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On determining the Point of no Return in Climate Change

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Abstract. Earth's Global Mean Surface Temperature (GMST) has increased by about 1.0°C over the period 1880-2015. One of the main causes is thought to be the increase in atmospheric greenhouse gases (GHGs). If GHG emissions are not substantially decreased, several studies indicate there will be a dangerous anthropogenic interference (DAI) with climate by the end of this century. However,

5 there is no good quantitative measure to determine when it is 'too late' to start reducing GHGs in order to avoid DAI. In this study, we develop a method for determining a so-called Point of No Return (PNR) for several GHG emission scenarios. The method is based on a combination of stochastic viability theory and uses linear response theory to estimate the probability density function of the GMST. The innovative element in this approach is the applicability to high-dimensional climate

10 models as is demonstrated by results obtained with the PLASIM climate model.

1 Introduction

In the year 2100, which is as far away (or as close) as 1932 in the past, mankind will be living on an Earth with, at least, a different climate than today. At that time, we will know the 2100-mean Global Mean Surface Temperature (GMST) value and its increase, say ΔT , above the pre-industrial GMST

15 value. From the then available GMST records, it will also be known whether this change in GMST has been gradual or whether it was rather 'bumpy'. If the observational effort will continue as of today, there will also be an adequate observational record to determine whether the probability of extreme events (e.g., flooding, heat waves) has increased.

The outcomes of these future observations, to be made by our children and/or grandchildren, will strongly depend on socio-economic and technological developments and political decisions which are made now and over the next decades. Fortunately, there is a set of rational tools available to inform decision makers: Earth System Models. These models come in different flavours, from global climate models (GCMs) aimed to provide details on the development of the ocean-atmosphereice-land system to integrated assessment models (AIMs) which also aim to describe the develop-

25 ment of the broader socio-economic system. During the preparation for the fifth assessment report

(AR5) of the Intergovernmental Panel of Climate Change (IPCC), GCM studies have focussed on the climate system response to GHG changes as derived by AIMs from different socio-economic scenarios; the data from these simulations is gathered in the so-called CMIP5 archive (http://cmip-pcmdi.llnl.gov/cmip5/).

- 30 Depending on the representation of fast climate feedbacks in GCMs, determining their climate sensitivity, the CMIP5 models project a GMST increase ΔT of 2.5-4.5°C over the period 2000-2100 (Pachauri et al., 2014). This does not mean that the actual measured value of ΔT in 2100 will be in this interval. For example, the GMST may be well outside this range because of current model errors which misrepresented the strength of a specific feedback. As a consequence, a transition occurred
- 35 in the real climate system during the period 2016-2100, which did not occur in any of the CMIP5 model simulations. Another possibility is that the GHG development eventually was far outside of the scenarios considered in CMIP5.

A crucial issue in 2100 will be whether a climate state has been reached where a dangerous anthropogenic interference (DAI) can be identified (Mann, 2009). In this case, present-day islands will have been swallowed by the ocean, extreme events have increased in frequency and magnitude

as envisioned in the Burning Embers diagram (Smith and Schneider, 2009). These effects are then very inhomogeneously distributed over the Earth and have lead to enormous socio-economic consequences. If this is the case in 2100, then there is a point in time where we must have crossed the conditions for DAI. This time, marking the boundary of a 'safe' and 'unsafe' climate state, obviously depends on the metrics used to quantify the state of the complex climate system.

In very simplified views, this boundary is interpreted as a threshold on pCO₂ (Hansen et al., 2008) or on GMST. The latter, in particular the $\Delta T_c = 2^{\circ}$ C threshold, has become an easy to communicate (and maybe therefore leading) idea to set mitigation targets for greenhouse gas reduction. Emission scenarios have been calculated (Rogelj et al., 2011) such that ΔT will remain below ΔT_c . Although

50 thresholds on GMST have been criticized for being very inadequate regarding impacts (Victor and Kennel, 2014), such a threshold forms the basis of policy making as is set forward in the Paris 2015 (COP21) agreement (the aim is $\Delta T_c = 1.5^{\circ}$ C).

Suppose that measures are being taken to keep $\Delta T < \Delta T_c$, does this mean that we are 'safe'? The answer is a simple no, as regionally still DAI may have occurred such as the disappearance

- of island chains due to sea level rise (Victor and Kennel, 2014). Hence, attempts have been made to define what 'safe' means in a more general way, such the Tolerable Windows Approach (TWA) (Petschel-Held et al., 1999) and Viability Theory (VT) (Aubin, 2009). These approaches also deal with general control strategies to steer a system towards 'safety' when needed. On a more abstract level, both TWA and VT start by defining a desirable (or 'safe') subspace V of a state vector x in a
- 60 general state space X. This subspace is characterized by constraints, such as thresholds on properties of x. For example, when x is a high-dimensional state vector of a GCM, such a threshold could be $\Delta T < \Delta T_c$ on GMST. When the time-development (or trajectory) of x is such that it moves outside

the subspace V, a control is sought to steer the trajectory back into V. Note that this is an abstract formulation of the mitigation problem, when the amplitude of the emission of greenhouse gases is

taken as control. Recently, Heitzig et al. (2016) have added more detail to regions in the space X which differ in their 'safety' properties and amount of flexibility in control to steer to 'safety'.

Giving a certain desirable subspace of the climate system's state vector (e.g., to avoid DIA) and a suite of control options, (e.g., CO_2 emission reduction) it is important to know when it is too late to be able to steer the system to 'safe' conditions, say at the year 2100. In other words, when is the Point

- 70 of No Return (PNR)? The TWA and VT approaches, and the theory in Heitzig et al. (2016), suffer from the 'curse of dimensionality' and cannot be used within CMIP5 climate models. For example, the optimization problems in VT and TWA lead to dynamic programming schemes which have up to now only been solved for model systems with low-dimensional state vectors. The approach in Heitzig et al. (2016) requires the computation of region boundaries in state space, which also
- 75 becomes tedious in more than two dimensions. Hence, with these approaches it will be impossible to determine a PNR using reasonably detailed models of the climate system.

In this paper, we present an approach similar to TWA and VT, but one which can be applied to high-dimensional models of the climate system. Key in the approach is the estimation of the probability density function of the properties of the state vector \mathbf{x} which determine the 'safe' subspace

- 80 V. The PNR problem is coupled to limitations in the control options (e.g. of emissions) and can be defined precisely using these options and stochastic viability theory. The more abstract formulation is presented in section 2, and just to illustrate the concepts, we apply the approach in section 3 to an idealized energy balance model with and without tipping behavior. In section 4, the application to a high-dimensional climate model follows, using data from the Planet Simulator (PLASIM, Fraedrich
- et al. (2005)). A summary and discussion in section 5 concludes the paper.

2 Methodology

Here we briefly describe the concepts we need from stochastic viability theory and then define the PNR problem, specifically in the climate change context.

2.1 Stochastic viability theory

90 Viability theory studies the control of the evolution of dynamical systems to stay within certain constraints on the system's state vector (Aubin, 2009). These constraints define a viable region V in state space. For a finite dimensional deterministic system, with state vector $\mathbf{x} \in \mathbb{R}^d$ and vector field $\mathbf{f}: \mathbb{R}^d \to \mathbb{R}^d$, given by

$$\frac{d\mathbf{x}}{dt} = \mathbf{f}(\mathbf{x}, t),\tag{1}$$

an initial condition $\mathbf{x}_0 = \mathbf{x}(t=0)$ is called viable if $\mathbf{x}(t) \in V$, for all $0 \le t \le t^*$, where t^* is a certain end time. The set of all these initial conditions forms the viability kernel associated with V. In a more general formulation of viability theory, an input is also considered in the right hand side of Eq. (1) which can be used to control the path of the trajectory $\mathbf{x}(t)$ in state space.

Stochastic extensions of viability theory consider finite dynamical systems defined by stochastic 100 differential equations

$$d\mathbf{X}_{t} = \mathbf{f}(\mathbf{X}_{t}, t)dt + \mathbf{g}(\mathbf{X}_{t}, t)d\mathbf{W}_{t},$$
(2)

where $\mathbf{X}_t \in \mathbb{R}^d$ is a multidimensional stochastic process, $\mathbf{W}_t \in \mathbb{R}^n$ is a vector of n-independent standard Wiener processes and the matrix $\mathbf{g} \in \mathbb{R}^{d \times n}$ describes the dependence of the noise on the state vector. The normalised probability density function (PDF) $p(\mathbf{x}, t)$ can be formally determined from the Fokker-Planck equation associated with Eq. (2).

A stochastic viability kernel V_{β} consists of initial conditions \mathbf{X}_0 for which the system has, for $0 \le t \le t^*$, a probability larger than a value β to stay in the viable region V (Doyen and De Lara, 2010). For example, in a one-dimensional version of Eq. (2) with state vector $X_t \in R$ and with a viable region V given by $x \le x^*$ a state X_t is called viable, with tolerance probability β_T , if

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$$\int_{-\infty}^{x} p(x,t) \, dx \ge \beta_T, \tag{3}$$

and otherwise, X_t is called non viable.

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2.2 The Point of No Return problem

In the climate change context, scenarios of GHG increase and the associated radiative forcing have been formulated as Representative Concentration Pathways (RCPs). In Pachauri et al. (2014),
there are four RCP scenarios (Fig. 1a) ranging from an increase in radiative forcing of 2.6 W m⁻² (RCP2.6) at 2100 (with respect to 2000) to a forcing increase of 8.5 W m⁻² (RCP8.5).

To define the PNR for each of these RCPs, a collection of mitigation scenarios on greenhouse gas emission can be considered. These mitigation scenarios will lead to changes in GHG concentrations, described by functions $F_{\lambda}(t)$, where λ is a parameter. For instance, the collection F_{λ} could result

- 120 from mitigation measures that lead to an exponential decay to different stabilisation levels (measured in CO₂ equivalent, or CO₂eq) within a certain time interval. An example of such a collection F_{λ} is shown by the dashed and dotted red lines in Fig. 1b. The most extreme member of F_{λ} is defined as the mitigation scenario (represented by a certain value of parameter λ) which has the steepest initial decrease at a certain time t (dashed curve in Fig. 1b).
- Along the curve of a certain RCP scenario, there will be a point in time where action will be taken to reduce emissions of GHG; this is indicated by a time of action t_b . Consider for example (Fig. 1b) that t_b is chosen as the first year where the state vector \mathbf{X}_t is not viable anymore. A reduction in emissions is, however, not immediately followed by a decrease in CO₂eq due to the long residence time of atmospheric CO₂. In addition, there is a delay to take action in emission reduction due to



Figure 1. (a) CO_2eq trajectories of the RCP scenarios used by the IPCC in CMIP5. (b) The solid red curve represents a typical RCP scenario. At the time t_b the climate state becomes non-viable, while at $t = t_c$ a CO_2eq reduction F_{λ} applies; at time t_e , the climate state is viable again.

- 130 technological, social, economic and institutional challenges. Hence, emission reduction will only start Δt_1 years after t_b . The CO₂eq will, even after emissions have been reduced, also still increase over a time Δt_2 . The time at which the CO₂eq starts to reduce according to F_{λ} is indicated by $t_c = t_b + \Delta t$, where $\Delta t = \Delta t_1 + \Delta t_2$. Eventually, \mathbf{X}_t may become viable again and this point in time is indicated by t_e (Fig. 1b).
- For a given RCP scenario, tolerance probability β_T , viable region V and collection F_{λ} , we define the PNR (π_t) as the first year t_c where, even when at that moment the most extreme CO₂eq reduction scenario F_{λ} applies,
 - (a) either \mathbf{X}_t will be non viable for more than τ_T years, where τ_T is a set tolerance time, or
 - (b) \mathbf{X}_t will be non viable in the year 2100.
- 140 The first PNR, which we will indicate below by π_t^{tol} , is based on limiting the amount of years that \mathbf{X}_t is non viable, since (during these years) society is exposed to risks from, for example, extreme weather events. The second PNR, which we will indicate below by π_t^{2100} imposes no restrictions on how long \mathbf{X}_t is non viable, but it only requires that \mathbf{X}_t is non viable at the end on this century. We will use both PNR concepts in the results below.

145 **3** Energy balance model

In this section, we illustrate the concepts and the computation of the PNR for an idealized energy balance model of Budyko-Seller type (Budyko, 1969; Sellers, 1969). We will also assume that the CO_2eq can be directly controlled, and hence no carbon cycle model is needed to determine CO_2eq from an emission reduction scenario.



Figure 2. Bifurcation diagram of the deterministic energy balance model for $\alpha_1 = 0.45$ ((a), monostable model) and $\alpha_1 = 0.2$ ((b), bistable model). The solid curve represents a stable equilibrium while a dashed curve represents an unstable equilibrium.

150 3.1 Formulation

We use the stochastic extension of the model formulation as in Hogg (2008). The equation for the atmospheric temperature T_t (in K) is given by

$$dT_t = \frac{1}{c_T} \left\{ Q_0(1 - \alpha(T_t)) + G + A \ln \frac{C(t)}{C_{ref}} - \sigma \epsilon T_t^{\ 4} \right\} dt + \sigma_s dW_t.$$
(4)

The values and meaning of the parameters in Eq. (4) are given in Table 1. The first term in the right hand side of Eq. (4) represents the short-wave radiation received by the surface and $\alpha(T)$ is the albedo function, given by

$$\alpha(T) = \alpha_0 H(T_0 - T) + \alpha_1 H(T - T_1) + \left(\alpha_0 + (\alpha_1 - \alpha_0) \frac{T - T_0}{T_1 - T_0}\right) H(T - T_0) H(T_1 - T).$$
(5)

This equation contains the effect of land ice on the albedo and $H(x) = 1/2(1 + \tanh(x/\epsilon_H))$ is a continuous approximation of the Heaviside function. When the temperature $T < T_0$, the albedo will

- 160 be α_0 and when $T > T_1$ it will be α_1 and the albedo is linear in T for $T \in [T_0, T_1]$. The second term in the right hand side of Eq. (4) represents the effect of greenhouse gases on the temperature. It consists of a constant part (*G*), and a part $(A \ln \frac{C(t)}{C_0})$ depending on the mean CO₂eq concentration in the atmosphere (indicated by C(t)). The third term in the right hand side of Eq. (4) expresses the effect of long-wave radiation on the temperature and the last term represents noise with a constant
- 165 standard deviation σ_s . The standard value of σ_s chosen as 3% of the value of G/c_T , hence about 0.3K/year. The variance in CO₂ concentration originates mostly from seasonal variations, and the 3% is on the high side. Nevertheless, we still use this value, because if we take values smaller than 3% the PDF of the GMST will almost be a delta function and concepts can not be illustrated clearly.

c_T	$5.0\times 10^8~{\rm Jm}^{-2}{\rm K}^{-1}$	Thermal inertia	ϵ	1.0	Emissivity
Q_0	$342 \mathrm{~Wm^{-2}}$	Solar constant/4	α_0	0.7	Albedo parameter
G	$1.5\times 10^2~\mathrm{Wm^{-2}}$	Constant	α_1	0.2 or 0.45	Albedo parameter
A	$2.05\times10^1~\mathrm{Wm^{-2}}$	Constant	T_0	263 K	Albedo parameter
C_{ref}	280 ppmv	Reference CO ₂ concentration	T_1	293 K	Albedo parameter
σ	$5.67 \ \mathrm{x} \ 10^{-8} \ \mathrm{Wm}^{-2} \mathrm{K}^{-4}$	Stefan Boltzmann constant	ϵ_H	0.273 K	Albedo parameter

Table 1. Value and meaning of the parameters in the energy balance model given by Eq. (4).

3.2 Results: stochastic viability kernels

- 170 When using the global mean CO₂eq concentration C in Eq. (4) as a time-independent control parameter, a bifurcation diagram can be easily (numerically) calculated for the deterministic case ($\sigma_s = 0$). In Fig. 2, such diagrams are plotted of C versus the equilibrium temperature T for two values of α_1 . To obtain realistic values for the temperature, the equilibrium temperature equilibria are shifted upwards by 30 K. This is done by substituting T with T - 30 and adapting the right hand side of
- 175 Eq. (4) such that the new temperature is a steady state. This is obviously a bit artificial here, but we justify it by our aim to only wanting to illustrate the methodology; results from more realistic models will follow in section 4 below. The diagram corresponding to $\alpha_1 = 0.2$ (Fig. 2a) has two saddle-node bifurcations which are absent for $\alpha_1 = 0.45$ (Fig. 2b). From now on, the energy balance model with $\alpha_1 = 0.45$ and $\alpha_1 = 0.2$ will be called the monostable and bistable case, respectively.

180 For $\sigma_s \neq 0$, we explicitly determine the normalised PDF p(x,t). Rewriting Eq. (4) as

$$dT_t = f(T_t, t)dt + \sigma_s dW_t, \tag{6}$$

with $f(T,t) = c_T^{-1}(Q_0(1-\alpha(T)) + G + A \ln \frac{C(t)}{C_{ref}} - \sigma \epsilon T^4)$, the Fokker-Planck equation of Eq. (6) is given by

$$\frac{\partial p}{\partial t} + \frac{\partial (fp)}{\partial x} - \frac{\sigma_s^2}{2} \frac{\partial^2 p}{\partial x^2} = 0.$$
(7)

185 This differential equation is solved numerically for p(x,t) under any prescribed function C(t) with boundary conditions $p(x_u,t) = p(x_l,t) = 0$, where $x_l = 270$ K and $x_u = 335$ K, and an initial condition p(x,0) (specified below) satisfying $\int_{x_l}^{x_u} p(x,0) dx = 1$.

We first show stochastic viability kernels for each initial condition T₀ and C₀, where C₀ is an initial CO₂eq concentration and T₀ is the expectation value of the initial PDF of T_t. As starting time,
190 we take the year 2030 and suppose that the climate system will be forced by a certain RCP scenario from 2030 till 2200. For every C₀, the original RCP scenario from Fig. 1a is adjusted such that its time development remains the same, but it has C₀ as CO₂eq concentration in 2030. The PDF of the GMST p(x,t=0) (t = 0 refers to the year 2030) has a prescribed variance (defined by σ_s²) and an expectation value T₀.



Figure 3. The stochastic viability kernels for the monostable and bistable cases forced by the RCP4.5 scenario. The viable region is defined as $T \le 293K$ and is indicated by the red dashed line. This plots show, for each combination of T_0 and C_0 , in which stochastic viability kernel these initial values are located. The numbers in the colourbar stand for the β in V_{β} . For convenience, the bifurcation diagram of the deterministic model is also shown.

- In Fig. 3, the stochastic viability kernels are plotted for the energy balance model forced by the RCP4.5 scenario and a viable region V defined by T ≤ 293 K. The results for the monostable and bistable cases are plotted in Fig. 3a and Fig. 3b, respectively. The colors indicate for each combination of T₀ and C₀ in which stochastic viability kernel the initial state (C₀, T₀) is located. For example, consider the bistable case and an initial condition of T₀ = 288 K and C₀ = 400 ppmv, then
 this initial condition is in the kernel V_β with β ≥ 0.9. This means that, with a probability larger than 0.9, a trajectory of the model starting at (C₀, T₀) will remain viable up to the year 2200, where C follows the RCP4.5 scenario. The white areas contain initial conditions that are in a stochastic
 - viability kernel V_{β} with $\beta < 0.5$.
- The sensitivity of the stochastic viability kernels with respect to RCP scenario, threshold defining 205 the viable region V and amplitude of the noise σ_s was also investigated (results not shown). The behaviour is as one can expect in that the area of the kernels becomes smaller (larger) when noise is larger (smaller), when the threshold temperature is smaller (larger) and when the radiative forcing associated with the RCP scenario is more (less) severe. For example for the RCP6.0 scenario, each combination of T_0 and C_0 (same range as in Fig. 3) is in a V_β with $\beta < 0.5$ for both mono- and bittels associated
- 210 bistable cases.

Results: Point of No Return 3.3

Again for illustration purposes, we assume that reduction of the emissions will have an immediate effect of the CO₂eq, such that effectively the CO₂eq is controlled. We choose the collection F_{λ} to consist of mitigation scenarios that exponentially decay to the preindustrial CO₂eq concentration,

215 which is 280 ppmv. For this exponential decay, we consider different e-folding times τ_d . The most extreme scenario has an exponential decay within 50 years, which corresponds to an e-folding time of $\tau_d = 9$ years. Hence, the collection F_{λ} is given by (for $\tau_d \ge 9$)

$$F_{\lambda}(t) = (C_{t_c} - 280) \exp\left(-\frac{t - t_c}{\tau_d}\right) + 280.$$
(8)

In this equation, t_c is the time at which the scenario is applied and C_{t_c} the associated CO₂eq concentration at that moment.

Next, we determine PNR values π_t^{tol} for the energy balance model when it is forced by the four different RCP scenarios using a tolerance probability of $\beta_T = 0.9$ and a tolerance time of $\tau_T = 20$ years. The π_t^{tol} values for a system forced with the RCP4.5, RCP6.0 and RCP8.5 scenarios are shown in Fig. 4 for both the monostable and bistable cases. As expected, the more extreme the

- 225 RCP scenario, the earlier the PNR. This can be easily explained by the fact that when the CO_2eq concentration is rising faster, the temperature will get non viable earlier. Consequently, the PNR will be earlier, since the GMST is only allowed to be non viable for at most τ_T years. When the model is forced with RCP2.6, there is no PNR for both models. The reason for this is that the CO_2eq concentration will remain low throughout the whole period and consequently the temperature will
- 230 stay viable. The value of π_t^{tol} of the bistable case is for each scenario earlier than the value of the monostable case. This can be clarified by the fact that the PDF of the temperature in the bistable case will leave the viable region at a lower CO_2 eq concentration because of the existence of nearby equilibria.

The sensitivity of π_t^{tol} versus the tolerance time τ_T and the tolerance probability β_T was also investigated and the results are as expected (and therefore not shown). A longer tolerance time will 235 shift π_t^{tol} to later times. For example, for the RCP4.5 scenario $\pi_t^{tol} = 2071$, 2088 and 2116 for $\tau_T = 0,20$ and 50 years for the bistable case (for fixed $\beta_T = 0.9$). With a fixed $\tau_T = 20$ years, the value of π_t^{tol} shifts to smaller values when the tolerance probability is increased. For example, for $\beta_T = 0.80$ and 0.99, the values of π_t^{tol} are 2127 and 2058, respectively, for the bistable case (for $\beta_T = 0.9, \pi_t^{tol} = 2088$, see Fig. 4). 240

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4 PLASIM

The results in the previous section have illustrated that a PNR can be calculated when an estimate of the probability density function can be calculated and a collection of mitigation scenarios is available. We will now apply these concepts to the more detailed, high-dimensional, climate model



Figure 4. The PNR π_t^{tol} for a system forced with different RCP scenarios, tolerance probability $\beta_T = 0.9$ and tolerance time $\tau_T = t_e - t_b = 20$ years and $\Delta t = 0$. The triangles indicate the point of no return for the bistable case and the squares for the monostable case. The dotted line is the most extreme scenario of F_{λ} with an exponential decay to 280 ppmv and an e-folding time of 9 years. Note that for both cases there is no PNR when the model is forced with the RCP2.6 scenario.

PLASIM, a General Circulation Model developed by the University of Hamburg. A main problem here is to determine a relation between the CO₂eq (and associated radiative forcing) and the GMST, i.e. a response function. Previous approaches have used a fit of a specific response function (e.g., a power law function) to available observations (Rypdal, 2016). This is more complicated for an approach using stochastic viability theory and hence we proceed along another path.

250 4.1 Linear response theory

In order to find the temporal evolution of the PDF of the global mean surface temperature GMST (indicated by T) under any CO₂eq forcing in PLASIM, we will use linear response theory (LRT). With this theory, the effect of any small forcing perturbation on the system state can be calculated by running the climate model for only one forcing scenario (Ragone et al., 2014).

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In LRT, the expectation value of an observable Φ , when forcing the system with a time-dependent function f(t), can be calculated by computing the convolution of a Green's function $G_{\langle \Phi \rangle}$ and the forcing f(t), according to

$$\langle \Phi \rangle_f(t) = \int_{-\infty}^{+\infty} G_{\langle \Phi \rangle}(\tau) f(t-\tau) d\tau.$$
(9)

To construct this Green's function, the property that the convolution in the time domain is the same

as point-wise multiplication in the frequency domain is used. The Fourier transform of Eq. (9) is given by

$$\langle \Phi \rangle_f(\omega) = \chi_{\langle \Phi \rangle}(\omega) \tilde{f}(\omega), \tag{10}$$

with χ_(Φ)(ω), (Φ̃)_f(ω) and f̃(ω) being the Fourier transforms of G_(Φ)(t), (Φ)_f(t) and f(t), respectively. Therefore, once the time evolution of the expectation value of an observable under a certain forcing is known, the Green's function of this observable can be constructed with Eq. (10) and consequently the linear response of the observable to any forcing can be calculated.

We use the same data as in Ragone et al. (2014), provided by F. Lunkeit and V. Lucarini (Univ. Hamburg, Germany). The difference with those in Ragone et al. (2014) is that the seasonal forcing is present, which results in a long-term increase of the GMST of 5 $^{\circ}$ C (instead of 8 $^{\circ}$ C in Ragone

- et al. (2014)) under a scenario where the CO_2 concentration doubles. The reason of this difference is not fully clear but probably result from seasonal rectification effects of nonlinear feedbacks. Data of GMST from two ensembles was used, each of 200 simulations, made with two different CO_2 forcing profiles (all other GHGs are kept constant). For both forcing profiles, the starting CO_2 concentration is set to a value of 360 ppmv, which is representative for the CO_2 concentration in 2000. During
- the first set of experiments, the CO₂ concentration is instantaneously doubled to 720 ppmv and kept constant afterwards. During the second set of experiments, the CO₂ concentration increases each year by 1 % until a concentration of 720 ppmv is reached. This will take approximately 70 years and afterwards the concentration is fixed. The total length of the simulations is 200 years. Furthermore, the forcing *f*(*t*) in Eq. (9) is the logarithm of the CO₂ concentration, since the radiative forcing scales approximately logarithmically with the CO₂ concentration.

To determine the PDF of GMST under any CO₂eq forcing, we make the assumption that at each point in time the PDF of the GMST is normally distributed. As we have 200 data points for the GMST at each time interval, a χ^2 test was used to analyse the PDFs. For each time, the value of $\chi^2 > 0.05$ and therefore the assumption that the PDF of the GMST is normally distributed appears justified.

- The Green's functions for the expectation value and variance of GMST have been calculated with the instantaneously doubling CO_2 profile and the associated ensemble. From the ensemble, at each point in time the expectation value and variance are calculated such that we get the temporal evolution of these two variables. Subsequently, we have found the Green's functions using (10). To check whether these Green's functions perform well, we compared the temporal evolution of the expectation value
- 290 and variance of the GMST under the 1 % forcing (calculated with (9)) with those directly generated with PLASIM (Fig. 5). The expectation value determined with LRT is close to the one directly generated by PLASIM. However, the variance of the ensemble generated by PLASIM is a lot noisier than the one calculated with LRT. Although the Green's function of the variance provides only a rough approximation, it has the right order of magnitude and we will use it to calculate the variance
- 295 of the GMST for other forcing scenarios.



Figure 5. (*a*) The expectation value and (*b*) variance of GMST generated by PLASIM (orange) and determined through LRT (blue) for the 1% CO₂ concentration increase.

4.2 Results: Point of No Return under CO₂eq control

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We first consider the case without a carbon cycle model, again assuming that the CO₂eq concentration can be controlled directly. The scenarios F_{λ} chosen for use in PLASIM are exponentially decaying to different stabilisation levels (varying between 400 and 550 ppmv, see Edenhofer et al. (2010)). This stabilization level is taken as the parameter λ . We assume that stabilisation happens within 100 years, which corresponds to an e-folding time τ_d of about 25 years; the mitigation scenarios F_{λ} are then given by

$$F_{\lambda}(t) = \left(C_{t_c} - \lambda\right) \exp\left(-\frac{t - t_c}{\tau_d}\right) + \lambda,\tag{11}$$

where t_c is again the time at which the mitigation scenario is applied and C_{t_c} the associated CO₂eq
concentration. The most extreme mitigation scenario in F_λ in terms of CO₂eq decrease is the one that stabilises at a CO₂eq concentration of 400 ppmv.

We next determine the PNR π_t^{2100} by requiring that the GMST must be viable in 2100 using a tolerance probability of $\beta_T = 0.90$. Furthermore, the viable region is set at $T \le 16.15^{\circ}$ C, which corresponds to temperatures less than 2°C above the preindustrial GMST. The values of π_t^{2100} for all

- the RCP scenarios are plotted in Fig. 6a. Solid curves show the RCP scenarios while dashed curves present the most extreme scenario F_{λ} . For RCP8.5, π_t^{2100} is 10 years earlier than for RCP6.0, since the CO₂eq concentration increases much faster for the RCP8.5 scenario. The mitigation scenario after the point of no return, represented by the dashed line, is the same for all RCP scenarios. This is related to our definition of π_t^{2100} , where it is required that the GMST is viable in 2100. The
- 315 mitigation scenario that is plotted is the ultimate scenario that guarantees this. It indicates that for each CO₂ scenario the associated π_t^{2100} is given by the intersection of that CO₂eq scenario and the mitigation scenario. This is because it is considered that an exponential decay to 400 ppmv



Figure 6. (a) The PNR π_t^{2100} for the RCP2.6, RCP4.5, RCP6.0 and RCP8.5 scenarios for a tolerance probability of $\beta_T = 0.9$ and $\Delta t = 0$. The solid lines represent the RCP scenarios and the dashed line the most extreme scenario from F_{λ} . Note that these dashed lines coincide. (b) The point of no return for RCP4.5 for different tolerance probabilities.

within 100 years is always possible, no matter the CO_2eq concentration at t_c . However, when this concentration becomes too high, this mitigation scenario is not very realistic anymore.

320 The influence of the tolerance probability on π_t^{2100} for the RCP4.5 scenario is plotted in Fig. 6b, where we only consider a tolerance probability of 0.8, 0.9 and 0.99. When the tolerance probability is higher, it takes longer before the GMST will be viable again and thus the PNR π_t^{2100} will be earlier. However, the differences are very small, since the mitigation scenarios that guarantee viability in 2100 for the different tolerance probabilities are very close.

325 4.3 Results: Point of No Return under emission control

Finally, we consider the more realistic case where emissions are controlled and a carbon model converts emissions to CO_2eq . A simple carbon model relating emissions E to concentrations C is given by

$$C_{CO2}(t) = C_{CO2,0} + \int_{0}^{t} G_{CO2}(\tau) \ E_{CO2}(t-\tau) \ d\tau,$$
(12)

330 where $C_{CO2,0}$ is the initial concentration. The Green's function for CO₂ is taken directly from Joos et al. (2013):

$$G_{CO2}(t) = a_0 + \sum_{i=1}^{3} a_i e^{t/\tau_i},$$
(13)

where the parameters are shown in Table 2. The quantity E_{CO2} is the CO₂ emission in ppm yr⁻¹ that has been converted from GtCyr⁻¹ using the Carbon molecular weight as E_{CO2}[ppm yr⁻¹] = γ E_{CO2}[GtC/yr], with γ = 0.46969 ppm GtC⁻¹. The emissions for the RCP scenarios are taken

from Meinshausen et al. $(2011)^{1}$. The carbon model underestimates CO₂ levels for very high emission scenarios as it does not include saturation of natural CO₂ sinks.

Following Table 8.SM.1 of Myhre et al. (2013) we obtain the changes in radiative forcing compared to preindustrial (in $W m^{-2}$) due to changes in CO_2 as

340
$$\Delta F_{CO2} = \alpha_{CO2} \ln \frac{C_{CO2}}{C_0},$$
 (14)

where C_0 is the pre-industrial (1750) CO₂ concentration.

We use the same PLASIM ensemble of instantaneous CO_2 doubling runs again to determine a Green's function that relates radiative forcing changes to temperature changes as

$$\Delta T(t) = \int_{0}^{t} G_T(\tau) \Delta F(t-\tau) d\tau$$
(15)

345 where G_T is the data-based function determined from LRT. The total radiative forcing is taken as $\Delta F_{tot} = A \Delta F_{CO2}$, where we introduce a scaling constant A to correct for the high climate sensitivity of the PLASIM model compared to typical CMIP5 models. Based on trial runs attempting to reconstruct mean CMIP5 RCP temperature trajectories with RCP CO₂ emissions we choose A = 0.6.

For PLASIM, the Green's function G_T , as determined through LRT, is well approximated by a one-time scale exponential:

$$G_T(t) \approx b_1 e^{-t/\tau_{b1}},\tag{16}$$

with $b_1 = 0.25 \,\mathrm{KW^{-1} m^2 yr^{-1}}$ and $\tau_{b1} = 4.69 \,\mathrm{yr}$. We determine a Green's function for the temperature variance in the same way.

Table 2. Model Parameters. No units are given for dimensionless parameters

<i>C_{CO2,0}</i> (ppm)	278	α_{CO2}	5.35	C_0 (ppm)	278
a_0	0.2173	A	0.6	$\gamma (\mathrm{ppm}\mathrm{GtC}^{-1})$	0.46969
a_1	0.2240	a_2	0.2824	a_3	0.2763
τ_1 (yr)	394.4	$ au_2$ (yr)	36.54	$ au_3$ (yr)	4.304

355

To compute the Point of No Return π_t^{2100} in the carbon-climate model we start from pre-industrial CO₂ concentrations and take the corresponding initial temperature perturbation as $\Delta T = 0$. We then prescribe the RCP emissions scenarios for RCP2.6, RCP4.5, RCP6 and RCP8.5 (that are identical up to the year 2005). At a year $t_b > 2005$ we start reduction of emissions at an exponential rate, i.e. for $t > t_b$ the emissions follow

$$E_{CO2}(t) = E_{CO2}(t_b) \exp\left(-\frac{t - t_b}{\tau_e}\right)$$
(17)

¹See the database at http://www.pik-potsdam.de/~mmalte/rcps/



Figure 7. (a) Warming in 2100 when starting exponential CO_2 emission reduction in a given year. (b) CO_2 concentration for the four RCP scenarios as computed by the model (solid) and following exponential mitigation starting from the Point of no Return (dashed).

360 where τ_e = 25 yr is the e-folding timescale of the emission reduction that we keep constant. Using the carbon model we compute the instantaneous CO₂ concentrations for each such scenario and use the Green's functions for GMST mean and variance to determine the PDF at year 2100 for each starting year t_b. Assuming Gaussian distributions (this is well satisfied for the original PLASIM ensemble), we can then easily determine the temperature threshold below which 90% of the values fall. The first year for which this threshold is above 2 K gives π_t²¹⁰⁰. Note that the value of t_c (in Fig.

1b) where the CO_2 starts to decrease is determined by the coupled carbon-climate model.

The warming in 2100 predicted by our simple climate model when starting exponential CO₂ emission reduction in a given year is shown in Fig. 7a. The intersections between the RCP curves (solid color) and the dashed line (giving 2 K warming) provide values of π_t^{2100} . Values do not differ

- 370 much for the different RCP (4.5, 6.0 and 8.5) scenarios and are before 2030. RCP2.6 does not have a Point of No Return as its emission scenario is sufficient to keep the warming safely below 2 K. The counter-intuitive lowering of the curve for RCP2.6 (also slightly for RCP4.5) is due to very fast emission reductions in these RCP scenarios. So starting emission reduction at later times may lead to lower total emissions (and hence, temperatures). The CO₂ concentration for the four RCP
- 375 scenarios as computed by the model (solid) and following the exponential mitigation starting from the Point of no Return (dashed) are shown in Fig. 7b. Note how emissions 'still in the pipeline' lead to CO_2 increases even after the reduction is initiated. Note that this approach does not factor in the uncertainty in the carbon model as we do not have a Green's function propagating the carbon uncertainty through the temperature model. Including this would very likely increase the variance in
- 380 the pdf and move the Point of No Return to an earlier year. On the other hand, the PLASIM variance is quite small, so the 90% threshold is not vastly different from the mean.

5 Discussion

Pachauri et al. (2014) stated with high confidence that: "Without additional mitigation efforts beyond those in place today, and even with adaptation, warming by the end of the 21st century will lead to

- 385 high to very high risk of severe, widespread and irreversible impacts globally". If no measures are taken to reduce GHG emissions during this century and neither will there be any new technological developments that can reduce GHGs in the atmosphere, it is likely that the GMST will be 4 °C higher than the preindustrial GMST at the end of the 21st century (Pachauri et al., 2014). Consequently, it is important that anthropogenic emissions are regulated and significantly reduced before widespread
- 390 and irreversible impacts occur. It would help motivate mitigation when we would know when it is 'too late'.

In this study we have defined the concept of the Point of No Return (PNR) in climate change more precisely, using stochastic viability theory and a collection of mitigation scenarios. For an energy balance model, as in section 3, the probability density function could be explicitly computed

- 395 and hence stochastic viability kernels could be determined. The additional advantage of this model is that one can easily construct a bistable regime, so one can investigate the effects of tipping behavior on the PNR. We used this model (where the assumption was made that CO_2 could be controlled directly instead of through emissions), to illustrate of concept of PNR which is based on a tolerance time for which the climate state is non viable. For the RCP scenarios considered, one finds that
- 400 the PNR is smaller in the bistable than in the monostable regime of this model. The occurrence of possible transitions to warm states in this model indeed cause the PNR to be 'too late' earlier.

The determination of the PNR in the relatively high-dimensional PLASIM climate model, however, shows the key innovation in our approach, i.e. the use of linear response theory (LRT) to estimate the probability density function. PLASIM was used to compute another variant of a PNR

- 405 based on only requiring that the climate state is viable in the year 2100. In the PLASIM results, we used a viability region that was defined as GMSTs lower than 2°C above the pre-industrial value, but with our methodology, the PNR can be easily determined for any threshold defining the viable region. The more academic case where we assume that GHG levels can be controlled directly provides PNR (for RCP4.5, RCP6.0 and RCP8.5) values around 2050 (section 4.2). However, the more
- 410 realistic case where the emissions are controlled (section 4.3) and a carbon model is used, reduces the PNR for these three RCP scenarios by about 30 years. The reason is that there is a delay between the decrease in GHG gas emissions and concentrations.

Although our approach provides new insights into the Point of No Return in climate change, we recognize there is potential to substantial further improvement. First of all, the PLASIM model has a

415 too high climate sensitivity compared to CMIP5 models. Although in the most realistic case (section 4.3), we somehow compensate for this effect, it would be much better to apply the LRT approach to CMIP5 simulations. However, a large ensemble such as that available for PLASIM is not available (yet) for any CMIP5 model. Second, in the LRT approach, we assume the GMST distributions to be

Gaussian. This is well justified in PLASIM, as can be verified from the PLASIM simulations, but it

420 might not be the case for a typical CMIP5 model. Third, for the more realistic case in section 4.3, we do not capture the uncertainties in the carbon model and hence in the radiative forcing.

Due to these shortcomings, one cannot attribute much importance to the precise PNR values obtained for the PLASIM model as in Fig. 7. However, we think that our approach is general enough for handling many different political and socio-economical scenarios combined with state-of-the-art

- 425 climate models when the necessary simulations have been done with CMIP5 models to determine the Green's function, using LRT. Hence, it will be possible to make better estimates of the PNR for the real climate system. We therefore hope that eventually these ideas on the point of no return in climate change will become part of the decision-making process during future debates about climate change.
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