

Dear ESD Editors:

Attached is the revised version of our manuscript entitled “Linking Resilience and Robustness and Uncovering their Trade-offs in Coupled Infrastructure Systems” in which we have incorporated and addressed the second round of comments and concerns raised by the two reviewers. We wish to thank the editor and the two referees for the constructive comments which improve the manuscript and provide useful ideas for future work.

Our point-by-point responses to other comments are listed below. With these revisions, we believe the manuscript is much improved and now ready for publication. We again thank the editor and reviewers for their constructive and useful comments. Please do not hesitate to let me know if you have questions.

Sincerely,

*Mehran Homayounfar*

On behalf of all authors

## Response to Referee #2

1- Page 1, line number 26: I suggest moving the (web page-based) reference to the bibliography at the end of the main text.

**Response 2-1.** Thank you for this suggestion. We have moved the reference to the bibliography at the end of the main text as suggested (P1:L26 and P11:L40).

2- Consider a scenario where the distribution of  $R_{\text{system}}$  is highly skewed towards the right. This would result in a high value of the mean, giving the impression of a high value of the resilience than it practically should. Also, a larger part of the distribution would be available for computing  $\mu_{R < \mu}$  which could subsequently yield a misleading value of the robustness. A related problem could arise when the distribution of  $R_{\text{system}}$  is highly skewed to the left. The measures proposed in the revised manuscript and the analysis revolving around them seem to be all affected by the aforementioned issue. Please clarify your stand on this problem and if necessary, add details in the next iteration of the manuscript for dealing with it.

**Response 2-2.** Thank you for this comment. The Reviewer raised a good point—one that applies generally to finding good measures of central tendency and risk for a random variable with a highly skewed distribution, not just our metric of resilience. This issue would be more critical if we were to use or interpret the *absolute* values of  $R_{\text{system}}$  directly. However, in this study, we use  $\mu_{R_{\text{system}}}$  and  $\mu_{R < \mu}$  to systematically compare across systems with different policies.

An example would help here. Consider a system with policy  $\{C_1, y_1\}$ , which yields, under 10,000 settings, 10,000 values of  $R_{\text{system}}$  that are more or less normally distributed. Consider another system with policy  $\{C_2, y_2\}$ . Suppose now that for 5,000 settings, this second system yields the same lower-half values of  $R_{\text{system}}$  as those of the first system. However, for the remaining 5,000 settings, the second system yields  $R_{\text{system}}$  values that are three times those of the first system. The distribution of the second system's  $R_{\text{system}}$  is thus skewed to the right, leading to higher  $\mu_{R_{\text{system}}}$  and  $\mu_{R < \mu}$ . We would then say that the second system is superior to the first both in terms of resilience and robustness, and it would be a logical conclusion, as under no settings is the second system less resilient than the first. (On the other hand, if the variance is used, one would say the second system is superior to the first in one dimension, i.e., that of robustness, which is somewhat illogical since, again, under no settings is the second system less resilient than the first.)

In sum, we believe that for their usage in our study—to compare and highlight trade-offs among systems with different policies— $\mu_{R_{\text{system}}}$  and  $\mu_{R < \mu}$  are useful and appropriate metrics.

3- Page 7, line number 10: What do the two local maxima of  $\mu_{R < \mu}$  actually reflect, i.e., are there greater implications of these maxima in the context of the stylized dynamical model considered in the manuscript?

**Response 2-3.** They simply reflect the nonlinear interplay between the model parameters and model structure and may affect the nature of the trade-off between robustness and resilience reported in Figures 4 and 5. We have added a remark on this issue on P7:L17.

- 4- Page 7, line number 33: The authors mention - ‘In particular, we use the standard deviation of the quantitatively defined resilience metric as the metric of robustness (low standard deviation means high robustness).’ I suspect that the authors have forgotten to revise this statement from the previous version of the manuscript. If yes, please appropriately revise it.

**Response 2-4.** Thank you for catching this typo! We have revised the manuscript accordingly (P8:L8).

### **Response to Referee #3**

Review of the revised version of “Linking Resilience and Robustness and Uncovering their Trade-offs in Coupled Infrastructure Systems” (formerly entitled “How robust is your system resilience?”). In general, the manuscript is rather clear and concise, and the work is relevant and interesting enough to merit publication. However, I do have some reservations that I would like to point out before acceptance.

First, although I wholeheartedly agree that we should put more effort in specifying and quantifying resilience and related properties, the study focuses on engineering resilience only. This is a result of the choice to use a dynamical system model in which the behavior of agents is included only in a very indirect and inflexible way. The assumption in this CIS model is that agents operate with perfect knowledge, foresight, and decision-making regarding investments in and yields from infrastructure (replicator dynamics). I have no problem with that, as long as it is recognized as such. I think there is a third type of resilience that is not recognized as such in this manuscript, and that is resilience resulting from adaptation. Agents in a CIS may have the capability of changing their behaviour in response to policy changes, environmental shocks, technical change, etc., which may also change the structure (and not only parameter values) of the CIS. It remains to be seen in how far the used methodology is applicable when agent behaviour is included explicitly, in particular when agents change their behaviour and decision-making after policy changes. That said, I do not feel the authors have to cover all aspects in one paper, but in my view paragraph 16-21 of the discussion on page 8 should be rewritten as it currently oversteps the application range of the resilience metric used in this manuscript. Exploring policy effects is very relevant, but should include a model construct that allows for agent behavioural change or at least an explicit description of the current assumptions on agent behaviour regarding policy.

**Response 3-1.** Thank you for the comment. Regarding the definition of resilience, the current study focuses in fact on “ecological resilience,” as indicated in the text P3, L31-33: “*Here, it is worth noting that, while it is possible to examine recovery-based resilience (or the so-called “engineering resilience”), this paper focuses on regime shift-based resilience (traditionally called “ecological resilience”) and its robustness.*” In any event, the Reviewer pointed out important issues regarding adaptation, which is related to resilience. We have added an additional paragraph below to discuss this issue in the ‘discussion and conclusions’ section (starting on P8:L30).

“Additionally, agents in a CIS may have the capacity to change their behaviour in response to changes in policy, environmental conditions, technological changes, and the like. In this study, strategic behaviour and decision-making process are assumed unchanged in the analysis. Adaptation in strategic behaviour of agents will subsequently alter the nature of resilience, its robustness, and their trade-offs. Capturing such effects of adaptation requires

structural changes to the model, e.g., in terms of specification of payoffs or even the formulation of the dynamical equations. With such adaptive social agents, how should one devise adaptive governance to enhance resilience and robustness of a CIS? Addressing such a question is a theoretically intriguing future research direction with great practical implications.”

Second, I have some questions about the model and the model analysis. Is there an overview of the model variables, parameters, and their nominal values, ranges, dimensions, units and meaning? This would help in getting a direct overview of the model, and I would welcome such an addition to the appendix on the model description.

**Response 3-2.** We provide brief description of the model in the Appendix. A more detailed description is available in an earlier paper (Muneepeerakul & Anderies, Earth’s Future 2017); we did not want—nor did we think the Editor would think it is appropriate—to repeat too many details from another already published paper.

Of somewhat bigger concern is the model analysis. The authors use Routh-Hurwitz criteria to determine the resilience regarding stability. The model however includes nonlinear terms, in particular an Allee model-like term  $rU(1 - U)(\pi_U - w)$ , and seems to resemble food web models from the field of theoretical ecology. Are there no bifurcations in the model other than tangent bifurcations that mark the boundaries of resilience? I find it hard to assess, as there are no parameter values reported, and I cannot analyse the model in-depth myself. I think this is not the case here, otherwise the landscape pictures would probably look differently, but I would think this is something that could be clarified given the potential relevance for the resilience assessment of the model.

**Response 3-3.** The Reviewer is right in that there may exist other interesting bifurcations in the model. However, here we focus on those related to the non-trivial sustainable long-term outcome. Analyzing other bifurcations related to other equilibria may be mathematically interesting, but it could make the manuscript less accessible and dilute the key message about the resilience-robustness trade-off of different policies aiming at keeping the system in the basin of attraction of the non-trivial sustainable long-term outcome. We have added a remark on this issue (P4:L32).

The parameter values are the same as those used in the previous work (Muneepeerakul & Anderies, Earth’s Future 2017) and now included in the revised manuscript (P13:L18). Thank you for pointing this out to us.

Minor comments:

Page 2, end of the top paragraph does not read very well. It refers to the desirability of resilience, but resilience has not been introduced yet. Maybe switch this piece of text with the first lines of the following paragraph in which resilience is explained?

**Response 3-4.** The ending part of the paragraph is a way to set up the research question of the study. It aims at invoking the reader’s curiosity about the potential trade-off between resilience and robustness—regardless of what the reader’s own definitions of these terms might be. Our strategy is to get the reader interested first and then follow by a more concrete treatment of these terms. We feel that this flow of narrative works well and would like to keep it.

Page 6, between lines 10 and 15, it could be explained already there to what values is referred to with “those values associated with ‘bad deviations’”. It becomes clear later on in the third paragraph of the page, but it does not read very well.

**Response 3-5.** Here, we agree with the Reviewer that referring to “bad deviations” here may cause unnecessary confusion and not read well. We have removed the phrase in the revised manuscript (P6:L13).

Figures 4 and 5: I think you could add a line explaining the black dots.

**Response 3-6.** Thank you for this comment. The figures’ captions have been updated accordingly (P7:L29-32 and P15 and 16).

Figure 5 caption typo:  $\mu_R$ -syste, without the m.

**Response 3-7.** We in fact could not find the typo.

Why is there variation in the description of UNRH(I\_HM)? Eq. (A3): NRUH; Eq. (A4): NURH; Fig. A1: UNRH.

**Response 3-8.** Thank you for this comment. The equations have been updated accordingly.

What is the meaning of the slashed O in Eq. (A6)?

**Response 3-9.** This was the Greek letter phi ( $\phi$ ), whose definition has been now included (P13-L18). Thank you for catching this oversight.

# Linking Resilience and Robustness and Uncovering their Trade-offs in Coupled Infrastructure Systems

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**Abstract.** Robustness and resilience are concepts in systems thinking that have grown in importance and popularity. For many complex social-ecological systems, however, robustness and resilience are difficult to quantify and the connections and trade-offs between them difficult to study. Most studies have either focused on qualitative approaches to discuss their connections or considered only one of them under particular classes of disturbances. In this study, we present an analytical framework to address the linkage between robustness and resilience more systematically. Our analysis is based on a stylized dynamical model that operationalizes a widely used conceptual framework for social-ecological systems. The model enables us to rigorously delineate the boundaries of conditions under which the coupled system can be sustained in a long run, define robustness and resilience related to these boundaries, and consequently investigate their connections. The results reveal the trade-offs between robustness and resilience. They also show how the nature of such trade-offs varies with the choices of certain policies (e.g., taxation and investment in public infrastructure), internal stresses and uncertainty in social-ecological settings.

## 1. Introduction

The concepts of “resilience” and “robustness” have grown considerably in popularity as desirable properties for a wide range of systems. Terms like “resilient communities” and “robust cities” have been used more frequently in public discourse (e.g., Chang and Shinozuka, 2004; Longstaff et al., 2010; Chang et al., 2014). The UK’s Water Act 2014 even included “primary duty to secure resilience” as one of the general duties of its Water Services Regulation Authority ([Water Act, 2014](#)). Growing with that popularity is some confusion and potential misuse of the terms “robustness” and “resilience” due to imprecision, vagueness, and multiplicity of their definitions. Such lack of consistency and rigor hinders advances in our understanding of the interplay between these two important system properties.

Relatively speaking, robustness has been defined more consistently and rigorously—as it can be linked to a more familiar concept of sensitivity. For example, according to Carlson and Doyle (2002), robustness in engineering systems refers to the maintenance of system performance either when subjected to external disturbances or internal uncertain parameters. In other words, in robust systems, performance is less sensitive to disturbances or uncertainty.

Robustness may very well be a desirable property of a system, but it seems to come with a price. Recent research shows that tuning a system to be robust against certain disturbance regimes almost always reduces system performance and likely increases its vulnerability to other disturbance regimes (Ostrom et al., 2007; Anderies et al., 2007; Bode, 1945; Csete and Doyle, 2002; Wolpert and Macready, 1997). Now, if resilience is also a desirable property of the same system, does it also come at the expense of performance and robustness? Put it another way, is there a trade-off among performance, robustness, and resilience? Such a trade-off, if exists, is a crucial consideration for governing and/or managing social-ecological systems (SESs).

But resilience, as alluded to above, is trickier to define. According to Holling (1973), resilience refers to the amount of change or disruption required to shift the maintenance of a system along different sets of mutually reinforcing processes and structures. In other words, resilience can be thought of as how far the system is from certain thresholds or boundaries beyond which the system will undergo a regime shift or a quantitative change in system structure or identity. Holling (1996) categorized resilience into two types, engineering resilience, which refers to the ability of a system to return to steady state following a perturbation, and ecological resilience, which refers to the capacity of system to remain in a particular stability domain in the face of perturbations. The latter category is used by many researchers to discuss resilience of SESs, or more generally, *coupled infrastructure systems* (CISs) (Carpenter et al., 2001; Folke, S et al., 2002; Anderies et al., 2006; Folke, 2006; Folke et al., 2010; Biggs et al., 2012; Barrett and Constanas, 2014; Redman, 2014; Walker et al., 2002; Gunderson et al., 1995; Berkes and Folke, 1998; Carpenter et al., 1999a, 1999b; Scheffer et al., 2000; Berkes et al., 2003; Walker et al., 2004; Carpenter and Brock, 2004; Janssen et al., 2004; Folke et al., 2002; Anderies et al., 2006; Folke et al., 2016; Cote and Nightingale, 2012; Mitra et al., 2015; Cumming and Peterson, 2017). The term coupled infrastructure systems (CISs) has been introduced to generalize the notions of coupled natural-human systems (CNHSs) and social-ecological systems (SESs); in this context, *infrastructure* is broadly defined to include human-made, social, and natural infrastructure (see, e.g., Anderies et al., 2016). The problem is that these CISs are complex and thus identifying thresholds and potential regime shifts associated with those thresholds is often difficult, if not impossible. In many cases, major aspects of resilience in CISs may not be directly observable and must be actualized indirectly via surrogate attributes (Carpenter et al., 2005; Kerner and Thomas, 2014). Recent significant advances have been made toward identifying early-warning signals that indicate whether a critical threshold is being approached for a wide class of systems (Scheffer et al., 2009 and 2012). Still, there are gaps in our understanding of how indicators of resilience and robustness will behave in more complex situations. This lack of a rigorous metric for resilience makes the investigation into their connections, interplay, and trade-offs with robustness and performance impossible.

But these knowledge gaps need to be filled if one wishes to make advances in understanding the interplay between social dynamics and planetary boundaries. Given the magnitude of impacts that human activities have on pushing Earth systems toward their planetary boundaries, we need clearer understanding of how social and biophysical factors come together to define the nature of these boundaries. This paper is a step in that direction. In particular, we will build on recent work that mathematically operationalizes the Robustness of SES framework (Anderies et al., 2004) into a formal stylized dynamical model (Muneepeerakul and Anderies, 2017). We will exploit the relative simplicity

of the model to rigorously define robustness and resilience of the coupled system. The modelled system will be subject to fluctuations in external drivers, which will affect the well-defined robustness and resilience, thereby enabling us to investigate the interplay and trade-offs between these important properties, as well as how the nature of the interplay and trade-offs are affected by policies implemented by social agents.

## 5    **2. Methods**

Here we analyse a mathematical model developed by Muneeppeerakul and Anderies (2017) by subjecting the coupled system to uncertainty in ecological and social factors. The model captures the essential features of a system in which a group of agents shares infrastructure to produce valued flows. Such a system is the archetype of most, if not all of human sociality: groups produce infrastructure that they cannot produce individually (security, defence, irrigation  
10    canals, roads, markets, financial systems, coordination mechanisms, etc.) that significantly increases productivity. The challenge is maintaining this shared infrastructure (e.g. decaying infrastructure is a major problem in the US at the time of writing (ASCER CIA Advisory Council, 2013). The model allows for mathematical definitions of the boundaries of policy domain that result in a sustainable system in which both human-made and natural infrastructure can be maintained over the long run. Based on these boundaries and uncertainty in the exogenous factors, we define  
15    metrics of resilience and robustness associated with each policy choice and investigate the trade-off between them. The basic model presented by Muneeppeerakul and Anderies (2017) is described in the Appendix.

Here a policy is defined as a combination of taxation level  $C$  and the proportion of tax revenue invested in infrastructure maintenance  $y$  that the public infrastructure providers (PIPs) decide to implement in the system. The infrastructure (e.g. canals) enable resource users (RUs) to produce valued goods from a natural resource. The two  
20    fluctuating exogenous factors are the replenishment rate of the natural resource  $g$  and the wage  $w$  that resource users (RUs) would earn from working outside the system—a combination of  $g$  and  $w$  defines a “social-ecological setting” or simply “setting.” There are two boundaries that, once crossed, will cause the system will collapse. The first boundary is called PIP participation constraint (PPC): when the PIPs must invest too much in maintaining the public infrastructure (exceeding the opportunity cost of  $w_p$ ) and/or cannot retain enough revenue for themselves, they will  
25    abandon the system for another. The second boundary is the stability condition of the non-trivial equilibrium point (i.e., the “society” in which both PIPs (e.g. the state) and RUs (e.g. citizens) participate in the system and public infrastructure is sufficiently maintained in a long run). Together, these two boundaries delineate a set of policies ( $C$ - $y$  combinations) that correspond to sustainable outcomes. The resilience metric to be developed below can be thought of as a metric of how far the system is from these boundaries. As the two exogenous factors defining settings, namely  
30     $g$  and  $w$ , fluctuate, the two boundaries and thus the resilience metric, too, fluctuate with them. How sensitive the resilience metric is to these fluctuating settings is used to define robustness. Here, it is worth noting that, while it is possible to examine recovery-based resilience (or the so-called “engineering resilience”), this paper focuses on regime shift-based resilience (traditionally called “ecological resilience”) and its robustness. Quantification of and trade-offs between resilience and robustness is a novel concept that requires expositional clarity. Presenting several metrics of  
35    resilience, let alone studying their trade-offs with potentially different metrics of robustness, may confuse the matter

and dilute the key messages we attempt to convey. As such, in what follows, we will focus on developing a metric for regime shift-based resilience. With the scope of analysis clarified and suitably bounded, we will now define resilience and robustness more formally.

## 2.1. Resilience metric

Direct measurement of above-mentioned resilience, as a specified form of resilience (Walker et al., 2004), in SES's is difficult because boundaries and thresholds that separate domains of dynamics for SES's are difficult to identify (Carpenter et al., 2005; Scheffer et al., 2009 and 2012). In this stylized model, however, such boundaries can be clearly identified by the stability condition (SC) and the PPC. Here, we are interested in the resilience of system's ability to provide sufficient livelihoods for the PIPs and resource users. The basin of attraction for system resilience is defined by those system states (i.e. infrastructure state) in which this is possible, and these system states are directly mapped to the SC and PPC. We will thus define resilience metrics based on the SC and PPC boundaries. Here our goal is to develop resilience metrics that can be meaningfully compared to one another. As such, we identify some desired properties that guide the definitions of these resilience metrics. First, they should be zero at their respective boundaries. Second, positive values indicate greater resilience of the system in a desirable state. These first two properties align with how resilience has been measured, i.e., the distance from the boundary of a basin of attraction (e.g., Anderies et al., 2002; S. R. Carpenter et al., 1999). Third, to facilitate the consideration of relative risks associated with different types of regime shifts that the system may be facing, the metrics should be comparable in magnitude. These properties guide us toward the following definitions of the resilience metrics.

We define the resilience of the system against abandonment by PIPs as follows:

$$R_{PPC} = (\pi_{PIP}/w_P) - 1, (1)$$

where  $\pi_P$  is the net revenue that PIPs collect and  $w_P$  is the opportunity cost that they will earn if they choose to work with another system. Positive values of  $R_{PPC}$  indicate that the system is resilient against being abandoned by PIPs, while negative values indicate that the system will eventually collapse due to the PIPs' abandonment. It is important to note that  $\pi_P$  results from the coupled dynamics of the CIS; this means that  $R_{PPC}$  has already integrated the dynamics of infrastructure, resource, and resource users (Eqs. A1, A4 and A5), making it a metric of the system, not of an individual component.

Numerical analysis of the model indicates that the equilibrium becomes unstable when the following Routh-Hurwitz condition (e.g., May, 2001; Kot, 2001) is violated:

$$D - T(J_{1,1}J_{2,2} + J_{2,1}J_{1,2} + J_{2,3}J_{3,2} + J_{1,3}J_{3,1}) > 0, (2)$$

where,  $D$ ,  $T$ , and  $J$ 's are determinant, trace, and entries, respectively, of the Jacobian matrix of the dynamical system (Eqs. A1, A4, and A5) evaluated at the nontrivial equilibrium point (Eq. A6)—when such an equilibrium point exists. Here, it is worth noting that we focus on the equilibrium point related to the non-trivial sustainable long-term outcome. Analyzing other bifurcations related to other equilibria may be mathematically interesting, but it could make the study less accessible and dilute its key message about the resilience-robustness trade-off of different policies aiming at keeping the system in the basin of attraction of the non-trivial sustainable long-term outcome.

Following the guideline provided by the three desirable properties above, we rearrange terms in Eq. (2) and define the resilience of the system against instability (increased probability of collapse of infrastructure) as follows:

$$R_{stability} = \frac{D}{|T(J_{1,1}J_{2,2} + J_{2,1}J_{1,2} + J_{2,3}J_{3,2} + J_{1,3}J_{3,1})|} - 1, (3)$$

This formulation is parallel to that of the first resilience metric (Eq. 1); it possesses the three properties:  $R_{stability}$  of zero indicates the boundary between stability and instability; positive  $R_{stability}$  means the system at the equilibrium point is stable; and the magnitudes of  $R_{stability}$  are comparable to those of  $R_{PPC}$  (see Fig. 1). Note that  $R_{stability}$ , too, is determined from the coupled dynamics of the CIS; this means that it has integrated the dynamics of infrastructure, resource, and resource users (Eqs. A1, A4 and A5).

This allows us to meaningfully define the overall system resilience as the minimum between the two resilience metrics, namely:

$$R_{system} = \begin{cases} \text{Min}[R_{PPC}, R_{stability}], & R_{PPC}, R_{stability} \geq 0 \\ 0, & \text{otherwise} \end{cases}, (4)$$

Equation (4) implies that  $R_{system}$  is positive only when the nontrivial equilibrium point (Eq. A6) exists and is stable; otherwise, the system is considered not resilient and denoted by  $R_{system} = 0$ .  $R_{system}$  thus represents the tension between the PIPs being too greedy (high  $C$ , low  $y$ ) whereby they get close to the stability boundary and “not greedy enough,” i.e., low  $C$  and high  $y$  whereby they get close to the PPC, given a particular choice for  $w_P$ . Note that the values of  $w$  and  $w_P$  represent the socio-economic embedding of the CIS. Therefore, the biophysical structure of the CIS along with the socio-economic context in which it is embedded co-determine the maximum resilience that can be achieved. Given that the nontrivial equilibrium point exists and is stable, if the system is at a greater risk of being abandoned by the PIPs (and eventually collapsing),  $R_{system} = R_{PPC}$ ; if the system is at a greater risk of becoming unstable (and eventually collapsing),  $R_{system} = R_{stability}$ . Figure 1 illustrates the relationships between  $R_{PPC}$ ,  $R_{stability}$ , and  $R_{system}$ .

Figure 1: Resilience metrics for a specific setting (a  $g$ - $w$  combination) inside the sustainable region in the policy space (i.e.,  $C$ - $y$  plane): (a)  $R_{PPC}$  contours; (b)  $R_{stability}$  contours; and (c)  $R_{system}$  contours. The black star in panels (a), (b), and (c) indicate the policy with the highest  $R_{PPC}$ ,  $R_{stability}$ , and  $R_{system}$ , respectively.

## 2.2. Linking Robustness and resilience

As discussed earlier, robustness can be thought of as the opposite of sensitivity. A commonly used measure of sensitivity is variance. Thus, variance of a given function under specific disturbance or uncertainty regimes may be used to indicate robustness of that function against those disturbance or uncertainty regimes (robustness of what to what). In this case, the system function of interest is the system resilience  $R_{system}$ . By choosing  $R_{system}$  as our function, we can usefully link these concepts. If we use a ball and cup metaphor for resilience, the robustness of

resilience refers to the degree at which the geometry of the cup changes as a result of external disturbances and/or parameter changes. However, as we will argue below, relating high variance of  $R_{system}$  to low robustness may be misleading and should not be used in evaluating a given policy. By definition, the variance treats “good deviations” and “bad deviations” from the mean equally. For functions with preferred values, such as resilience or profit, values greater than the mean and those lower should not be treated in the same way. Specifically, contribution to a high variance from a heavy tail in the good direction should not be translated to less robustness. This problem also arises in assessing financial risk: what makes an asset risky is the values on the “bad tail” of the distribution (i.e., low or negative profits). This has motivated more and more analyses to switch to considering other measures of risk, such as the conditional value at risk, in evaluating their portfolios of investment (Rockafellar and Uryasev, 1999; Krokhamal et al., 2001; Sarykalin et al., 2008; Zymler et al., 2013). Intuitively, this means that the shape of the “cup” can be asymmetric and we need to take this into account.

Following this logic, we propose to use a “below-mean mean” as a new robustness metric: the mean of all resilience values lower than the mean, ~~i.e., those values associated with the “bad deviations.”~~ This new definition of the robustness metric has several desirable features. First, it can now be appropriately thought of as a robustness metric in the sense that the higher the value, the more robust the system (unlike the variance for which low variance means high robustness). Second, by using the mean as the threshold value for bad deviations, we remove some arbitrariness associated with prescribing a certain quantile (e.g., 5<sup>th</sup> or 10<sup>th</sup> quantile) in calculating the conditional value at risk. Third, it still carries some information about the sensitivity of the resilience metric to outside factors—the information that variance conveys; that is, the higher the “below-mean mean” (i.e., the bad deviations from the mean are small and the below-mean mean is close to the mean), the less sensitive—and thus more robust—the resilience metric.

In this study, we subject the modelled system to uncertainty in one natural factor and one social factor, namely, the natural replenishment rate of the resource  $g$ , and the payoff that a RU earns from working outside the system  $w$ . Thus, we are computing how the resilience of the system to shocks/variation in state variables changes as the parameters  $g$  and  $w$  change (i.e., we are uncertain about the underlying social-ecological setting of the system). In particular, we assume that  $g$  is uniformly distributed over the range [75, 125] and  $w$  is uniformly distributed over the range [0.75, 1.75]. A social-ecological setting, or setting, is defined as a combination of  $g$  and  $w$ . For a given policy (a  $C - y$  combination), we calculate  $R_{system}$  for 10,000 settings (i.e., 10,000  $g - w$  combinations) (see Fig. 2). Then, from these 10,000 values of the resilience metric  $R_{system}$ , we calculate the mean,  $\mu_{R-system} = E[R_{system}]$ , and use it as the **resilience metric** of the coupled system with a given policy, and the below-mean mean,  $\mu_{R < \mu} = E[R_{system} \vee R_{system} < \mu_{R-system}]$ , as the **metric for robustness of resilience**. This metric measures the robustness of the capacity of the system to cope with variation in state variables  $I_{HM}$ ,  $R$ , and  $U$  to fundamental uncertainty about the underlying setting of the system.

Figure 2: Variation of  $R_{system}$  of a CIS with a fixed policy  $(C, y)$  over 10,000 settings associated with uncertainty characterized by  $\{g \in [75, 125], w \in [0.75, 1.25]\}$ : (a)  $R_{system}$  surface and (b)  $R_{system}$  contours. The values of  $R_{system}$  are used to calculate the mean,  $\mu_{R_{system}}$ , and the below-mean mean,  $\mu_{R < \mu}$ . In this particular case, the resilience does not change much when  $g$  is greater than about 100, but becomes more sensitive to both  $g$  and  $w$  when  $g$  is lower than 100.

### 3. Results

The surfaces and contours of the system resilience metric,  $\mu_{R_{system}}$ , and associated with different policies  $(C - y)$  over the policy space are shown in Figs. (3a and b), respectively. The policies with sustainable outcomes are located in the middle of the policy space, with  $\mu_{R_{system}}$  peaking in the center and declining as policies become more extreme in either direction. Our analysis also shows that  $\mu_{R_{system}}$  is more or less proportional to the fraction of settings ( $g$ - $w$  combinations) under which the system with that particular policy (a  $C$ - $y$  combination) results in a sustainable outcome ( $R_{system} > 0$ ). A similar concept has been used in the robust decision making literature (e.g., Groves and Lempert, 2007; Bryant and Lempert, 2009).

The surfaces and contours of the robustness of,  $\mu_{R < \mu}$ , associated with different policies over the policy space are shown in Figs. (3c and d), respectively. The  $\mu_{R < \mu}$  “landscape” is more irregular, having two local maxima with one being more dominant than the other. The region with high robustness appears to be in the same general areas as the region with high resilience. These features reflect the nonlinear interplay between the model parameters and model structure and may affect the nature of the trade-off between robustness and resilience reported in Figures 4 and 5.

Figure 3: the mean,  $\mu_{R_{system}}$ , and the below-mean mean of  $R_{system}$ ,  $\mu_{R < \mu}$ , over entire decision space: (a) Surface of the resilience metric,  $\mu_{R_{system}}$ ; (b) Contours of  $\mu_{R_{system}}$ ; (c) Surface of the robustness, the below-mean mean ( $\mu_{R < \mu}$ ); (d) Contours of the robustness, the below-mean mean ( $\mu_{R < \mu}$ )

We explore the interplay between  $\mu_{R_{system}}$  and  $\mu_{R < \mu}$  in Fig. 4. Figure 4 shows that there are no perfect policies in the sense that no policies yield both maximum resilience and maximum robustness. Recall that the robustness indicates how sensitive  $R_{system}$  itself is to uncertainty in the underlying setting of the system (e.g.,  $g$  and  $w$ ). The best policies are those along the Pareto frontier in the resilience-robustness space: among this set of Pareto-optimal policies, an increase in resilience is necessarily accompanied by a decrease in robustness, clearly illustrating the trade-off between robustness and resilience. Fig. 5 illustrates where the Pareto-optimal policies are located in the policy space.

Figure 4: Resilience-robustness trade-off. Each point represents,  $\mu_{R_{system}}$  and  $\mu_{R < \mu}$  of the coupled system with a given policy. The trade-off is only apparent at the Pareto-optimal frontier (the black dots represent a set of Pareto-optimal policies).

Figure 5: Pareto optimal policies, represented by (the black dots,) with high—in the policy space  $(C$ - $y$  plane), superimposed with resilience ( $\mu_{R_{system}}$ ) contours (a) and robustness ( $\mu_{R < \mu}$ ) contours (b).

#### 4. Discussion and conclusions

In this paper, we exploit the simplicity of a stylized model to quantitatively link resilience and robustness by computing how the CIS's resilience to shocks in state variables changes with parameters. In this way, we compute the robustness of CIS resilience to uncertainty in the underlying CIS setting. The resilience metric developed here is a measure of how far the CIS is from the boundaries beyond which it will collapse. The model affords us with expressions of these boundaries, which clearly show how social and biophysical factors interplay to define these boundaries. With a concrete definition of resilience, resilience itself can be considered as the “of what” in the “robustness of what to what” notion. In particular, we use the ~~below-mean mean standard deviation~~ of the quantitatively defined resilience metric as the metric of robustness (~~low standard deviation means high robustness~~). Consequently, this enables us to rigorously investigate the interplay between the two important, but not always well-defined, system properties. A key finding is the fundamental trade-off between resilience and robustness: there are no perfect policies in governing a CIS, only Pareto-optimal ones. Specifically, policies designed to maximize the resilience of a CIS to shocks on timescales at which the state variables play out may be very sensitive to being wrong about our understanding of the underlying dynamics of the CIS in question.

Importantly, we hope this work will stimulate further advances in rigorous studies of CISs that address such subtle, policy-relevant questions, a few of which we briefly discuss here. More dimensions can be considered in defining Pareto-optimality. Figure 5 may give an impression that the set of Pareto-optimal policies is confined to a small region in the policy space, which would imply that PIPs do not have that many choices—even in a simple CIS like the one studied. But that would be a wrong impression. In addition to resilience and robustness (as defined here), a policy maker or a social planner may be interested in other types of robustness with different “of what” and “to what” components. She may also be concerned about other system properties, e.g., productivity, user participation, etc. As more dimensions are considered, the set of Pareto-optimal policies grow. In the same spirit as that of the work done here, these other dimensions should be defined rigorously.

This work also lends itself to more rigorous studies of “adaptive governance.” In the present study, the governance structure, represented by a policy (a combination of  $C$  and  $y$ ), is fixed. A natural next step is to explore if a policy is allowed to change, how one may improve the resilience and robustness of a CIS and/or alter the nature of their inherent trade-offs. For example, if  $C$  and  $y$  are to be functions of other factors, e.g., resource availability and outside incentives, what functional forms should they take to improve the system's resilience and robustness? Indeed, in the absence of transparent metrics, attempts to explore such adaptive policies are severely limited.

Additionally, agents in a CIS may have the capacity to change their behaviour in response to changes in policy, environmental conditions, technological changes, and the like. In this study, strategic behaviour and decision-making process are assumed unchanged in the analysis. Adaptation in strategic behaviour of agents will subsequently alter the nature of resilience, its robustness, and their trade-offs. Capturing such effects of adaptation requires structural changes to the model, e.g., in terms of specification of payoffs or even the formulation of the dynamical equations.

With such adaptive social agents, how should one devise adaptive governance to enhance resilience and robustness of a CIS? Addressing such a question is a theoretically intriguing future research direction with great practical implications.

In keeping with the theme of “social dynamics and planetary boundaries in Earth system modelling,” our results shed light on how social and biophysical factors may interplay to define “boundaries” of a sustainable coupled infrastructure system. While the modelled system here is admittedly simple, our methodology and results constitute a step toward quantitatively and meaningfully combining social and biophysical factors into indicators of boundaries of more complex systems. Just as in this work, once those boundaries are clearly defined, calculation and discussion of resilience and robustness can become concrete.

**Author contribution:** MH, RM, JMA, and CPM designed the study. MH carried out the analysis. MH and RM prepared the manuscript with contributions from all co-authors.

**Competing interests:** The authors declare that they have no conflict of interest.

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## APPENDIX

- 5 **Basic model.** Here we briefly describe the basic model presented by Muneepeerakul and Anderies (2017). The model shows dynamic behaviour of three principal variables, namely, the state of the public infrastructure,  $I_{HM}$ , resource level,  $R$ , and the fraction of time user makes use of infrastructure,  $U$ , through Eq's (A1, A4 and A5). The schematic diagram of this system of equations is shown in Figure A1.

Figure A1: Schematic diagram of the dynamical system model. Taken from Muneepeerakul and Anderies (2017).

- 10 In this context,  $I_{HM}$  depends on PIPs in term of maintenance cost and has a positive relationship with the capacity of users to create resource flows. Eq. (A1) illustrates the dynamics of  $I_{HM}$  as follows:

$$\frac{dI_{HM}}{dt} = M(\dots) - \delta H(I_{HM}), \quad (A1)$$

- where,  $\delta$  is the infrastructure's depreciation rate and  $H(I_{HM})$  states functional relationship of public infrastructure and productivity of each resource user. According to Muneepeerakul and Anderies (2017) many shared infrastructures  
15 can be modelled by threshold functions. Given that  $H(I_{HM})$  shows threshold behavior, they used a piecewise linear function to capture such behavior through Eq. (A2).

$$H(I_{HM}) = \begin{cases} 0, & I_{HM} < I_0 \\ h \frac{I_{HM} - I_0}{I_m - I_0}, & I_0 \leq I_{HM} \leq I_m, \\ h, & I_{HM} \geq I_m \end{cases} \quad (A2)$$

- where,  $h$  represents maximum amount of harvest by each user under no restriction and  $I_0$  and  $I_m$  are lower bound and upper bound thresholds of  $I_{HM}$  respectively. Also,  $M(\dots)$  is maintenance function (Eq. A3) and depends on  
20 social structure of the system.

$$M(\dots) = \mu_2 \gamma C p \text{RUN} H(I_{HM}), \quad (A3)$$

- In Eq. (A3), given the number of users  $N$ ,  $\text{RUN} H(I_{HM})$  is the total harvest from the natural infrastructure. The resource users sell total harvest at price  $p$  to generate revenue. Subsequently, they assign a proportion  $C$  of revenue to PIP's for their contribution. Meanwhile, the PIP's spend proportion  $\gamma$  of  $C$  on maintaining public infrastructure  
25 through the maintenance function  $M(\dots)$ . Also,  $\mu_2$  is maintenance effectiveness of PIP's investment.

The second variable is resource level,  $R$ . They assumed the dynamics of resource to be:

$$\frac{dR}{dt} = G(R) - \text{RUN} H(I_{HM}) \quad (A4)$$

Natural infrastructure is assumed to invoke the conservation law comprising of regenerating capacity ( $G(R) = g - dR$ ) and total unit of harvest,  $RH(I_{HM})$ . The definition presented for  $G$  is the simplest model for natural infrastructure where  $g$  and  $d$  are the natural replenishment and the loss rates, respectively.

The strategic behavior of the resource users ( $RU$ 's) is captured by employing replicator equation. Indeed, replicator dynamics provide modeler with simple, realistic social mechanism where agents follow and replicate better-off strategies. The two possible strategies considered for  $RU$ 's are staying inside system with the associated payoff of  $\pi_U = (1 - C)pRH(I_{HM})$  or leaving system with the payoff of  $\pi_w = w$ . According to replicator equation:

$$\frac{dU}{dt} = rU(1 - U)(\pi_U - w) \quad (A5)$$

Replicator equation discuss the fraction of time that  $RU$ 's assign to working inside system given  $C$  and  $y$ . Like  $RU$ 's, there is also two alternatives for  $PIP$ 's, working inside system or working for another CIS which leads to system failure. Meanwhile,  $C$  and  $y$  characterize the strategy or policy of  $PIP$ s. The  $PIP$ s will participate in this coupled system only when  $\pi_p = (1 - y)pCRH(I_{HM}) \geq \pi_w$ . In other words, the  $PIP$ s maintain in the system when they are better-off than working outside. This condition is termed the  $PIP$  Participation Constraint (PPC).

Based on the system of three differential equations (Eqs. A1, A4 and A5), the sustainable equilibria, i.e., long-term system outcomes that satisfy the stability condition and  $PIP$  Participation Constraint (PPC), can be expressed as follows:

$$i_{HM}^* = \frac{yCU^*NR^*}{g}H(I_{HM}^*); R^* = \frac{g}{d}\left(1 - \frac{i_{HM}^*}{yC}\right); U^* = \frac{(1-C)}{yC}\phi_1 i_{HM}^*, \quad (A6)$$

where  $i_{HM}^* := \frac{I_{HM}^*\delta}{\mu_2 pg}$  ( indicates dimensionless) and  $\phi_1 = \frac{pg}{wN}$ , a dimensionless group representing the relative lucrativeness of the system, namely the ratio of potential income—with the entire resource flow turned into income—relative to outside wage. The results reported in this study are based on the following parameter values:-  $h = 0.0005$ ;  $\delta = 0.1$ ;  $I_0 = 0.3$ ;  $I_m = 3$ ;  $g = 100$ ;  $d = 0.02$ ;  $N = 1000$ ;  $r = 0.15$ ;  $p = 10$ ;  $w = 1$ ;  $w_p = 100$ ;  $\mu_2 = 0.001$ .

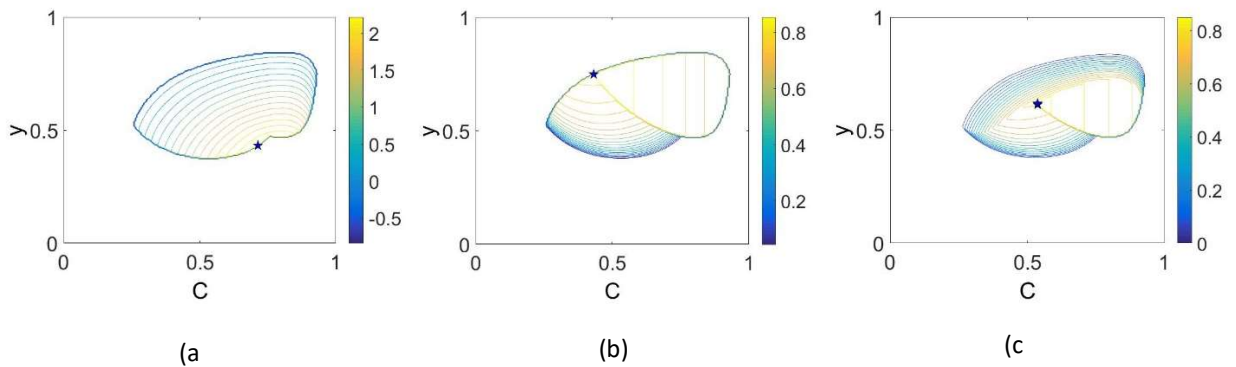


Figure 1: Resilience metrics for a specific setting (a  $g$ - $w$  combination) inside the sustainable region in the policy space (i.e.,  $C$ - $y$  plane): (a)  $R_{PPC}$  contours; (b)  $R_{Stability}$  contours; and (c)  $R_{System}$  contours. The black star in panels (a), (b), and (c) indicate the policy with the highest  $R_{PPC}$ ,  $R_{Stability}$ , and  $R_{System}$ , respectively.

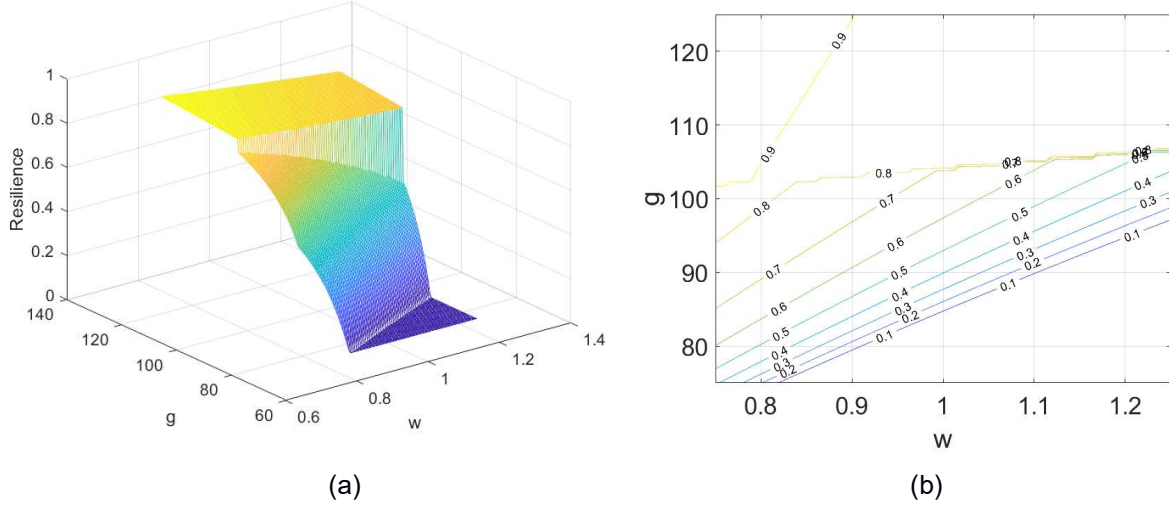
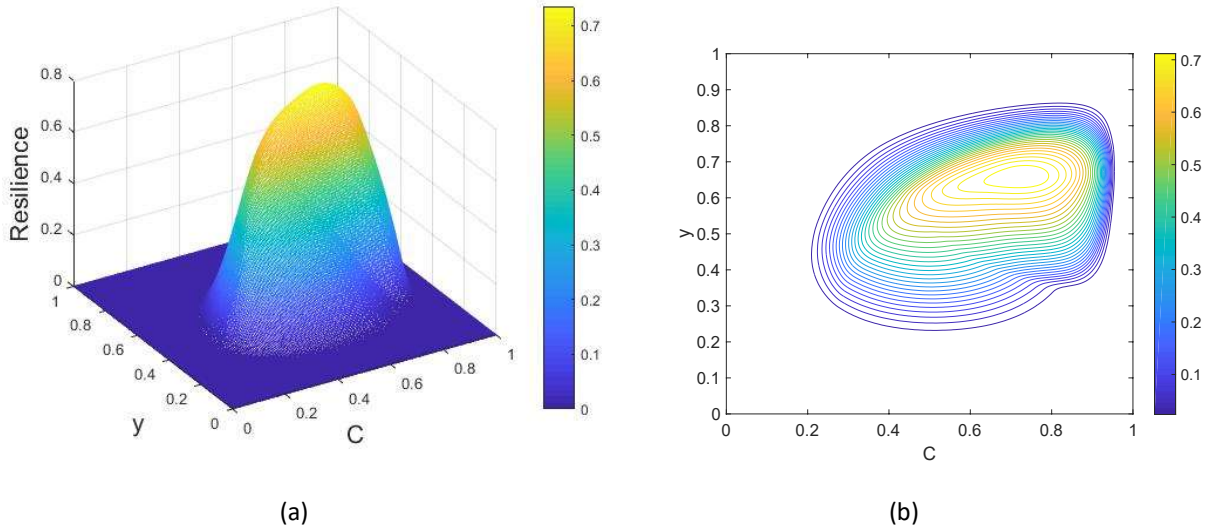


Figure 2: Variation of  $R_{system}$  of a CIS with a fixed policy ( $C, y$ ) over 10,000 settings associated with uncertainty characterized by  $\{g \in [75,125], w \in [0.75,1.25]\}$ : (a)  $R_{system}$  surface and (b)  $R_{system}$  contours. The values of  $R_{system}$  are used to calculate the mean,  $\mu_{Rsystem}$ , and the below-mean mean,  $\mu_{R<\mu}$ . In this particular case, the resilience does not change much when  $g$  is greater than about 100 but becomes more sensitive to both  $g$  and  $w$  when  $g$  is lower than 100.



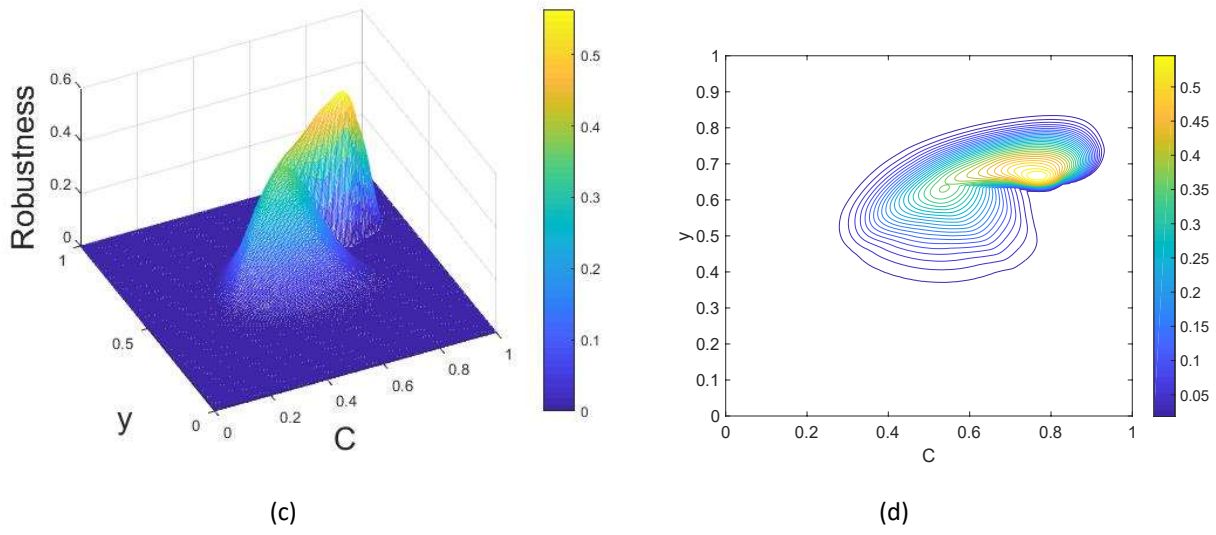


Figure 3: the mean,  $\mu_{R-system}$ , and the below-mean mean of  $R_{system}$ ,  $\mu_{R<\mu}$ , over entire decision space: (a) Surface of the resilience metric,  $\mu_{R-system}$ ; (b) Contours of  $\mu_{R-system}$ ; (c) Surface of the robustness, the below-mean mean ( $\mu_{R<\mu}$ ); (d) Contours of the robustness, the below-mean mean ( $\mu_{R<\mu}$ )

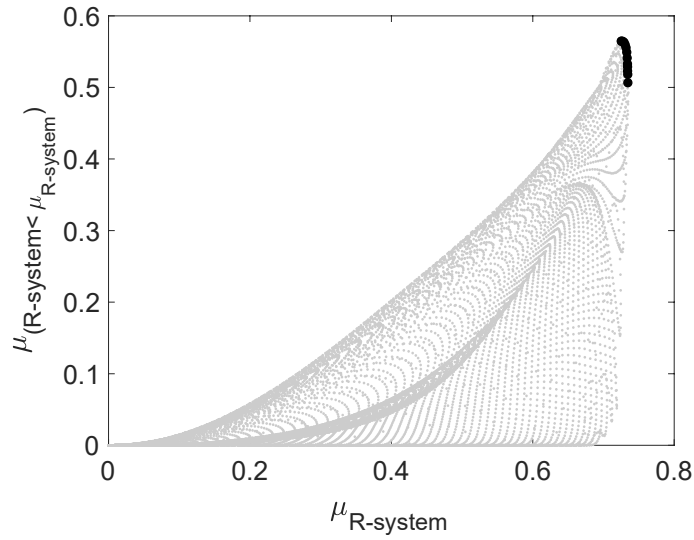


Figure 4: Resilience-robustness trade-off. Each point represents,  $\mu_{R-system}$  and  $\mu_{R<\mu}$  of the coupled system with a given policy. The trade-off is only apparent at the Pareto optimal frontier (the black dots represent a set of Pareto optimal policies).

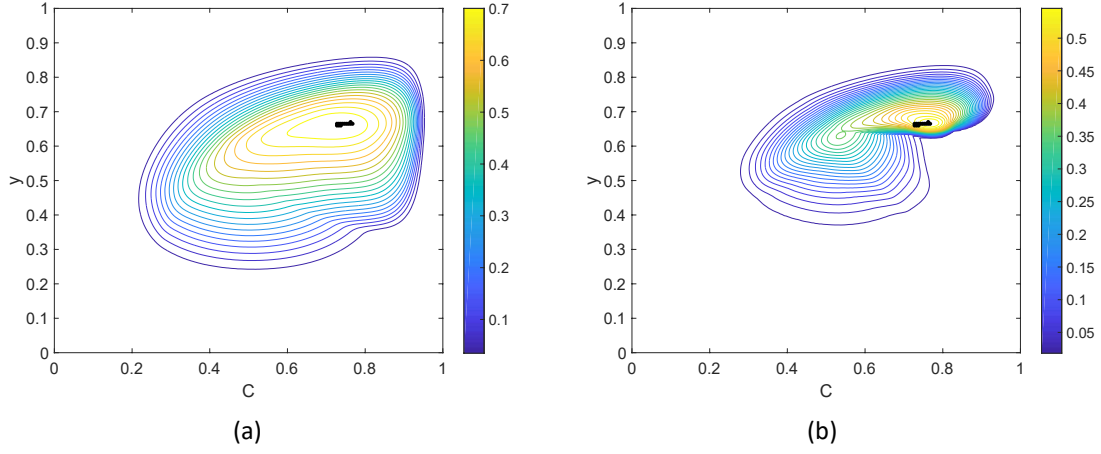


Figure 5: Pareto optimal policies, represented by (the black dots,) with high  $\mu$  in the policy space ( $C$ - $y$  plane), superimposed with resilience ( $\mu_{R-system}$ ) contours (a) and robustness ( $\mu_{R < \mu}$ ) contours (b).

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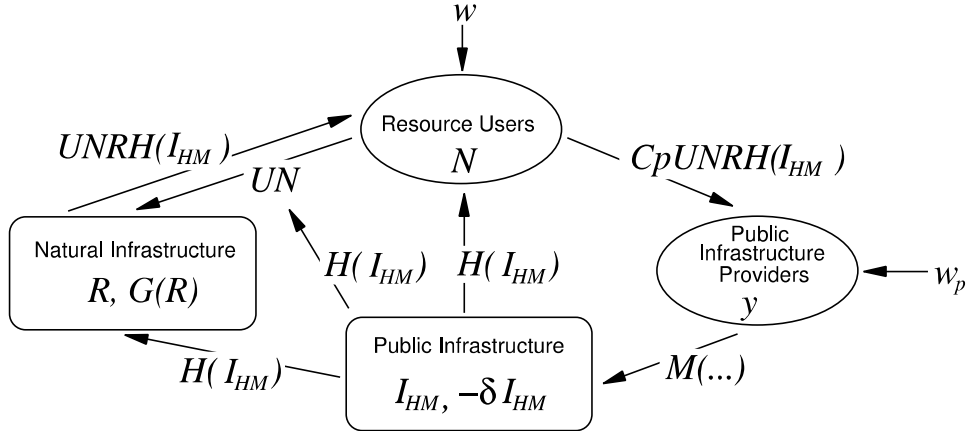


Figure A1: Schematic diagram of the dynamical system model. Taken from Muneeppeerakul and Anderies (2017).