On the importance and significance of Intermittency

Rebuttal of Section 3 of Rypdal and Rypdal 2016

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Intermittency and volcanism

Although this was originally motivated as a response to Rypdal and Rypdal 2016 (R+R below) we well this as an opportunity to clarify some important points about intermittency in general and volcanic intermittency in particular. Since intermittency is not well known outside the turbulence community, these clarifications could be of wide interest to the community. The response to the first part of R+R will be given in a separate comment.

The nonlinearity of the atmospheric response to strong, "spikey", intermittent forcings such as volcanism was noticed over twenty years ago by [*Clement et al.*, 1996] who found that in numerical climate models, there is a high sensitivity to small forcings and a low sensitivity to large forcings. Nonlinearity response to volcanism was also claimed by [*Mann et al.*, 2005], indeed, it was a main purpose of his simulations of the interaction between El Nino and volcanism that were analysed in [*Lovejoy and Varotsos*, 2016] (henceforth L+V). Therefore, the primary contribution of the corresponding section 4 in L+V is to quantify this. The L+V method is straightforward, it simply exploits the known fact that linear transformations of a time series can only make linear changes to the exponent scaling function $\xi(q)$, they cannot affect the nonlinear part that is associated with the intermittency (see e.g. ch. 5 of [*Lovejoy and Schertzer*, 2013]).

This simple and fundamental result has been known since at least the 1980's and was explained in the original L+V text; it is correct. Although R+R complain that L+V paper had long-winded and unnecessary reviews of theory, they would be advised to look more carefully at the discussion of the trace moment analysis that quantified the intermittency in section 4 of L+V – or better still – to consult chapters 3-5 of [*Lovejoy and Schertzer*, 2013]. Trace moment analysis was originally developed thirty years ago as a sensitive way to quantify multifractal intermittency in turbulence. The very first step in the analysis is precisely a nonlinear transformation of the series so as to obtain an estimate of the underlying driving fluxes: the absolute values of the first or second differences are commonly used as flux estimates. The reason that trace moment analysis is so effective is that it removes the linear qH term in the structure function exponent so that the nonlinear $K(q)$ part can be studied directly. Had R+R noticed this fact, they would not have bothered to develop the linear analysis (eqs. 13 to 22) which is irrelevant to the trace moment analyses presented in L+V and to its conclusions.

But even if the trace moment analysis had been a linear one, the R+R analysis would still be of little interest since it makes unnecessarily restrictive assumptions (and incorrectly imputes them to L+V)! For example R+R's affirmation II, that an assumption of multifractality is needed: "i.e., that these processes belong to the multifractal class" is not true. It is enough to recall that for a scaling process it is enough that the *dominant statistics* vary as power laws with scale, and that this is usually accompanied by all kinds of sub-exponential corrections and these are missing in eqs. 15- 22.

Interestingly - although hardly surprisingly – volcanism is an excellent example of a strongly intermittent multifractal process. This fact was first pointed out in [*Lovejoy and Schertzer*, 2012] and [*Lovejoy*, 2014] used the estimated multifractal exponents to produce the highly realistic multifractal simulations reproduced fig. 1a which includes one real series. Can the reader spot the fakes?

R+R complain that because the volcanic forcing series may be approximated by a distribution of Levy type (power law) spikes, (each more or less uniformly distributed along the time axis) that this somehow contradicts the multifractality of the process. This is a misunderstanding. Although cascade processes are the generic multifractal process, there are many, many ways of generating a multifractal process and their proposal to distribute Levy eruptions uniformly along the time axis is indeed one such a proposal (note that for more generality, these could be distributed over a fractal set). No matter which model is closer to reality - the Levy spikes – or a cascade process, they are both strongly intermittent, multifractal and the L+V conclusions hold. In any case, fig. 1a shows that cascades can approximate volcanic processes quite well. And the L+V theorem that linear filters can only make linear changes in structure function exponents – and hence cannot change the intermittency - remains valid.

Just to complete the picture – to demonstrate without using trace moments - the strong differences in multifractal intermittency between the volcanic forcings and the responses, we have added fig. 1b which shows the ratio of the first order moment to the RMS moment of the Haar fluctuations. A nonintermittent, quasi-Gaussian process would be completely flat; nonzero slopes are consequences of intermittency. The reference slopes quantify the intermittency (for log-normal multifractals, the slope is the codimension of the mean, C_1 but at least for the volcanic forcings, this is a poor approximation). The values indicated (0.05, 0.30) are very different: the former is roughly the typical value of the intermittency of wind in atmospheric turbulence whereas the latter value corresponds to precipitation (see Box 4.1 “Overview of the horizontal scaling properties of atmospheric fields” in [*Lovejoy and Schertzer*, 2013]).

Finally, the fact that adding a white noise to the ZC volcanic response can lead to an apparently low intermittency multifractal process (R+R’s fig. 4c) only shows that when the intermittency is low and the range of scales is not so large, care must be taken in interpreting the results (they could see a useful discussion of this in appendix 4A in [*Lovejoy and Schertzer*, 2013], or the curved quasi-Gaussian envelopes in fig. 6c). In any case there is a huge difference in the intermittency between R+R’s fig. 4b and 4c (the volcanic forcing and response), just look at the range of the fluctuations at the large scale ratios (the small resolutions) at the right of the diagrammes (a factor of about 30)!

Erronneous Structure functions analyses:

It is unfortunate that the R+R comment was marred not only by an inappropriately harsh tone, but also by some real difficulties with the data analysis. We would not dwell on it except that it demonstrates some of the pitfalls of scaling analyses, and it may therefore be of general interest.

Consider R+R's treatment of the volcanic forcings, their Auto Correlation Function (ACF) and structure function analyses (fig. 3c, d). First, the decrease of the ACF with scale (fig. 3b) proves little. However, if the ACF decreases, then – at least for stationary processes (and their structure function analysis eq. 23 assumes this), the structure functions must simply asymptote to a maximum value: in obvious notation: $\left\langle \Delta F(\Delta t)_{diff}^{2} \right\rangle = 2\left(\left\langle F(t)^{2} \right\rangle - \left\langle F(t)F(t-\Delta t) \right\rangle \right)$. From this standard equation, it is obvious that the decrease of the ACP ($\left\langle F(t)F(t-\Delta t) \right\rangle$) at large Δt leads the difference structure function ($\left\langle \Delta F(\Delta t)_{diff}^{2} \right\rangle$ to asymptote to the series variance ($\left\langle F(t)^{2} \right\rangle$) which (in this case) is determined by the high frequency details of the series. That this is indeed the relevant explanation is shown in the (correct) difference based structure function analysis in fig. 2a below. Somehow, R+R must have made an error since their fig. 3c shows an impossibly increasing difference structure function. Indeed – as pointed out in the recapitulative part of L+V – when $H<0$, one needs to define the fluctuations differently, Haar fluctuations being a convenient method (see the details in L+V). Fig. 2b below compares the Haar fluctuations with the appropriately modified DFA method, both of which confirm the analysis of L+V and contradict R+R's fig. 3c, d. Just to make things perfectly clear, we have added a spectral analysis (fig. 2b) that again confirms that the second order structure function exponent $\xi(2) \approx -0.90$ (spectral exponent $\beta = 1+\xi(2)=0.1$) which is quite contrary to R+R who claim $\xi(2) \approx 1$ and hence $\beta = 2$, (they even suggest that this Brownian motion value has fundamental significance!). In all analyses, the R+R exponent is very far from the observations.

Although over thirty years ago, multifractal intermittency was a fundamental breakthrough in turbulence theory, its understanding and importance are not sufficiently appreciated outside the turbulence community. We would therefore like to thank R+R for giving us this opportunity to clarify this important question for the benefit of climate scientists.

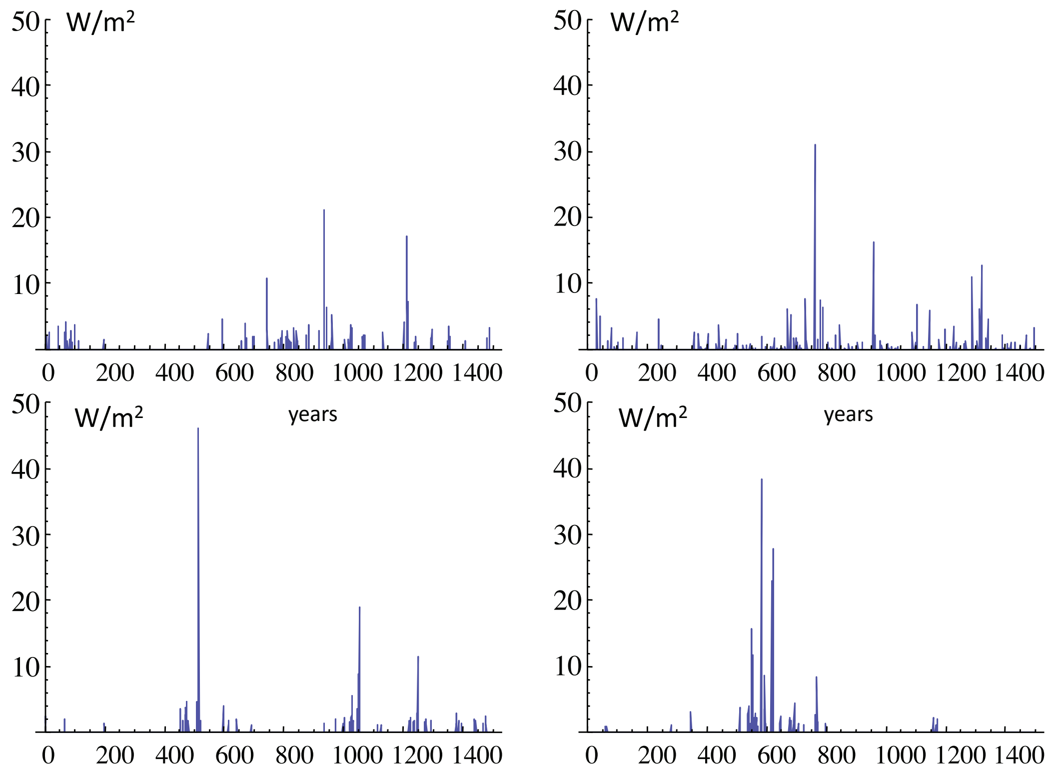

Fig. 1a: This figure is reproduced from [*Lovejoy*, 2014]. It shows three "fake" volcanic reconstructions produced by highly intermittent multifractal simulations as well based on the [*Gao et al.*, 2008] volcanic reconstruction. The fakes are normalized as to have the same mean forcing (the absolute forcings are shown). To find which is the real series, the reader may consult the original paper.

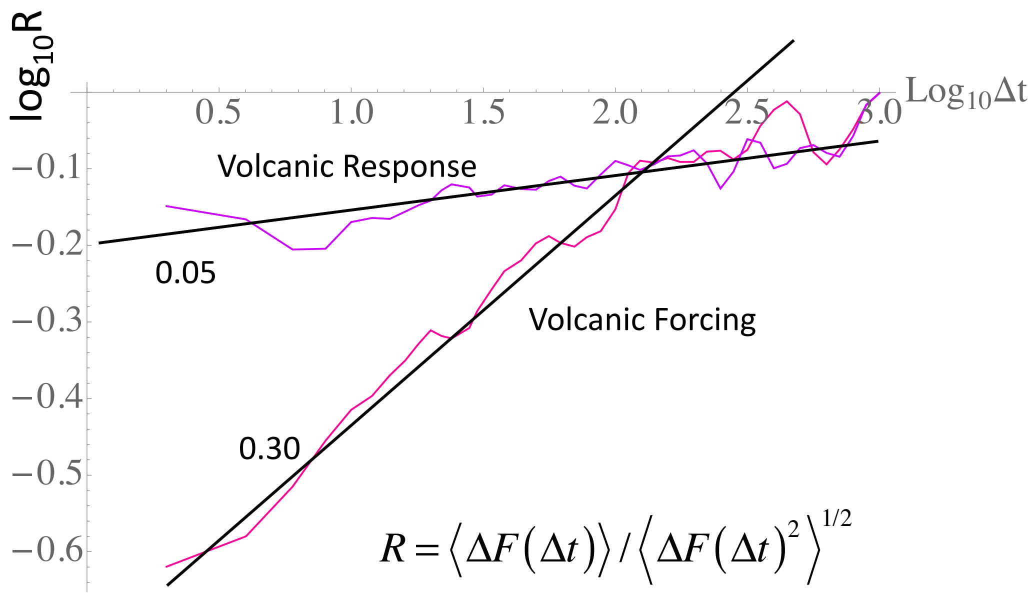

Fig. 1b: This shows the ratios R of the first order (q=1) fluctuations with respect to the root mean square (RMS) fluctuations for a series $F(t)$. Top is for F = the ZC temperature response to the volcanic response, the bottom is for F = the volcanic forcing. A nonintermittent, quasi-Gaussian process would be completely flat, nonzero slopes are consequences of intermittency.

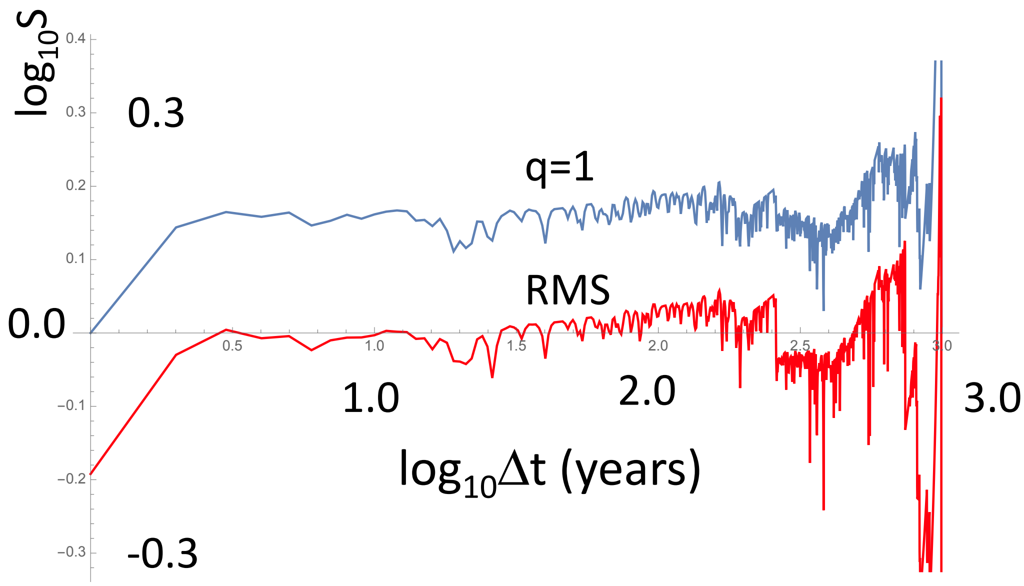

Fig. 2a: The first and second order structure functions for all intervals showing that they do roughly indeed asymptote to a constant as expected since H<0. Compare this to R+R's fig. 3c.

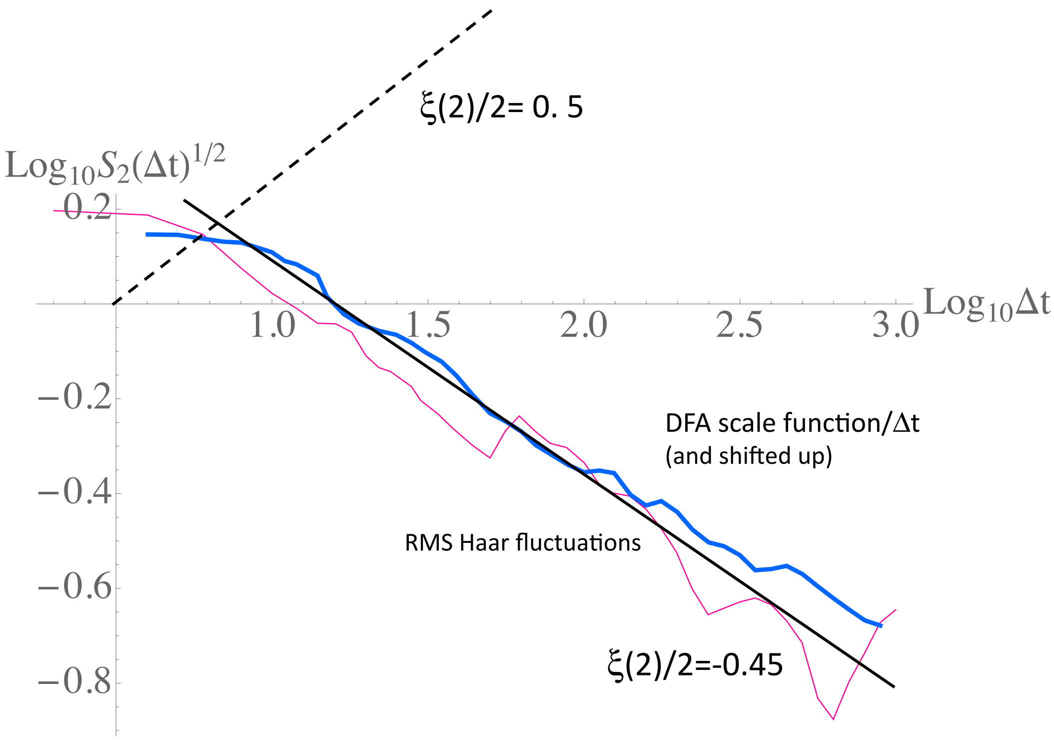

Fig. 2b: RMS Haar fluctuation analysis and the DFA scale function divided by Δt (so as to correspond to the series rather than its integral/ running sum) and shifted in the vertical by a factor ≈70. The solid reference slope corresponding to $\xi(2)$ = -0.90 is shown for reference as well as the dashed reference corresponding to $\xi(2)=1$ (the value from fig. 3d in R+R).

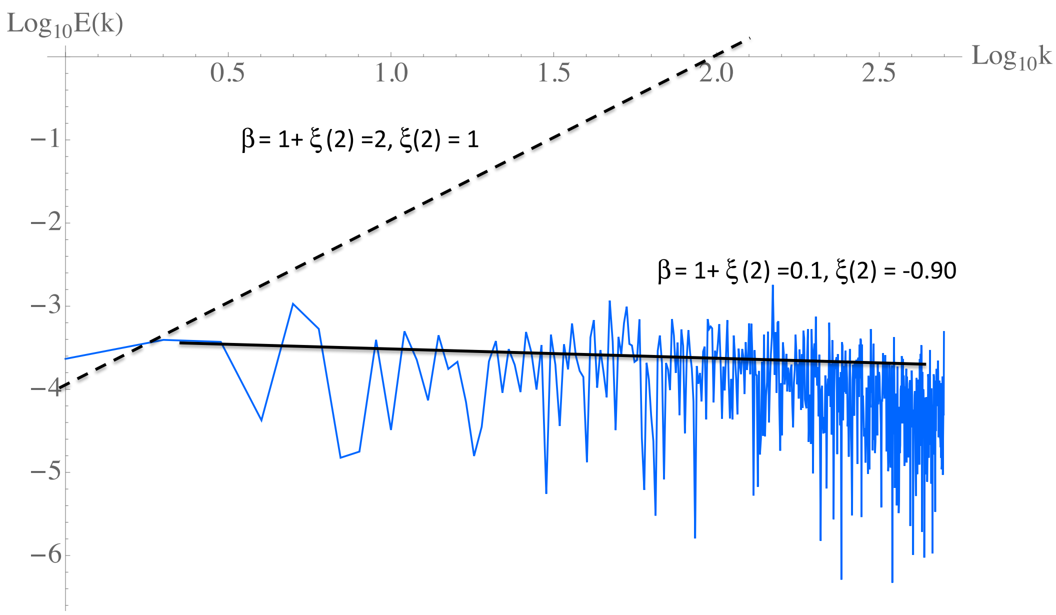

Fig. 3: The actual volcanic forcing spectrum (blue) compared to the slope (solid reference line) inferred from the Haar and DFA analysis (fig. 2b) as well as the inferred slope from R+R's structure function, their fig. 3d (dashed).

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