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Interactive comment on "Global warming projections derived from an observation-based minimal model" *by* K. Rypdal

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1   Introduction

The linear response framework used in Rypdal(2015) offers an interesting avenue to produce simple, observation- based surface temperature models and projections. The use of power-law Green's functions (*transfer functions*) - which we call *Climate Response Functions* –(CRF), is particularly interesting since over a wide range of scales there exists a scaling symmetry respected by both the dynamics and boundary conditions, which justify its use. Power law CRF's can be used to relate responses to forcings from internal variability as well as externally forced variability including solar, volcanic and anthropogenic.

However, power-law CRF's need to be treated with care since divergence issues arise at either high or low frequencies depending not only the scaling exponent, but also on either the high or – the case relevant here – the low frequency properties of the forcings. In Rypdal(2015), the author uses a power law CRF to make global warming projections from an observation-based minimal model. The problem lies in using the chosen exponent since a step function – or other finite increase in the forcing that lasts for long enough – will result in a divergence of the temperature. Therefore, in Rypdal's treatment, if the CO_2 levels were to be maintained at current levels forever,

the temperature of the earth would diverge; this is analogous to the dreaded *runaway greenhouse effect*, or in this case a *runaway Green's function effect*. In this case, the author uses forcings of finite duration that lead to only finite changes, but the exponents used are nevertheless unphysical. While it is true that the power law can be truncated to yield only a the finite effect, any results will depend crucially on the truncation time scale: it is a physical, not a technical, mathematical issue. Although for a different range of exponents divergences also arise at high frequencies, this will be dealt with elsewhere, here we discuss the low frequency issues relevant to Rypdal(2015). To illustrate this, we will first introduce the scaling CRF, then we will discuss low-frequency convergence for different forcing types and finally we will discuss physical implications of this divergence.

2 A Scaling Climate Response Function

Green's functions provide a general method of solving inhomogeneous linear differential equations. Consider the equation:

$$\mathcal{L}(T(t)) = F(t) \tag{1}$$

where the function $T(t)$ is taken to be surface temperature for the purpose of this paper, with certain boundary conditions, $\mathcal{L}$ is a linear differential operator and $F(t)$ is the forcing. The Green's function $G(t)$ is the solution of:

$$\mathcal{L}(G(t)) = \delta(t) \tag{2}$$

where $\delta(t)$ is the Dirac function and the solution $G(t)$ is subject to the same boundary conditions as the original equation. Therefore the solution of the original equation is:

$$T(t) = \int_{-\infty}^{\infty} G(t - t')F(t')dt' \tag{3}$$

Notice that the initial conditions are important: the same linear operator with different initial conditions will lead to a different Green's functions. Due to the scaling symmetry respected by the dynamics, and boundary conditions, over a wide range of scales, we take the basic Green's function as a power law:

$$G(t) = g_{H'} t^{H'-1} \Theta(t) \tag{4}$$

where $g_{H'}$ is a convenient constant, H' is the scaling exponent and $\Theta(t)$ is the Heaviside function, which is 1 for a positive t and 0 otherwise; this is needed to ensure causality of the response. Notice that for convenience we took a slightly different notation in this commentary for the scaling exponent, with H' instead of β_T; the two are related by $\beta_T = 2H'$.

With this, we obtain:

$$T(t) = g_{H'} \int_{-\infty}^{t} (t - t')^{H'-1} F(t') dt' \tag{5}$$

This is the solution to the fractional order differential equation:

$$\frac{d^{H'}}{dt^{H'}} T(t) = g_{H'} \Gamma(H') F(t) \tag{6}$$

where $\Gamma(H')$ is the usual Gamma function.

3 Low-Frequency Divergence

Power laws are famous for both high and low frequencies divergences; in physics, two notable cases are infrared and ultraviolet "catastrophes". We will start by reviewing the criterion for low-frequency convergence.

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3.1 Deterministic Forcing of a Finite Duration

We consider a step forcing $F(t)$ which is finite in duration and magnitude such that it is zero before $t = 0$ and after $t = \tau$; we obtain :

$$T(t) \propto (t^{H'} - (t - \tau)^{H'}) \tag{7}$$

where $t \geq \tau \geq 0$. If we consider the very large times such that $t >> \tau$, we obtain:

$$T(t) \propto t^{H'-1} \tag{8}$$

It is easy to see what happens if we take $t \to \infty$:

$$T(t) = \begin{cases} 0 & H' < 1 \\ \infty & H' > 1 \end{cases} \tag{9}$$

Therefore, to obtain convergence under any finite forcing of finite duration, we only need $H' < 1$, or alternatively $\beta_T < 2$.

3.2 Deterministic Forcing of Infinite Duration

Now we consider a step forcing of infinite duration, i.e. we take $\tau \to \infty$ in equation 7, and we obtain :

$$T(t) \propto t^{H'} \tag{10}$$

The condition for convergence is now restrained to $H' < 0$, or $\beta_T < 0$. As pointed out by the author, the positive β_T exponent for the Greens function is synonymous with a divergence of the surface temperature even when the carbon dioxide concentration is stabilized, i.e. a runaway greenhouse effect.

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3.3 Stochastic Forcing

Now we consider for the forcing Gaussian white noise that is a square integrable function $\gamma(t)$ with mean zero such that:

$$F(t) = \gamma(t) \tag{11}$$

$$\langle\gamma(t)\rangle = 0 \tag{12}$$

$$\langle\gamma(t)\gamma(t')\rangle = \delta(t - t') \tag{13}$$

where "$\langle\rangle$" indicates an ensemble average. This is appropriate for forcing due to internal variability of the climate system. In this case, since $\langle F(t)\rangle = \langle\gamma(t)\rangle = 0$, the low frequency constraint on H' for finite $T(t)$ is $H' < 1$. However, to avoid divergences, the variance should remain finite such that we have:

$$\langle T(t)^2\rangle \propto \int_{-\infty}^{t} (t - t')^{2H'-2} dt' < \infty \tag{14}$$

which implies $H' < \frac{1}{2}$, or $\beta_T < 1$. We now introduce the more convenient scaling exponent H which is defined from the scaling of the first order moment such that $\beta_T = 1 + 2H - K(2)$, where $K(2)$ is the multifractal "intermittency" correction to the second order moment($K(q)$ is the moment scaling function). In this we can have a situation where the absolute mean fluctuation diverges (depending on the first moment exponent, H) while the second moment (depending on beta) on the contrary converges. This is relevant for strongly strongly intermittent processes such as volcanism for which $K(2) \approx 0.2 - 0.3$. On the other hand, it is not important for solar forcing which has low intermittency and thus $K(2) \approx 0$. A Gaussian process $T(t)$ with $-1 < H < 0$ and $K(2) = 0$, and thus $-1/2 < H' < 1/2$ or equivalently $-1 < \beta_T < 1$, is a *fractional Gaussian noise*, fGn. When $H > 0$, the process $\langle T(t)^2\rangle$ diverges because of the low frequencies (the stochastic runaway Green's function effect), however differences

$\Delta T(\Delta t) = T(t) - T(t - \Delta t)$ have convergent variances: $\langle \Delta T(\Delta t)^2 \rangle \propto \Delta t^{2H}$, this is *fractional Brownian motion*, fBm. We note that when $H < 0$, $\langle T(t)^2 \rangle$ diverges at the high frequencies. However, if we truncate the Green's function at small scale τ, or if we use the smoothed $T_\tau(t)$, that is averaged over a scale τ, then $\langle T_\tau(t)^2 \rangle \approx \tau^{2H}$ $(-1 < H < 0)$, which is finite.

Now consider a fractional noise of order H_f :

$$\gamma_{H_f}(t) = I_{H_f}\gamma(t) \tag{15}$$

where I_{H_f} denotes a fractional integration of order H_f and $\gamma(t)$ is again a Gaussian white noise. The power-law CRF amounts to performing a second fractional integration and since $I_{H_1}(I_{H_2}) = I_{H_1+H_2}$, the criterion for finite variance changes because now:

$$\langle T(t)^2 \rangle \propto \int_{-\infty}^{t} (t - t')^{2(H'+H_f')-2} dt' < \infty \tag{16}$$

and the criterion for convergence thus becomes $H' < \frac{1}{2} - H_f'$ or, equivalently, $\beta_T < 1 - \beta_f$ where $\beta_f = 2H_f'$. This is relevant since if we take volcanic forcing, for example, which was found to have a scaling exponent $H_f \approx -0.2$ with $K(2) \approx 0.2$, yielding $\beta_f \approx 0.4$ and $H_f' \approx 0.2$, in Lovejoy et al.(2013), then its associated response under a power-law CRF will converge only if $\beta_T < 0.6$. In Rypdal(2015), the exponents considered as upper and lower bounds are $\beta_T = 0.35$ and $\beta_T = 0.75$, adding it is likely to be closer to the higher value; this means that that under this treatment volcanic forcing would be enough to make the variance diverge for the higher β_T value proposed. In contrast, using the first order moment criterion, we find the more restrictive convergence criterion: $H + H_f < 0$ so that $H < 0.2$. A more complete review of these issues is found in Lovejoy et al.(2015) and Lovejoy et al.(2013).

3.4 Summary of Criteria for Low-Frequency Convergence

- For a deterministic forcing function over a finite interval : $H' < 1$, $H < \frac{1}{2}$ or $\beta_T < 2$

- For a deterministic forcing function over an infinite interval : $H' < 0$, $H < -\frac{1}{2}$ or $\beta_T < 0$

- For a square integrable white noise forcing function with zero mean over an infinite interval : $H' < \frac{1}{2}$, $H < 0$ or $\beta_T < 1$

- For a square integrable fractional noise forcing function or order H_f with zero mean over an infinite interval : $H' < \frac{1}{2} - H'_f$, $H < -H_f$ or $\beta_T < 1 - 2\beta_f$

When intermittency is important, then the first and second order criteria (i.e. using H and β_T respectively) are no longer equivalent.

4 The Runaway Green's Function Effect

Therefore, the condition for convergence depends on the type of forcing considered and the scaling exponent. In the paper, the author argues for a positive β_T for a deterministic forcing based on a previous estimation. The scenarios shown all started with a business as usual scenario, i.e. a projection of the recent exponential trend in emissions, followed by a slow or rapid decline in emissions, 1% per year or 5% per year respectively, starting in 2030, 2070 or 2110. The *Representative Concentration Pathways*(RCP) scenarios involve a stabilization of the forcing at a given level, which, under this model, eventually lead to a divergence of surface temperature; this is the runaway greenhouse effect, or in this case, the *runaway Green's function effect*. In their extensive review on the matter, Goldblatt and Watson find that, although it cannot be completely ruled out, it is unlikely to be possible that such a phenomenon could be induced by addition of carbon dioxide to the atmosphere. It would be useful that the author puts his model into this contexts and specify the limits of this model upon stabilization of carbon dioxide concentration; otherwise, his result implies that we must

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eventually return to pre-industrial carbon dioxide concentration to prevent the earth from eventually becoming another Venus.

In a previous publication(Rypdal and Rypal 2014), the author argued that non-linear effects could produce a low-frequency truncation in the power law climate response function model which would prevent such divergence, but ruled out any truncation τ_L less than 100 years from comparison with millennial reconstructions, thus arguing the question is quasi-irrelevant for projections up to 2100. Here the projections are made up to 2200, are we to assume now that the truncation is likely to be larger than 200 years? Following a step-wise increase in carbon dioxide radiative forcing, the equilibrium temperature response under this power law climate response function will be proportional to τ_L^H; the resulting equilibrium change in temperature depends entirely on the choice of outer cutoff. If the cutoff is shown to be larger than the duration of the forcing, its exact value will not be affect the final result since the finite length of the forcing will act as an outer cutoff. If we take the year 1850 as the beginning of significant carbon dioxide forcing, to make projections to 2100 and 2200, τ_L needs to be at least 250 and 350 years respectively to be neglected.

5 Conclusion

We discussed the different criteria for convergence of responses to power-law CRF based on the scaling exponent, and the type of forcing. A Green's function scaling exponent $\beta_T < 1$ is appropriate for white noise, but not for sustained anthropogenic radiative forcing, the model presented by the author thus leads to divergence. Although the model is simple, it is important to make sure it is physically sound and well defined. In the context of the transfer function from emissions to concentration, it could be argued that sustained high emissions beyond a removal threshold would lead to a diverging concentration, but nothing indicates that it is the case for the temperature

response to concentration. A physically based low-frequency cutoff should be formally introduced and justified in the model if a positive exponent is to be used for the temperature response to deterministic forcings. We agree that detrended temperature series have properties of not too far from fGn, although to some limit of the order of centuries, but there is no reason that the same scaling exponent should be used to produce the fGn from Gaussian white noise forcing and the long-term projections from deterministic forcing, especially since the properties of the response are so dependent on the properties of the forcing. Finally, we pointed out that when intermittency is important - as in the case of volcanic forcing - that first and second order convergence criterion are not the same, and that the first order criterion should be used since it is more stringent.

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