Referee Report on Heitzig & Kittel:
'Topology of sustainable management in dynamical Earth system models with desirable states'

The ms 'Topology of sustainable management in dynamical Earth system models with desirable states' by Heitzig & Kittel suggests that Earth system analysis & management should be based on a lexicographic preference order, characterized by 'sunny' and 'dark' system states. The ms lays out possible topologies of dynamical systems, given this binary relation. They show that already this binary relation induces a rather complex set of phenomena when asking how the sunny region or even its invariant kernel ('shelter') could be reached. The authors put their rather abstract content properly into the context of ESD by applying it to archetypical systems that are crucial for Earth system analysis, such as the carbon cycle, competing species, combined population & resource dynamics, and mitigation. They argue that the topology were of primary importance rather than an economically defined welfare optimum.

To my taste, the content delivered by the authors clearly deserves publication in ESD. Topologies as defined in this ms are only rarely discussed by the Earth system community. An exception are bifurcations or tipping points in general that can be regarded a special case of the richer set of phenomena outlined in the present ms.

However the ms should not be published in its present form. Firstly, the introductory statements on economics could be misunderstood as economic theory being systemically ad odds with the insights presented. While this may hold for standard economics being based on expected utility (maximization) theory, in particular in the climate context a major fraction of the community has produced the majority of results in a lexicographic manner (see e.g. IPCC AR5 WGIII Ch6). IPCC AR5 WGIII Ch2 in turn points to the price one has to pay when deviating from expected utility maximization. While lexicographic preferences have a certain appeal for system-theory trained researchers, they deliver very unattractive features when being applied in the context of dynamic decision-making. Hence the authors should at least consider the following 4-phase type of analysis: (i) topological analysis as presented, (ii) putting up a welfare measure that is sensitive to this rich topology, (iii) optimization, (iv) clarifying to what extent this could still be interpreted as a market equilibrium, and if not, inclusion of suitable policy instruments. In particular the authors seem to be ignorant of the work on economics of tipping points as being performed in the groups of Klaus Keller (Penn State U), Tim Lenton (U Exeter), Martin Quaas (U Kiel).

Secondly, the ms delivers a great service to the community in making rather abstract content accessible to a wider audience. However it is not always so clear (i) what is new in terms of content / theorems, (ii) what is merely a didactic service, a transfer of knowledge from one community to another. In particular for me it was not so clear in what it differs from previous work by Aubin and his group, or Schellnhuber.

Thirdly, the ms while in general tackling a rather abstract subject (compared to other papers in ESD) very professionally, at some instances either the mathematical treatment or its illustration do not appear as fully 'baked through'. The ms might have been submitted a bit hastily (now summarizing remarks along the ms):

1. The intro should also cite the tolerable windows approach (Petschel – Held et al., Climatic Change, 1999; Bruckner & Zickfeld, 2008).
2. 1.1 tends to introduce terms on a non-technical level first. However I find this confusing. Keep to the standard scheme: definition first (that should be illustrated, indeed), then usage of that term.
3. Figures should be added to 1.1 and following subsections.
4. I liked the condensing § at the top of p440.
5. P440: Has 'safe' been defined before usage?
6. P442: We need a figure on C. The whole technical concept at the center of that page should be shifted to the appendix, and the essence should be illustrated.
7. While Fig1 is extremely helpful in general, I found a couple of aspects confusing. Hereby I assumed there was a water flow from the left to the right:
    a. What is the difference between solid and dashed thick lines? Can we have a fixed point for the latter?
    b. Why is 'sunny downstream' downstream? How can we return along a thick line to the manageable region?
    c. Why is a thick line before 'a' a subset of the shelter? If the flow is always downstream how can the boat stay in front of 'a' w/o management? Is there a fixed point?
    d. Are the colors at the boat's arrows messed up?
    e. Why are the glades as illustrated here not a subset of the shelter?
8. Figure 3 is cited before Figure 2.
9. I think the logic in the § before Eq6 is messed up. Should it not read somehow like: 'Now we turn to the region from where one cannot avoid ending up in the trenches. We define the abysses Y (uppercase greek letter Upsilon) as the closure of this region, in order to robustify it against infinitesimal perturbations. …' Then , of course, minus trenches. My point is: 'if one has to fear' comes too early.
10. Eq7: Clearly indicate that you are moving beyond Fig1.
11. Example 2: clearly say whether a(t) is management or system.
12. Motivate the structure of Ex3 better.
13. § below Eq16: Why is the last term in U^- a (+)? Why do we always end up in (+) w/o management?
14. Ex4: Where are anthropogenic emissions? Also the last § should be expanded in view of the audience of ESD.
15. Last sentence before Ex6: wording: an algorithm is not from a theory.

Overall a very inspiring ms!

For reasons of the realities of academic life, I could not check all of the topological proofs or system-analytic aspects of the examples, although accessible in principle, when being given enough time. In my view the Editor must make sure that both aspects will

have been checked line-by-line by 2 experts, one with an everyday-practice in topology, one in the analysis of dynamical systems, before final decisions are taken.