On testing the additivity of Zebiak-Cane model response to volcanic and solar forcing.

Rebuttal of Section 2 of Rypdal and Rypdal 2016

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Introduction

A rebuttal of Section 3 of Rypdal and Rypdal (2016) (R+R, below), in which we clarified some important points about intermittency in general and volcanic intermittency in particular, just appeared (Lovejoy and Varotsos, 2016a, as well as a further comment Lovejoy and Varotsos, 2016c). We now proceed to a rebuttal of Section 2 of R+R by examining the additivity of Zebiak-Cane (ZC) model response to volcanic and solar forcing using the data available from ftp://ftp.ncdc.noaa.gov/pub/data/paleo/climate_forcing/mann2005/mann2005.txt that have been also analyzed in the original paper by Lovejoy and Varotsos (2016b) (L+V below).

To situate the debate, recall that whereas at short enough time scales, when the external forcings are small enough, then theoretically we may expect the atmospheric response to be approximately linear, however, at long enough time scales, due to temperature – albedo feedbacks, the response is expected to become nonlinear. At the same time, it is possible that at long enough time scales, due to quite different detailed surface and atmospheric interactions, that solar and volcanic external forcings combine nonlinearly. Indeed this was one of the motivations for the Mann 2005 modelling study. In our paper L+V, we used both Mann's Zebiac-Cane model (100 realizations) as well as NASA GISS E2-R (full blown GCM) simulations to argue that for scales longer than about 50 years that there was evidence that solar and volcanic forcings combine subadditively. The fundamental difficulty - that we noted - was that neither had the full suite of simulations (i.e. control runs, solar only, volcanic only and combined solar and volcanic responses) needed in order to definitively answer the question. In the case of the Z-C model, the missing element was the control runs which meant that the internal variability was only indirectly estimated. R+R's criticism was based on the possibility that the internal variability – if large enough - might lead to a spurious subadditivity where in fact there was none. Although we believe that the original paper adequately answered this possibility, since this R+R's criticism, we have gone back to the ZC model and found a nearly 200 year period (at the beginning of the simulation series) where there was exactly zero volcanic forcing. Therefore this 200 year segment of the volcanic only response series was effectively a "control run" and could be used to estimate the magnitude of the internal variability. As we show below, at least for the Z-C model outputs, this settles the issue, vindicating the original L+V paper. However, in the last few days, R+R have found a totally different model and suite of results that they claim displays no subadditivity. Obviously – if valid - this new analysis of yet another model is an interesting contribution to the science since it would show that some models are subadditive while others are not. However, it is not directly relevant to the validity or otherwise of the original L+V paper.

The R+R linear response null hypothesis test fails for the ZC model response

R+R state (see lines 328-330): "*Our main focus in this comment, however, is not on the incorrect multifractal interpretation of the scaling analysis, but on the incorrect conclusions drawn from this analysis when it comes to nonlinearity in the response.*", here we deal with their main focus. Along these lines, we will check the validity of the linear response null hypothesis test that R+R suggested. In brief, by employing the notation "Δ" for the fluctuation and the subscripts s, v, and s+v for solar, volcanic, and combined solar and volcanic response, respectively, R+R suggest the following test:

If $\Delta T_{s+v}-\Delta T_s-\Delta T_v\equiv\Delta\varepsilon$, i.e., their Eq.(6), then $\Delta\varepsilon$ has the same distribution as $\sqrt{3}\,\Delta E$ (cf. their Eq.(7)), where E is the internal variability which is unaffected by the forcing (see R+R lines 61-62). E is defined though their Eq.(1):

$T(t)=T^{det}(t)+E(t), \qquad T^{det}(t)=\hat{L}[F(t)],$ **[1]**

where $\hat{L}[F(t)]$ is the linear response operator to the global forcing F(t). In their notation (c.f. they use a lower case epsilon (ϵ) that is different from ε whereas we use capital E for the reader's convenience) one should have: " $\Delta\varepsilon \overset{d}{=} \sqrt{3}\,\Delta\epsilon.$ " (which is their Eq.(7)) for linear response and they conclude (see R+R lines 92 to 96) that:

"*Hence, a prediction based on the linear response hypothesis is that the difference between the temperature driven by combined solar and volcanic forcing and the sum of the temperatures driven by solar and volcanic forcing is realisation of a noise process which is $\sqrt{3}$ times the internal variability process. In the next subsection we shall test this prediction on the data from the ZC model. If the prediction is inconsistent with the data the linear response hypothesis is rejected for this model.*"

We will show below that the above test suggested by R+R fails.

According to their Eq.(9) (R+R line 126), for solar forcing we have:

$E(t)=T_s(t)-\Delta T^{det}_s(t,\Delta t)$ **[2]**

where $\Delta T^{det}_s(t,\Delta t)$ is linearly related to the solar forcing $F_s(t)$ (see R+R Eq.(8) in line 117) through their Eq.(8):

$\Delta T^{det}_s(t,\Delta t)=-S\,\Delta F_s(t-\tau,\Delta t)$ **[3]**

where S is the climate sensitivity and τ some time lag. Using now the fact (that was not noticed earlier!) that during the first 195 years of the ZC modelling the volcanic forcing is $F_v(t)=0$, Eqs.**[2]** and **[3]** applied for the volcanic response during the same 195 years yield:

$\Delta E=\Delta T_v$ **[4]**

Hence, we can obtain ΔE from Eq. **[4]** and examine the validity of the linearity of the model by comparing whether $\Delta\varepsilon$ ($=\Delta T_{s+v}-\Delta T_s-\Delta T_v$) equals to $\sqrt{3}\Delta E=\sqrt{3}\Delta T_v$ for the first 195 years of the available ZC modelling.

Figure 1 shows that for the period of the first 195 years $\Delta\varepsilon$ (= ΔT_{s+v}-ΔT_s-ΔT_v) is well below $\sqrt{3}\Delta E$ when using the Detrended Fluctuation Analysis (DFA) and hence the suggested test of linearity by RR fails. It also shows that even when considering all the 1000 years of the simulation $\Delta\varepsilon$ is well below $\sqrt{3}\Delta E$.

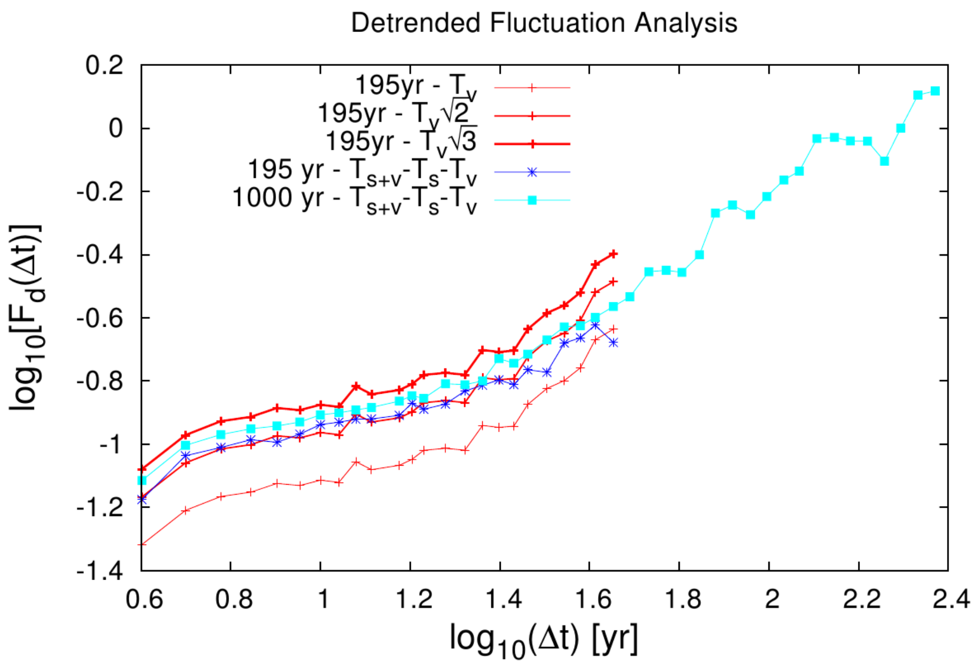

Figure 1 Results of the DFA when using Eq.**[4]** to estimate the internal variability E. Both for the period of the first 195 years (blue) as well for the whole period of 1000 years (cyan) ΔT_{s+v}-ΔT_s-ΔT_v (=$\Delta\varepsilon$) is well below $\sqrt{3}\Delta E$ = $\sqrt{3}$ ΔT_v (thickest red line, top).

The same results are obtained when using the Haar wavelet analysis employed by L+V which are shown in Figure 2.

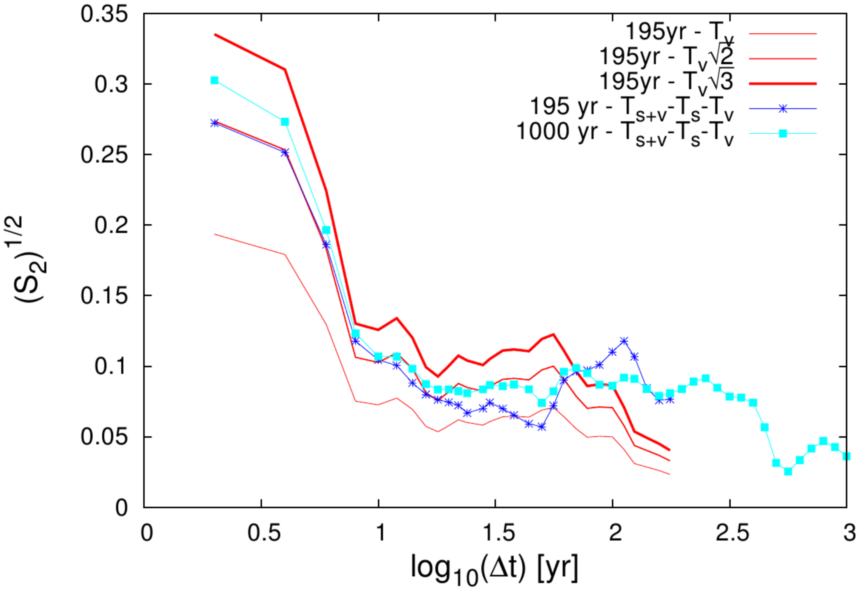

Figure 2 Results from the Haar wavelet analysis used by L+V for the same time-series as in Fig.1, (in degrees C). We observe that for scales up to $10^{1.7} \approx 50$ years the RMS structure function of T_{s+v}-T_s-T_v (either for the first 195 years or for the whole study period) never exceeds $\sqrt{3}\Delta E$ (the thickest red line) but rather remains close to $\sqrt{2}\Delta E$.

Hence, the linearity check suggested by R+R fails pointing to the non-linearity of the responses as suggested by L+V.

Other deficiencies of R+R are: a) they make use of statistical independence in their Eq. (12) - thus increasing the effect of the internal variability E to the ratio R - while they criticize L+V for doing so (see lines 162-164) and b) they claim (in lines 164-166) that L+V admit that their analysis is wrong, which of course is not the case.

Finally, in the references section of R+R the correct doi for L+V is 10.5194/esd-7-133-2016.

References:

Lovejoy, S., and C. Varotsos (2016a), Interactive comment on “Comment on "Scaling regimes and linear/nonlinear responses of last millennium climate to volcanic and solar forcing" by S. Lovejoy and C. Varotsos” by K. Rypdal and M. Rypdal, *Earth Syst. Dynam. Discuss.*, doi:10.5194/esd-2016-10-SC1.

Lovejoy, S., and C. Varotsos (2016b), Scaling regimes and linear/nonlinear responses of last millennium climate to volcanic and solar forcings, *Earth Syst. Dynam.*, *7*, 1–18, doi:10.5194/esd-7-133-2016.

Lovejoy, S., and C. Varotsos (2016c), Trained Eye deceived by fractal clustering, Interactive comment on “Comment on: Scaling regimes and linear/nonlinear responses of last millennium climate to volcanic and solar forcing by S. Lovejoy and C. Varotsos” by K. Rypdal and M. Rypdal , http://www.earth-syst-dynam-discuss.net/esd-2016-10/esd-2016-10-SC2-supplement.pdf, Interactive comment on Earth Syst. Dynam. Discuss., Earth Syst. Dynam. Discuss. doi: doi:10.5194/esd-2016-10, 2016.

Rypdal, K., and M. Rypdal (2016), Comment on “Scaling regimes and linear/nonlinear responses of last millennium climate to volcanic and solar forcings” by S. Lovejoy and C. Varotsos, *Earth Syst. Dynam. Discuss.*, doi:10.5194/esd-2016-10, 2016.