (Bopp et al., 2013). We represent this climate-induced weakening in a single approximately linear factor, so that the rate of carbon transported out of the upper ocean mixed layer by the biological pump to lower deep sea layers is given by

$$B(\Delta T) = B_0(1 - B_T \Delta T).$$

As on land, we assume a pre-industrial equilibrium where the biological pump was balanced by transport of carbon back to the mixed layer by ocean circulation. We neglect deposition of organic carbon to the sea floor and the long time-scale variations
15 in the biological pump that may have contributed to glacial-interglacial cycles (Sigman and Boyle, 2000). We therefore add an additional term $B(\Delta T) - B(0)$ to the transport of carbon from the ocean mixed layer to the deep ocean. Organic carbon that does not sink to the deep ocean is rapidly respired back to forms of inorganic carbon; the ocean mixed layer stock of organic carbon is therefore small, around 3 PgC (Ciais et al., 2013), and we do not count it in the model's carbon balance.

By combining the expressions for the solubility and biological pumps with ocean-atmosphere carbon dioxide diffusion, we
20 obtain the rate of change of ocean mixed layer DIC, c_m :

$$\frac{dc_m}{dt} = \frac{Dc_{m0}}{rp(c_{m0}, 0)}(c_a - p(c_m, \Delta T)) - w_0(1 - w_T \Delta T)(c_m - c_{m0}) - B(\Delta T) + B(0), \quad (6)$$

The coefficient of the first term was chosen such that $1/D$ is the time scale on which carbon dioxide diffuses between the atmosphere and the ocean mixed layer (that is, derivative of the first term with respect to c_m , evaluated at the pre-industrial equilibrium, is D).

25 The carbon content of the deep ocean does not explicitly enter Eq. (6). To evaluate ocean carbon feedbacks, however, we require the change in total ocean carbon content c_M compared to pre-industrial conditions. We calculate this as ocean mixed layer carbon plus carbon transported to the deep ocean by the solubility and biological pumps:

$$\Delta c_M = \Delta c_m + \int_0^t [w_0(1 - w_T \Delta T)(c_m(t) - c_{m0}) + B(\Delta T) - B(0)] dt \quad (7)$$

We do not explicitly model factors such as the thickness of ocean stratification layers, spatial variation of stratification, nutri-
30 ent limitations to NPP, or changes in ocean circulation due to wind forcing, freshwater forcing or sea-ice processes (Bernardello et al., 2014).

2.3 Atmosphere

Carbon is conserved within the We define c_s to be the total carbon in our 'system' comprised by the ocean mixed layer, atmospheric and terrestrial carbon stocks. , that is,

$$c_a + c_t + c_m = c_s. \quad (8)$$

- 5 The only processes that affect the total carbon in the model are human emissions of fossil carbon into the atmosphere, $e(t)$, and export of carbon into the deep ocean by the solubility and biological pumps, giving

$$c_a + c_t + c_m \frac{dc_s}{dt} = c_{a0} + c_{t0} + c_{m0} + Ee(t) - \int_0^t w_0(1 - w_T \Delta T)(c_m - c_{m0}) dt - \int_0^t (B(\Delta T) - B(0)) dt., \quad (9)$$

in which the initial value of c_s is $c_{a0} + c_{t0} + c_{m0}$. To obtain the dynamics of atmosphere carbon stocks, we therefore solve the differential equation (9) and then use the carbon balance equation (8) to find c_a .

- 10 Increasing atmospheric carbon dioxide levels c_a cause an change in global mean surface temperature, ΔT , compared to its pre-industrial level. To model the response of ΔT , we follow the formulation of Kellie-Smith and Cox (2011), which includes a logarithmic response as per the Arrhenius law and a delay of time scale τ . Physically, this time delay is primarily due to the heat capacity of the ocean.

$$\frac{dT}{dt} \frac{d\Delta T}{dt} = \frac{1}{\tau} \left(\frac{\lambda}{\log 2} \log \left(\frac{c_a}{c_{a0}} \right) - \Delta T \right). \quad (10)$$

- 15 The climate sensitivity λ specifies the increase of temperature in response to a doubling of atmospheric carbon dioxide levels. The climate sensitivity accounts for energy balance feedbacks such as from clouds and albedo. We use the transient climate sensitivity (Collins et al., 2013) as this specifies the response of the climate system over an approximately 100-year time scale (see section 3).

Table 1. Model parameters.

Name	Symbol	Value	Reference/Notes
Pre-industrial atmospheric carbon	c_{a0}	589 PgC	Ciais et al. (2013)
Pre-industrial soil and vegetation carbon	c_{t0}	1875 PgC	1325 PgC of soil organic carbon in top metre of soil (Köchy et al., 2015) plus midrange of vegetation carbon estimate by the Ciais et al. (2013).
Pre-industrial ocean mixed layer carbon	c_{m0}	900 PgC	Ciais et al. (2013)
Climate sensitivity (TCR)	λ	1.8 K	Multi-model mean transient climate response (Flato et al., 2013)
Climate lag	τ	4 yr	Calculations on ocean heat uptake, the primary cause of climate lag, indicate a response time (e -folding time) of 4 yr for time scales up to centuries, before deep ocean heat uptake dominates at millennial time scales (Gregory et al., 2015). This result is consistent with simulations that indicate that maximum warming after a CO ₂ pulse is reached after only a decade (Ricke and Caldeira, 2014) and with results from impulse response model experiments (Joos et al., 2013).
Atmosphere-ocean mixed layer CO ₂ equilibration rate	D	1 yr ⁻¹	Time scale of approximately 1 year, although highly spatially dependent (Jones et al., 2014).
Revelle (buffer) factor	r	12.5	Williams et al. (2016)
Solubility temperature effect	D_T	4.23%/K	Takahashi et al. (1993); Ciais et al. (2013, p498)
Pre-industrial biological pump	B_0	13 PgC/yr	Ciais et al. (2013)
Temperature dependence of biological pump	B_T	3.2%/K	12% decrease (Bopp et al., 2013, Fig 9b) after approximately 3.7 K climate change (Collins et al., 2013)
Solubility pump rate	w_0	0.1 yr ⁻¹	DIC flux rate from ocean mixed layer divided by DIC stock in mixed layer (Ciais et al., 2013)
Weakening of overturning circulation with climate change	w_T	10%/K	Approximate fit to values reported by Collins et al. (2013, p1095)
Terrestrial respiration temperature dependence	Q_R	1.72	Raich et al. (2002); Xu and Shang (2016). Based on soil respiration, which contributes the majority of terrestrial ecosystem respiration.
Pre-industrial NPP	NPP_0	55 PgC/yr	Wieder et al. (2015); Sitch et al. (2015)
Fertilisation effect	K_C	0.3	Estimated by substituting recent NPP \approx 60 PgC/yr (Wieder et al., 2015; Sitch et al., 2015) and recent terrestrial carbon stocks, $c_t \approx c_{t0} + 240$ (Ciais et al., 2013), into Eq. (1). Alexandrov et al. (2003) found that values between 0.3 and 0.4 are compatible with results from a process-based global NPP model.

Table 2. Model validation. Historical changes are carbon stocks in 2011 relative to stocks in 1750 (Ciais et al., 2013) and temperatures in 2012 relative to temperatures in 1880 (Hartmann et al., 2013). Predicted future changes are carbon stocks in 2100 compared to 2012 (Collins et al., 2013) and global mean surface temperatures (GMST) averaged over 2081–2100 relative to 1986–2005 (Collins et al., 2013), under the range of RCP scenarios.

	Ocean carbon changes (PgC)		Land carbon changes (PgC)		GMST change, ΔT (K)	
	IPCC AR5	Model result	IPCC AR5	Model result	IPCC AR5	Model result
Historical	155 \pm 30	95	-30 \pm 45	26	0.85 [0.65 to 1.06]	0.82
RCP2.6	150 [105 to 185]	174	65 [-50 to 195]	67	1.0 [0.3 to 1.7]	0.5
RCP4.5	250 [185 to 400]	243	230 [55 to 450]	135	1.8 [1.1 to 2.6]	1.2
RCP6	295 [265 to 335]	278	200 [-80 to 370]	168	2.2 [1.4 to 3.1]	1.7
RCP8.5	400 [320 to 635]	340	180 [-165 to 500]	207	3.7 [2.6 to 4.8]	2.4

3 Model parameterisation and validation

Our climate-carbon cycle model has twelve parameters, four state variables and three nontrivial initial conditions (by definition, the initial value of ΔT is 0). We choose to parameterise each process with the best available knowledge about that process, rather than try to force the model to fit historical data. This is in line with our stated model purposes of understanding and learning, rather than prediction. Parameters for the response of climate to carbon dioxide (λ , τ) and two parameters of the response of the ocean to changing temperature (B_T and w_T) were set based on the output of atmosphere-ocean global circulation models. For the climate sensitivity λ , transient climate response was used. All other parameters are based on historical observations of the global carbon cycle (Table 1).

Unless otherwise noted, we perform emissions-based model runs using harmonized historical data and future RCP scenarios on fossil fuel emissions [$E(t)e(t)$] and land use emissions [LUC(t)] (Meinshausen et al., 2011b). While the focus of our study is on future climate change, from the present day until 2100, we begin simulations in 1750 to compare our model against historical observations. Time series of the model output are displayed in Fig. 2. Model solutions were approximated in continuous time.

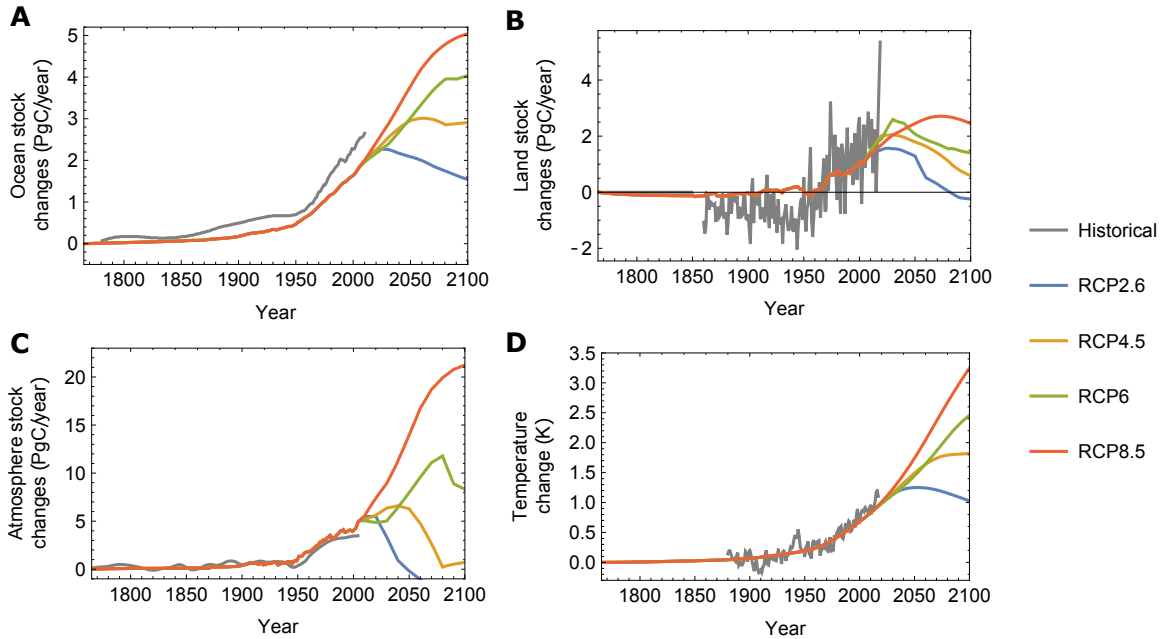


Figure 2. Model output under forcing from different RCP scenarios: (a) land carbon stock change, (b) ocean carbon stock changes, (c) atmospheric carbon stock change, and (cd d) global mean surface temperature change. Historical changes in carbon stocks are from Le Quéré et al. (2016) and historical temperature anomalies are from NOAA (2018). The historical temperature dataset of NOAA (2018), which is relative to the period 1901-2000, has been offset to match the model's average temperature anomaly over the same period.

4 Feedback analysis

Our climate-carbon cycle model is sufficiently simple that the strengths of its feedbacks can be estimated analytically. Such computations are useful since the resulting symbolic expressions can be used to identify how parameters of interest affect feedback strengths and model dynamics. In this section we introduce definitions of feedback strengths, calculate climate-carbon cycle feedbacks analytically and numerically, and estimate feedback nonlinearities.

4.1 Definitions

There are multiple measures of carbon cycle feedbacks currently in use. We here review three of the most common measures.

Consider an emission of E PgC over some time period to the atmosphere. In the absence of carbon cycle feedbacks, the atmospheric carbon content would increase by $\Delta c_a^{\text{off}} \equiv E$. With a feedback switched on, the atmospheric carbon content would actually change by Δc_a^{on} . The *feedback factor* is (Zickfeld et al., 2011)

$$F = \frac{\Delta c_a^{\text{on}}}{\Delta c_a^{\text{off}}}. \quad (11)$$

Out of the total atmospheric carbon change Δc_a^{on} , the carbon cycle feedback contributes (Hansen et al., 1984)

$$\Delta c_a^{\text{feedback}} = \Delta c_a^{\text{on}} - \Delta c_a^{\text{off}}. \quad (12)$$

Gain is the change in a feedback to atmospheric carbon content caused by changes in atmospheric carbon content:

$$g = \frac{\Delta c_a^{\text{feedback}}}{\Delta c_a^{\text{on}}}. \quad (13)$$

5 Gain and feedback factor are related by

$$F = \frac{1}{1 - g}. \quad (14)$$

An alternative formalism, introduced by Friedlingstein et al. (2006), allows feedbacks to be characterised from carbon cycle model output. Climate models are not required, except as a forcing to the carbon cycle model. The formalism relates the changes in terrestrial and marine carbon stocks to changes in global mean temperature and atmospheric carbon dioxide as

10 follows:

$$\Delta c_t = \beta_L \Delta c_a + \gamma_L \Delta T \quad (15)$$

$$\Delta c_M = \beta_O \Delta c_a + \gamma_O \Delta T. \quad (16)$$

Here the β_L and β_O feedback parameters are the land and ocean, respectively, carbon sensitivities to atmospheric carbon dioxide changes Δc_a . Likewise, γ_L and γ_O are the land and ocean, respectively, carbon sensitivities to temperature changes ΔT . Note that c_M denotes the total marine carbon stock, both mixed layer and deep ocean. The differences Δc_a , etc., are usually calculated over the duration of a simulation. To isolate the β and γ feedback parameters, simulations are conducted with biogeochemical coupling only and with radiative coupling only (Gregory et al., 2009).

In both the formalisms introduced thus far, the feedback measures are calculated by examining the changes in carbon stocks at the end point of model simulations. In contrast, Boer and Arora (2009) estimate sensitivities Γ and B of the instantaneous carbon *fluxes* from atmosphere to land and ocean:

$$\frac{dc_t}{dt} = B_L \Delta c_a + \Gamma_L \Delta T \quad (17)$$

$$\frac{dc_M}{dt} = B_O \Delta c_a + \Gamma_O \Delta T. \quad (18)$$

These feedback parameters B and Γ are usually computed for all time points during a simulation, again using biogeochemically coupled and radiatively coupled simulations.

25 The two sets of parameters (B, Γ) and (β, γ) are related by

$$\beta \Delta c_a = \int B \Delta c_a dt \quad (19)$$

$$\gamma \Delta T = \int \Gamma \Delta T dt. \quad (20)$$

Accordingly, Boer and Arora (2013) refer to B and Γ as *direct feedback parameters* and to β and γ as *time-integrated feedback parameters*.

4.2 Analytical feedback strengths based on equilibrium changes

Analytical approximations to the strengths of carbon cycle feedbacks in our model require choosing a time scale on which the feedbacks will be calculated. Numerically estimated feedback factors [Eq. (11)] and time-integrated feedback parameters [Eqs. (15-16)] are conventionally calculated using carbon stock changes over 100 years or more. Responses on the longest 5 time scales of our model are therefore most relevant if our analytical approximates are to approximate numerically calculated values. While recognising that the Earth's climate system is presently far from equilibrium, we use changes in the equilibrium state of the model to approximate model responses over long time scales.

We analytically calculate the gains associated with each of the feedback loops in Fig. 1 as follows. We calculate the sensitivity (mathematically, partial derivative) of the equilibrium value of each quantity in the feedback loop with respect to the preceding 10 quantity in the loop. We form the product of the derivatives (as per the chain rule of differentiation) to estimate the gain of that feedback loop. For example, to calculate the land climate-carbon gain we calculate the sensitivity of equilibrium temperature with respect to changes in atmospheric carbon content ($\partial T^*/\partial c_a$), multiplied by the sensitivity of equilibrium terrestrial carbon with respect to changes in temperature ($\partial c_t^*/\partial T$), multiplied by the sensitivity of equilibrium atmospheric carbon with respect to changes in terrestrial carbon ($\partial c_a^*/\partial c_t$).

$$\begin{aligned}
 15 \quad \text{Land climate-carbon equilibrium gain} \quad g_{TL}^* &\equiv \frac{\partial T^*}{\partial c_a} \frac{\partial c_t^*}{\partial T} \frac{\partial c_a^*}{\partial c_t} \\
 \text{Land concentration-carbon equilibrium gain} \quad g_L^* &\equiv \frac{\partial c_t^*}{\partial c_a} \frac{\partial c_a^*}{\partial c_t} \\
 \text{Ocean climate-carbon equilibrium gain} \quad g_{TO}^* &\equiv \frac{\partial T^*}{\partial c_a} \frac{\partial c_m^*}{\partial T} \frac{\partial c_M}{\partial c_m} \frac{\partial c_M}{\partial T} \frac{\partial c_a^*}{\partial c_M} \\
 \text{Ocean concentration-carbon equilibrium gain} \quad g_O^* &\equiv \frac{\partial c_m^*}{\partial c_a} \frac{\partial c_M}{\partial c_m} \frac{\partial c_M}{\partial c_a} \frac{\partial c_a^*}{\partial c_M}
 \end{aligned}$$

The subscript T denotes that the feedback involves temperature. Asterisks (*) denote equilibrium quantities. From these gains, 20 the feedback factors F_{TL}^* , F_L^* , F_{TO}^* and F_O^* can be calculated using Eq. (14). We label these gains and feedback factors g^* and F^* , respectively, to denote they are based on an equilibrium approximation, not directly from transient simulations as estimated by Zickfeld et al. (2011). We calculate $\frac{\partial c_m^*}{\partial c_a} \frac{\partial c_M}{\partial c_m}$ rather than simply $\frac{\partial c_M^*}{\partial c_a}$ as it is c_m that is a state variable in our model, from which we then estimate c_M .

The derivatives of c_a^* are trivial to calculate: by carbon balance, $\frac{\partial c_a^*}{\partial c_t} = \frac{\partial c_a^*}{\partial c_M} = -1$. To calculate $\frac{\partial c_m^*}{\partial T}$ we set $0 = \frac{dc_m}{dt} \equiv f(c_m^*, c_a, T)$, 25 use the chain rule to obtain $0 = \frac{\partial f}{\partial c_m^*} \frac{\partial c_m^*}{\partial T} + \frac{\partial f}{\partial T}$, and then solve for $\frac{\partial c_m^*}{\partial T}$ in terms of the partial derivatives of f . Similar procedures provide $\frac{\partial c_m^*}{\partial c_a}$ and the derivatives of T^* and c_t^* . the derivatives of c_t^* , we set $0 = \frac{dc_t}{dt}$, solve for c_t and calculate the necessary derivatives. A similar procedure provides $\frac{\partial T^*}{\partial c_a}$.

The remaining derivative is $\partial c_M/\partial c_m$ derivatives are $\frac{\partial c_M}{\partial T}$ and $\frac{\partial c_M}{\partial c_a}$. Carbon sunk into the deep ocean is substantial and cannot be neglected. Deep ocean carbon storage equilibrates on time scales of millennia or more, however, far longer than the time 30 scales of interest in this model (we therefore write $\partial c_M/\partial c_m$ not $\partial c_M^*/\partial c_m$ derivatives of c_M rather than c_M^*). We therefore cannot use the same equilibrium approach as for the other variables. Instead, we use derive approximations to Eq. (7) with the following approximations as follows. First, we neglect the temperature dependence of the biological pump and the rate of the overturning circulation, as for this derivative we are

primarily interested in the effects of changing carbon stocks, not temperatures. Second, let us assume a scenario where the trajectory of ocean mixed layer DIC c_m observe that in the SRES A2 scenario used below both $c_m(t)$ and $\Delta T(t)$ can be approximated by a linear increase from c_{m0} to c_m as linear increases, starting at $c_m = c_{m0}$ and $\Delta T = 0$ respectively, over a time interval t_{lin} . We estimate this time interval by $t_{\text{lin}} = (c_m(t_{\text{end}}) - c_{m0})/c'_m(t_{\text{end}})$ using the value c_m and gradient c'_m at the end of the simulation period. Using this approximation and

5 Eq. ,

$$\frac{\partial c_M}{\partial c_m} \approx 1 + M$$

where $M = w_0 t_{\text{lin}}/2$. The value of M will be strongly scenario-dependent We obtain

$$\Delta c_M \approx c_m - c_{m0} + w_0 \left(\frac{1}{2} - \frac{1}{3} w_T \Delta T \right) (c_m - c_{m0}) t_{\text{lin}} - \frac{1}{2} B_0 B_T \Delta T t_{\text{lin}}. \quad (21)$$

We use this equation to calculate the derivatives $\frac{\partial c_M}{\partial T}$ and $\frac{\partial c_M}{\partial c_a}$. Evaluating these derivatives will involve the derivatives $\frac{\partial c_m}{\partial T}$ and $\frac{\partial c_m}{\partial c_a}$. Since partial pressures across the air-sea interface equilibrate rapidly on the time scale of the model ($D = 1\text{yr}^{-1}$, Table 1), we assume that $c_a \approx p(c_m, \Delta T)$, rearrange for c_m and then calculate the appropriate derivatives from the resulting equation.

We analytically estimate equilibrium versions of the time-integrated feedback parameters of Friedlingstein et al. (2006) using a similar approach:

$$15 \quad \gamma_L^* = \frac{\partial c_t^*}{\partial T}$$

$$\beta_L^* = \frac{\partial c_t^*}{\partial c_a}$$

$$\gamma_O^* = \frac{\partial c_m^*}{\partial T} \frac{\partial c_M}{\partial c_m} \frac{\partial c_M}{\partial T}$$

$$\beta_O^* = \frac{\partial c_m^*}{\partial c_a} \frac{\partial c_M}{\partial c_m} \frac{\partial c_M}{\partial c_a}.$$

20 Since the ocean component of the model has multiple processes that respond to temperature, some analytical forms were too complicated for easy visual inspection (Table A1). We derived approximate analytical feedbacks by comparing the magnitudes of terms in the numerator and denominator of the feedback measures , and by expanding numerators by expanding in power series of $D_T T$ and c_a/c_{a0} .

4.3 Analytical feedback strengths based on carbon fluxes

We estimate the direct feedback parameters as follows:

$$\begin{aligned}
 \Gamma_L^* &= \left. \frac{dc_t}{dt} \right|_{c_a=c_{a0}} \frac{1}{\Delta T} \\
 B_L^* &= \left. \frac{dc_t}{dt} \right|_{\Delta T=0} \frac{1}{c_a - c_{a0}} \\
 5 \quad \Gamma_O^* &= \left. \frac{dc_M}{dt} \right|_{c_a=c_{a0}} \frac{1}{\Delta T} \\
 B_O^* &= \left. \frac{dc_M}{dt} \right|_{\Delta T=0} \frac{1}{c_a - c_{a0}}.
 \end{aligned}$$

Here dc_t/dt and dc_M/dt denote the atmosphere-land and atmosphere-ocean fluxes. The subscript $\Delta T = 0$ denotes a biogeochemically coupled (and radiatively decoupled) simulation and $c_a = c_{a0}$ denotes a radiatively coupled (and biogeochemically decoupled) simulation. **We use the**

- 10 The values of the feedback parameters are strongly scenario-dependent (Arora et al., 2013). To calculate the direct feedback parameters, we assume a standard CO₂-quadrupling concentration pathway in order to compare our results with Arora et al. (2013). This scenario has $c_a(t) = c_{a0}a^t$ where $a = 1.01$. In this scenario, $\frac{1}{c_a} \frac{dc_a}{dt} = \log a$ and, ignoring an initial exponential transient, $\frac{dT}{dt} = \lambda \log a / \log 2$.

- For the atmosphere-land carbon flux, the calculation is straightforward under the following assumptions. We assume that
 15 $NPP_0/c_{t0} \gg \log a$ so that c_t tracks its carrying capacity $c_t \approx K$ [Eq. (2)]. We also ignore land use change, so that $\frac{dc_t}{dt} \approx \frac{dK}{dt}$. Then we calculate $\left. \frac{dK}{dt} \right|_{c_a=c_{a0}} = \frac{\partial K}{\partial T} \frac{dT}{dt}$ and $\left. \frac{dK}{dt} \right|_{\Delta T=0} = \frac{\partial K}{\partial c_a} \frac{dc_a}{dt}$.

- While the atmosphere-ocean flux could be read off directly from the first term of Eq. (6), this form is however not particularly useful. As it involves a small difference between two large quantities, c_a and $p(c_m, \Delta T)$, the size of the difference can only be estimated from numerical results and gives no immediate insight into how it depends on parameters. Furthermore, we seek to
 20 compare our analytical results to the results presented by Arora et al. (2013), in which the feedback parameters are presented as functions of c_a or ΔT only (not c_m).

- We instead derive an approximation based on time scale separation as follows. The characteristic time **scales scale** of atmosphere-ocean diffusion **, is much faster than the** solubility pump, biological pump **and or** human emissions into the atmosphere **are** D ,
 25 w_0 , B_0/c_{m0} and $\log a$ respectively. These rates are ordered $D \gg w_0 \gg \log a, B_0/c_{m0}$. ($D \gg w_0, B_0/c_{m0}, \log a$). Since atmosphere-ocean diffusion is the fastest process, ocean mixed layer carbon content rapidly gains an equilibrium $c_m = p^{-1}(c_a, \Delta T)$ with respect to atmospheric carbon content, where $p^{-1}(c_a, \Delta T)$ is the solution to $c_a = p(c_m, \Delta T)$. **Ocean and atmosphere partial pressures are kept out of equilibrium by the next fastest process: the solubility pump.** On the time scale of our model, the atmosphere-ocean flux is therefore controlled by the solubility **pump and biological pumps**, with diffusion providing a rapid coupling between ocean mixed layer and atmosphere. An approximation for the atmosphere-ocean flux is therefore $dc_M/dt \approx w_0(1 - w_T \Delta T)(p^{-1}(c_a, \Delta T) - c_{m0})dc_M/dt \approx$
 30 $w_0(1 - w_T \Delta T)(p^{-1}(c_a, \Delta T) - c_{m0}) - B_0 B_T T$, which upon substitution into the definitions of B_O^* and Γ_O^* gives the forms in Table A1. Taylor series expansions and L'Hôpital's rule were then used to derive the approximate forms in Table 3.

4.4 Numerical estimation of feedback strengths

In addition to the analytical approximations to carbon cycle feedbacks derived from our model, we also estimate feedback factors from direct numerical simulations of our model. To compare the results of our model to previous studies, we use the following scenarios. To compare our results with the time-integrated feedback parameters reported by Friedlingstein et al. (2006) and the feedback factors and gains of Zickfeld et al. (2011), we employ the SRES A2 emissions scenario used in those articles. To compare our results with the direct feedback parameters of Arora et al. (2013), we use the **doubling CO₂ concentration same** scenario used in that article **in which CO₂ concentration increases 1% per year until it quadruples (approximately 140 years)**. For each scenario, we perform **three four** simulations:

1. Fully coupled simulation.
2. Completely uncoupled simulation, giving $c_a^{\text{off}}(t) = c_{a0} + \int^t E(t)dt$ for the emissions-driven scenario and the specified concentration pathway for concentration-driven scenario.
3. Biogeochemically coupled simulation. We switch off feedbacks involving temperature response to atmospheric CO₂, by setting $\lambda = 0$. **Since our model contains no radiative forcing other than changes in CO₂, temperature $\Delta T = 0$ in this simulation.** From this simulation we estimate the carbon-concentration feedback factors via land $F_L = \Delta c_a^{\text{on}} / \Delta c_a^{\text{off}} = 1 - \Delta c_t / \Delta c_a^{\text{off}}$ and ocean $F_O = \Delta c_M^{\text{on}} / \Delta c_M^{\text{off}} = 1 - \Delta c_M / \Delta c_M^{\text{off}}$, time-integrated feedback parameters $\beta_L = \Delta c_t / \Delta c_a$ and $\beta_O = \Delta c_M / \Delta c_a$, and direct feedback parameters $B_L(t) = \frac{dc_t}{dt} / (c_a - c_{a0})$ and $B_O(t) = \frac{dc_M}{dt} / (c_a - c_{a0})$.
4. Radiatively coupled simulation. We switch off feedbacks involving the carbon cycle, by setting $K_C = 0$ and changing the c_a in Eq. (6) to c_{a0} . From this simulation we estimate the carbon-climate feedback factors $F_{TL} = 1 - \Delta c_t / \Delta c_a^{\text{off}}$ and $F_{TO} = 1 - \Delta c_M / \Delta c_M^{\text{off}}$, time-integrated feedback parameters $\gamma_L = \Delta c_t / \Delta T$ and $\gamma_O = \Delta c_M / \Delta T$ **following Arora et al. (2013)**, and direct feedback parameters $\Gamma_L(t) = \frac{dc_t}{dt} / \Delta T$ and $\Gamma_O(t) = \frac{dc_M}{dt} / \Delta T$.

4.5 Feedback nonlinearity

Zickfeld et al. (2011) found, in emissions-driven scenarios, that the fully coupled simulation sunk more terrestrial and marine carbon than the sum of the biogeochemically and radiatively coupled scenarios. They refer to this difference as the non-linearity of the feedback, with the land sink contributing 80% of the nonlinearity and the ocean sink 20%. Our analytical expressions for the feedbacks can be used to obtain an alternative measure of feedback nonlinearity.

We evaluate the nonlinearity in the ocean and land climate-carbon feedbacks by $F_{TO}^*(c_a, c_m, c_t, \Delta T) - F_{TO}^*(c_{a0}, c_{m0}, c_{t0}, \Delta T)$ and $F_{TL}^*(c_a, c_m, c_t, \Delta T) - F_{TL}^*(c_{a0}, c_{m0}, c_{t0}, \Delta T)$, respectively, where the $F^*(c_{a0}, c_{m0}, c_{t0}, \Delta T)$ are analytical approximations of feedback factors from a radiatively coupled simulation (all carbon stocks are fixed at pre-industrial levels). We evaluate the nonlinearities in the ocean and land concentration-carbon feedbacks by $F_O^*(c_a, c_m, c_t, \Delta T) - F_O^*(c_a, c_m, c_t, 0)$ and $F_L^*(c_a, c_m, c_t, \Delta T) - F_L^*(c_a, c_m, c_t, 0)$, respectively, where the $F^*(c_a, c_m, c_t, 0)$ are analytical approximations of feedback factors from a biogeochemically coupled simulation (temperature is fixed at its pre-industrial level). These expressions indicate

Table 3. Feedback analysis. Gains (g), feedback factors (F), time-integrated feedback parameters (γ and β) and direct feedback parameters (Γ and B) were calculated analytically and numerically. Analytical ocean feedbacks are approximations of the exact forms in Table A1 (see Sec. 4.2. Exact forms were used to calculate numerical values. In this table, $p \equiv p(c_m, T)$. Units of the climate-carbon integrated feedback parameters are PgC/K and concentration-carbon integrated feedback parameters are PgC/ppm. Ranges for analytical results are written in the form (value at start of simulation) to (value at end of simulation). Emissions scenarios are as indicated; land use emissions were assumed to be zero. [From the results of simulations using the SRES A2 scenario we use \$t_{\text{lin}} \approx 100\$ corresponding to a period between the years 2000 and 2100.](#)

Feedback measure	Land climate-carbon feedback	Ocean climate-carbon feedback	Land conc.-carbon feedback	Ocean conc.-carbon feedback
Gain, analytical expression	$\frac{\lambda c_{t0} \left(1 + K_C \log \frac{c_a}{c_{a0}}\right) \log Q_R}{10 c_a Q_R^{\Delta T/10} \log 2}$	$\frac{\lambda D D_T (1+M) c_m}{c_a \log 2 \left(\frac{B_0 B_T}{2} + \frac{c_m D_T w_0}{2r} + \frac{w_0 w_T (c_m - c_{m0})}{3} \right)}$	$-\frac{c_{t0} K_C}{c_a Q_R^{\Delta T/10}}$	$-\frac{(1+M) c_m}{c_a r}$
Feedback factor (numerical scenario: SRES A2) (> 1 amplifies climate change; < 1 dampens climate change)				
- estimate from analytical gain	1.81 to 1.18	1.07 to 1.03 1.01 to 1.09	0.51 to 0.81	0.59 to 0.80 0.89 to 0.84
- from simulation	1.15	1.10	0.74	0.67
- Zickfeld et al. (2011)	1.25	1.22	0.66	0.71
Time-integrated feedback parameter (numerical scenario: SRES A2) (< 0 amplifies climate change; > 0 dampens climate change)				
- analytical expression	$-\frac{c_{t0} \log Q_R}{10 Q_R^{\Delta T/10}}$	$-\frac{D(1+M) c_m D_T}{B_0 B_T} \frac{t_{\text{lin}}}{2} - \frac{c_m D_T w_0 t_{\text{lin}}}{2r} - \frac{t_{\text{lin}}}{3} w_0 w_t (c_m - c_{m0})$	$\frac{c_{t0} K_C}{c_a}$	$\frac{(1+M) c_m}{c_a r}$
- estimate from analytical form	-102 to -86	-15 to -21 -3 to -67	2.04 to 0.60 0.51	1.48 to 0.51 0.26 to 0.42
- from simulation	-74	-48	0.73	1.04
- Zickfeld et al. (2011)	-129	-32	1.12	0.86
- Friedlingstein et al. (2006)	-79 (-20 to -177)	-30 (-14 to -67)	1.35 (0.2 to 2.8)	1.13 (0.8 to 1.6)
Direct feedback parameter (numerical scenario: CO ₂ doubling) (< 0 amplifies climate change; > 0 dampens climate change)				
- analytical expression	$-\frac{c_{t0} \lambda \log Q_R \log a}{10 \Delta T \log 2}$	$-\frac{w_0 c_{m0} D_T}{r} - B_0 B_T$	$\frac{c_{t0} K_C \log a}{c_a - c_{a0}}$	$\frac{w_0 c_{m0} c_a}{r c_{a0} (c_a - c_{a0})} \frac{w_0 c_{m0} c_a^{\frac{1}{2}-1}}{r c_{a0}^{\frac{1}{2}}}$
- estimate from analytical form	Fig. A1a	Fig. A1b	Fig. A1a	Fig. A1b
- from simulation	Fig. A1a	Fig. A1b	Fig. A1a	Fig. A1b
- Arora et al. (2013)		see text		
Nonlinearity	-0.43	-0.06 -0.11	0.03	0.02 0.03

the effect of temperature and atmospheric carbon on the concentration-carbon and climate-carbon feedbacks, respectively, We used the SRES A2 scenario.

5 Results and Discussion

5.1 Model evaluation

5 Most predictions of our model are within the range of model predictions produced for the IPCC's Fifth Assessment Report (Table 2). Our model estimates around 55 PgC more historical land carbon uptake than the IPCC multi-model mean, possibly due to our simplification to a single land carbon pool. Because it omits radiative forcing due to greenhouse gases other than CO₂, our model consistently underestimates future temperature changes, although in all except the RCP8.5 scenario the projections are within the IPCC model range. The purpose of our model is not to precisely predict future climate change, but to
10 serve as an approximate, mechanistically based emulator of the carbon cycle component of Earth System Models (see Sec. 2). We conclude that the model emulates historical observations and future projections of Earth System Models with sufficient accuracy for this purpose.

5.2 Feedback analysis

Both All feedback measures calculated directly from our model simulations and measures estimated from analytical approximations match well with stylised
15 model simulations, as well as most of our analytically estimated feedback measures, are within a factor of 2 of the mean output from Earth System Models reported by Friedlingstein et al. (2006) and Zickfeld et al. (2011) [Table 3; compare also Fig. A1 with figures 4-5 of Arora et al. (2013) for direct feedback parameters]. This agreement is a remarkably good agreement considering the highly stylised nature of our model. Furthermore, all of the numerically time-integrated feedback parameters from our stylised model are within the multi-model range reported by Friedlingstein et al. (2006). This agreement
20 serves as additional validation of our model as well as validation of the approximations used to calculate analytical feedback factors.

An exception to the close agreement is the ocean concentration-carbon feedback, with analytically estimated feedback factor and time-integrated feedback parameter indicating a weaker negative feedback than the numerical estimates from our stylised model or ESMs. This mismatch is primarily due to two approximations: one in the numerical simulation
25 and one in the analytical approximation. The numerical approximation is that disconnecting climate feedbacks in the biogeochemically coupled simulation leaves less carbon available to be sunk into the ocean, placing the ocean feedback at a different point in the highly nonlinear (as parameterised by the Revelle factor) ocean carbon uptake dynamics. The analytical approximation is that Eq. (21) underestimates carbon sunk into the deep ocean.

The approximate analytical expressions for the three different feedback measures all have similar dependences on state
30 variables and parameters. All measures of the land climate-carbon feedback have dependence of the form $c_{t0} \log Q_R/Q_R^{\Delta T/10}$. The ocean climate-carbon feedbacks all have the form $w_0 D_T c_{m0}/r$ (since $1 + M \sim w_0$ and ocean mixed layer carbon $c_m \approx c_{m0}$ to within 10% over the

duration of the simulation) terms of the form $B_0 B_T$ and $w_0 D_T c_m / r$. The land concentration-carbon feedback has the form $c_{t0} K_C / c_a$ and the ocean concentration-carbon feedbacks have the form $w_0 c_m / r c_a$ (since $p(c_m, \Delta T) \approx c_a$). We conclude that for each type of carbon cycle feedback, all three feedback formalisms detect similar features of the climate-carbon system.

The analytical expressions provide rapid insight into how feedback strengths depend on state variable and parameter values that could otherwise only be studied numerically or by qualitative reasoning. The analytical forms show that increasing Revelle factor r , as is likely to occur in an increasingly acidic ocean (Sabine et al., 2004), will decrease the strengths of ocean climate-carbon and concentration-carbon feedbacks. Weakening overturning circulation, via w_0 , would also decrease the strength of the ocean carbon cycle feedbacks. On land, the parameters Q_R and K_C control the terrestrial carbon cycle feedbacks.

We can compare likely trends in feedback strengths from the analytical expressions. In the ocean for the direct feedback parameters. According to our model, the destabilising ocean climate-carbon feedback is almost constant, while the ocean concentration-carbon feedback weakens with c_m (since $c_m / p(c_m, \Delta T) \sim c_m^{1-r} c_m / c_a \sim c_m^{1-r}$). Similarly, according to our model the destabilising land climate-carbon feedback will weaken less than the stabilising concentration-carbon feedback (under CO₂ doubling, $\sim Q_R^{-\Delta T/10}$ weakens by 9% at the new temperature equilibrium while $\sim 1/c_a$ weakens by 50%). This difference between the land climate-carbon and concentration-carbon feedbacks stems from the differing curvatures of $K(c_a, \Delta T)$ as a function of ΔT (close to linear) and c_a (concave). We conclude that under continued carbon emissions, according to our model, both land and ocean feedbacks will overall become more positive.

Where multiple processes contribute in parallel to a feedback, inspection of analytical forms can indicate the relative contributions of the different processes to the feedback. In the ocean component of the model, CO₂ solubility, the biological pump, and the solubility pump are all temperature-dependent and therefore contribute to the ocean climate-carbon feedback. Terms in the numerators of the exact forms of g_{TO} and γ_O (Table A1) correspond to these three processes. Substituting parameters and typical values for state variables into these three terms show that the Remarkably, all three processes contribute temperature dependences of a similar magnitude; we therefore list all three in the approximate analytical gain and time-integrated feedback parameter in Table 3. The three terms represent temperature dependence of the biological pump, CO₂ solubility contributes most to these climate-carbon feedbacks, and the solubility pump, respectively.

25 5.3 Feedback nonlinearity

As shown in Sec. 4.5, our analytical feedback expressions enable a new way of estimating feedback nonlinearities that is not possible from direct numerical simulation. Since the sum of the four nonlinearities is negative (Table 3), we conclude that summing feedbacks found by individual decoupled simulations will overestimate the atmospheric carbon levels, that is, underestimate land and ocean sinks. This result matches the findings of Zickfeld et al. (2011) and Matthews (2007). Terrestrial feedbacks contributed 91.83% of the total nonlinearity in our model, compared to 80% reported by Zickfeld et al. (2011). Furthermore, we can distinguish the nonlinearities in the climate-carbon and concentration-carbon feedbacks. We found that the nonlinearity in the terrestrial carbon-climate feedback was almost ten four times larger than any other (Table 3). By inspecting the analytical derivation of the gains we conclude that this dominance is likely due to a combination of two three reasons: First, due to the sensitivity of temperature to carbon dioxide, $\partial T / \partial c_a = \lambda / c_a \log 2$, the carbon-climate feedbacks are much

more sensitive to c_a than the concentration-carbon feedbacks are to ΔT . Second, the nonlinearity in the land climate-carbon feedback is larger than the ocean climate-carbon feedback because its feedback factor is larger and therefore more sensitive to changes in gain (see Eq. (13)). **Third, the century time-scale of the simulation prevented ocean carbon dynamics, which generally take place on longer time scales, from being exhibited.** We conclude that care must be taken when summing results
5 for feedbacks from decoupled simulations, especially for simulations involving land feedbacks.

6 Conclusions

Earth System Models span a wide variety of complexity. Here, we constructed a highly stylised, globally aggregated climate-carbon cycle model. Despite the model’s simplicity—just four state variables—the model emulated globally aggregated historical trends and future projections of Earth System Models. The model’s simple form allowed climate-carbon cycle feedbacks
10 to be estimated analytically, providing mechanistic insight into these processes. For example, we showed that carbon-climate feedbacks are less sensitive than carbon-concentration feedbacks; on land, this is due to the shape of $K(c_a, \Delta T)$. The simple but accurate climate-carbon cycle model could be a starting point for model-based investigations of Earth system processes that are too poorly understood to be incorporated in more **complex comprehensive** models.

Stylized models such as ours have significant value in policy contexts. When confronted with difficult policy decisions
15 involving long time periods and significant uncertainty, collaborative work with scientists allows policy makers to identify and clarify the impacts of various policy actions. In this context, the utility of a model is to increase stakeholders’ understanding of a system and its dynamics under various conditions (Voinov and Bousquet, 2010; Anderies, 2005). This is in stark contrast to the use of more **complex, detailed comprehensive** models to predict impacts of policies where mechanisms underlying dynamics and trade-offs are not transparent, and quick explorations with stakeholders are not practical. The utility of a stylised model is
20 in facilitating a learning process rather than in ‘accurately’ predicting outcomes.

We foresee at least two strands of valuable future research based on the climate-carbon cycle model developed in this paper. First, our climate-carbon cycle model could be extended by including further processes relevant on different time-scales of interest for Earth system analysis. This would enable a more in-depth analytical analysis of the feedback strengths and gains relating to other aspects of Earth system dynamics, such as the Earth’s energy balance, **carbon storage in the
25 tropics compared to extra-tropics**, albedo changes, the cryosphere, nutrient cycles, and even societal feedbacks. The task of characterizing the Anthropocene as an epoch could thus move beyond qualitative comparison of human-impact trends to an improved characterisation of the feedbacks that maintain different Earth system ‘regimes’. **The effects on feedback strengths of different functional forms, such as the fertilisation effect K_C , and how to constrain these functional forms from data could also be investigated and could yield insight into the continued divergence of ESM projections.**

Second, the model could comprise a ‘workbench’ for the systemic understanding of planetary boundary interactions and, hence, generate crucial insights into the structure of the safe operating space for humanity delineated by the planetary boundaries (Rockström et al., 2009; Steffen et al., 2015). Such extensions should focus on linking the core abiotic and biotic dimensions of the planetary boundaries framework. The present lack of well-developed model representations of the dynamics and

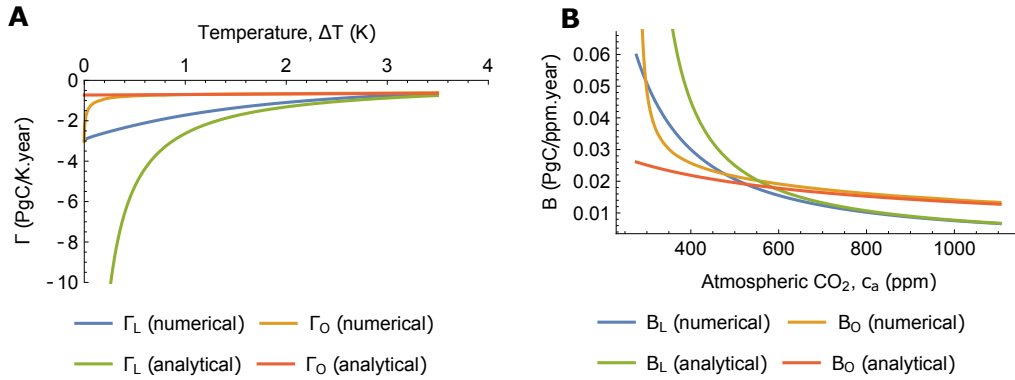


Figure A1. Direct feedback parameters, (a) concentration-carbon climate-carbon feedbacks and (b) climate-carbon concentration-carbon feedbacks.

Table A1. Exact forms for ocean feedbacks.

Feedback measure	Ocean climate-carbon feedback	Ocean concentration-carbon feedback
Gain	$\frac{\lambda(1+M)}{c_a \log 2} \frac{\frac{pc_{m0}DDT}{c_{a0}r(1-D_T\Delta T)} - B_0B_T - w_0w_T(c_m - c_{m0})}{\frac{pDc_{m0}}{c_m c_{a0}} + w(1-w_T\Delta T)}$ $+ \frac{\lambda}{c_a \log 2} \left[\frac{B_0B_T t_{lin}}{2} + \frac{t_{lin}}{3} w_0 w_t (c_m - c_{m0}) \right]$ $+ \frac{c_m D_T}{r(1-D_T\Delta T)} \left(1 + w_0 t_{lin} \left(\frac{1}{2} - \frac{w_T \Delta T}{3} \right) \right)$	$- \frac{c_{m0}D}{c_{a0}r} \frac{1+M}{\frac{pDc_{m0}}{c_m c_{a0}} + w(1-w_T\Delta T)}$ $- \frac{c_m}{c_a r} \left(1 + w_0 t_{lin} \left(\frac{1}{2} - \frac{w_T \Delta T}{3} \right) \right)$
Time-integrated feedback parameter	$- \frac{c_m D_T}{r(1-D_T\Delta T)} \left(1 + w_0 t_{lin} \left(\frac{1}{2} - \frac{w_T \Delta T}{3} \right) \right)$ $- \frac{B_0B_T t_{lin}}{2} - \frac{t_{lin}}{3} w_0 w_t (c_m - c_{m0})$	$\frac{c_{m0}D}{c_{a0}r} \frac{1+M}{\frac{pDc_{m0}}{c_m c_{a0}} + w}$ $\frac{c_m}{c_a r} \left(1 + w_0 t_{lin} \left(\frac{1}{2} - \frac{w_T \Delta T}{3} \right) \right)$
Direct feedback parameter	$\frac{w_0(1-w_T\Delta T)c_{m0} \left((1-D_T\Delta T)^{\frac{1}{r}} - 1 \right)}{\Delta T} - B_0B_T$	$\frac{w c_{m0} \left(\left(\frac{c_a}{c_{a0}} \right)^{\frac{1}{r}} - 1 \right)}{c_a - c_{a0}}$

ecosystem structure of biosphere diversity, heterogeneity and resilience, despite ongoing efforts in this direction (Purves et al., 2013; Bartlett et al., 2016; Sakschewski et al., 2016), emphasises the need for a more conceptual understanding of biosphere integrity, its vulnerability to anthropogenic perturbation, and its role for Earth system resilience.

Author contributions. SJL, JMA, SEC, JFD, IF, KR, JR and WS designed the research. SJL, JFD, IF, TG and CB constructed the model.

5 SJL analysed the model. All authors wrote the paper.

Competing interests. The authors declare that they have no conflict of interest.

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